Introduction to Market Structures
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Module 7: Cournot Competition
Lecture 25: Cournot Duopoly

Hello everyone, welcome to my course, Introduction to Market Structures. We have completed the game theory portion. Now, we are going to apply that game theory tools to study the market behavior and how the firms decide, okay.

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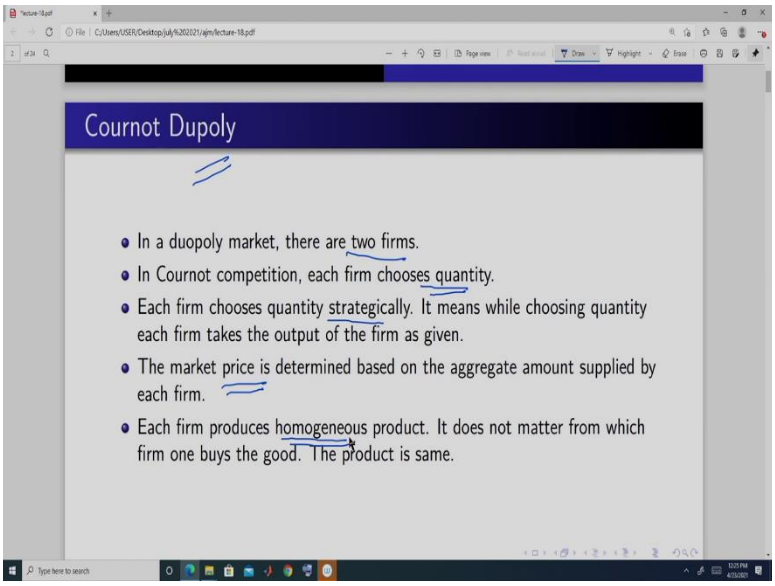

So first model that we are going to do is Cournot duopoly. In Cournot duopoly, there are two firms, okay. And in Cournot competition, each firm chooses quantity, okay. So they will decide how much amount of output they want to produce or they want to sell in the market, okay.

And now while deciding these outputs, they are going to decide it strategically. What do we mean by strategically? That means when firm 1 decides its output, it will take the output of firm 2 as given, that some amount of output is going to be produced by firm 2. Similarly, when firm 2 decides its output that is q2 it will take the output of firm 1 as given some amount. So in this way, they behave strategically.

But in the previous models of markets that we have done in a competitive market, they take the market price as given. So how other firms are behaving it does not matter in that model or in monopoly there is only one firm, so it does not matter how other firms are because there does not exist any other firm but here there are two firms and so while deciding output, they will take the output of other firm as given. So this information is known, okay.

Now again based on this price output the aggregate we will get the aggregate output and that aggregate output is going to determine the market price, okay. And here we make one more assumption and that assumption is that each firm produces homogenous product. It means that whether a consumer buys from a firm 1 or from firm 2, it does not matter. They are going to **get** the same product or same good, okay.

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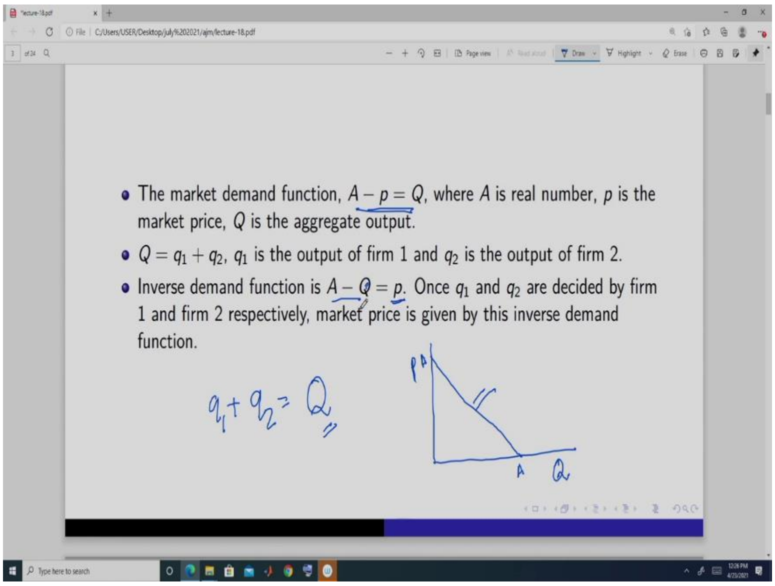

And first we will assume that the market demand is this-$A - p = Q$. So it is a downward sloping demand curve. So if we take output, aggregate output here, price here then this is something like this where this is A and this point is A. So this is our market demand curve, okay. And this Q is the aggregate output. So it is sum of output of firm 1 that is q1 and output of firm 2 that is q2 that gives us the aggregate output that is capital output**,** Q, i.e $Q = q_1 + q_2$, okay.

Now here once this q1 and q2 are decided, q1 by firm 1 and q2 by firm 2 then we plug in this we get this - $A - Q = p$ and from this inverse demand curve, we get the market price. So market price is determined based on the aggregate market demand curve, okay. So each firm decide the output it is going to sell and then that determines the aggregate output and that aggregate output determines the market price. So the market price is not decided by the firm. Market price is an outcome of the output that is chosen by each firm, okay.

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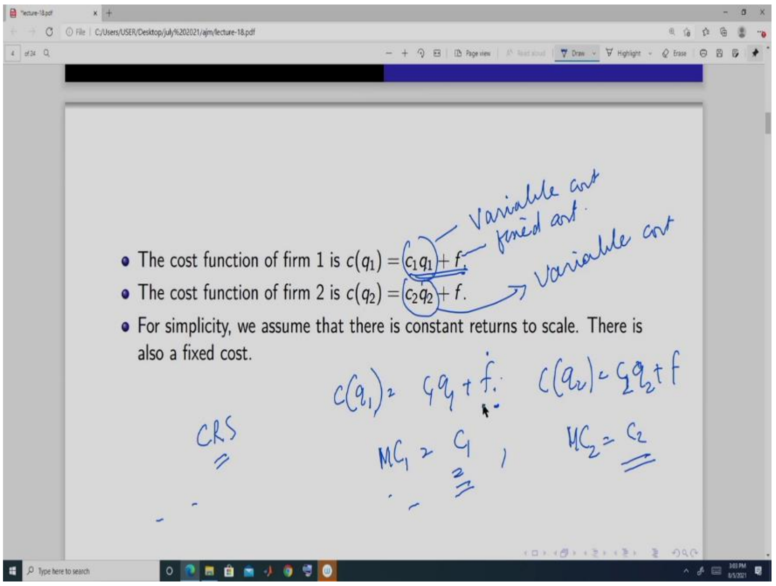

Now we specify the cost function because there are firms and to produce output they will require input and so they have a cost and we have done this already. So we assume that the cost function is of this form for firm 1- $c(q_1) = c_1q_1 + f$ so this is c1 q1 plus a. q1 is the output so this portion-c_1q_1 is the variable cost and this- f is the fixed cost.

And same here this portion- c_2q_2 is the variable cost and this- f is the fixed cost. And we assume that the variable costs are not same for each firm. Why, because they may have some differences in technology or they may say different wages or many reasons, okay or they may be using a different combination, so that is why, okay.

So here if we take this cost function of firm 1 in this then the marginal cost of firm 1 is c1. And marginal cost of firm 2 we will get in this form-c2. So it is constant, so we are assuming that there is CRS production function, okay. And this fixed cost may arise from some factor which is fixed like you can take **a** rent that we pay for the land.

So we are mainly varying machines and labor but land is supposed fixed. We do not vary. We suppose this land size is sufficiently big and we can keep on expanding our capacity so that in that case we can, the rent that we pay it can be a form of fixed cost. Another fixed cost can be like the license fee that we pay to set up a firm so those kind of things will constitute this fixed cost.

Now here we are making I should have specified. Here we are making that firm 1 knows the cost function, okay. We will do this later, not now. So we have got this specification of cost of each firm, okay.

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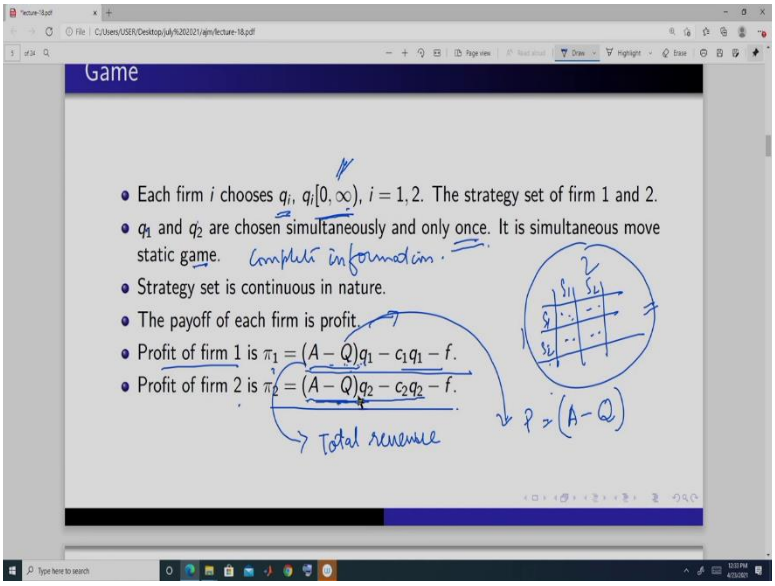

Now let us talk about the game. Now here what is happening, each firm choose qi firm i, i 1 and 2 so that is 2 firm, firm 1 and firm 2. And they can choose q from this range from 0 to infinity, okay. Infinity is not included, okay. So this you can say is the strategy set, okay of firm 1 and firm 2. And the strategies are to choose q1 of firm 1 and q2 of firm 2. They simultaneously choose this and they choose it only once. So you can think of this as a simultaneous move static game, okay.

So both these firms are taking the decision to produce output simultaneously at the same point in time or at the same time and they are choosing it only once, okay. Right? now here we have so this is a part of complete information. Now this is a simultaneous move static. Now this is also a complete information game, complete information. Complete information in the sense that firm 1 knows the payoff function of firm 2. So this- $\pi_2 = (A - Q)q_2 - c_2 q_2 - f$ is known to firm 1 and firm 2 also knows this- $\pi_1 = (A - Q)q_1 - c_1 q_1 - f$. What are these, I will come to it, okay.

Now here we have done the normal form game in the game theory portion. There we have assumed like this kind of games, right? So this is player 1, this is player 2, their strategy 1, strategy 2. So they have 2 strategies or two actions and these are the payoffs. We have not specified here. So this is kind of normal form or strategic form game that we have done.

Here this strategy space or the strategy set is a continuous thing. It lies here, okay. Instead of this discrete action or discrete strategies, we have now continuous strategies, okay. This is the difference from the thing that we have done earlier and the payoffs of each firm we have to specify so we have specified the strategies or the actions that strategies set.

Now we have to specify the pay-off so payoff is a profit, okay and for firm 1 it is this-$\pi_1 = (A - Q)q_1 - c_1q_1 - f$. This portion-$(A - Q)$ is what, this portion is price because price is equal to A minus aggregate output so this is price then this price into q1 so this is a total revenue. This portion is total revenue-$(A - Q)q_1$. This the total cost- $c_1q_1 + f$, we have specified the cost so total revenue minus total cost it gives me the profit. So this is the profit of firm 1.

Now this is the price because market price is going to be the same for each firm. So this is the price into output of firm 2 q2 minus this total cost so this is the profit of firm 2. So firm 1 maximizes this-$\pi_1 = (A - Q)q_1 - c_1q_1 - f$, firm 2 maximizes this-$\pi_2 = (A - Q)q_2 - c_2q_2 - f$, okay.

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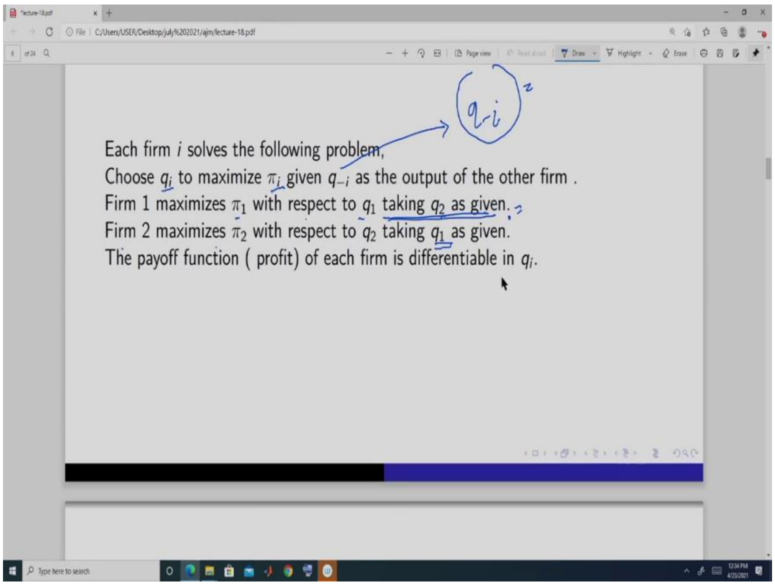

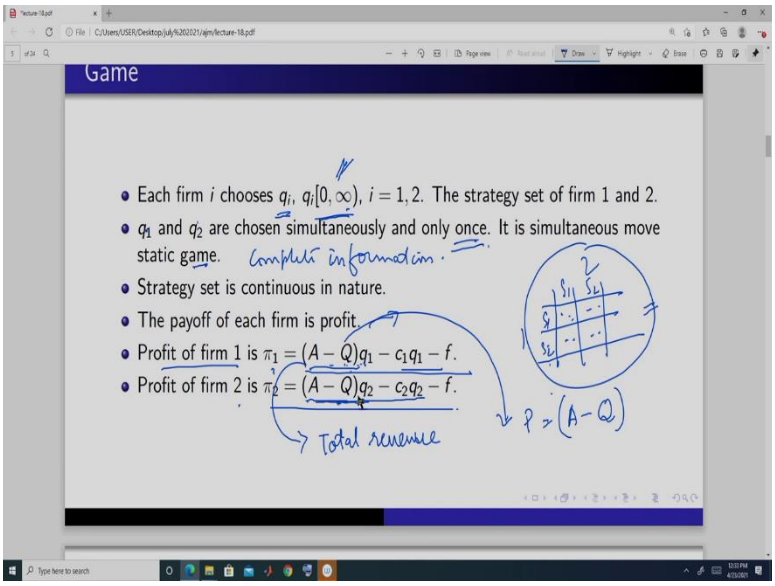

Now what is firms solve? They solve this problem. So firm i will choose qi to maximize this profit assuming that the there is a some given level of output of the other firm and this we represent it in this way- q_i. This is the output of other firm, okay.

So firm 1 maximizes profit pi one, i.e π_1 with respect to q1, taking q2 as given firm 2 so this portion is bringing in the strategic aspect, okay. And firm 2 maximizes profit that is pi 2 with respect to q2 taking q1 as given, okay and here if you look at this profit function it is obvious that they are differentiable, okay.

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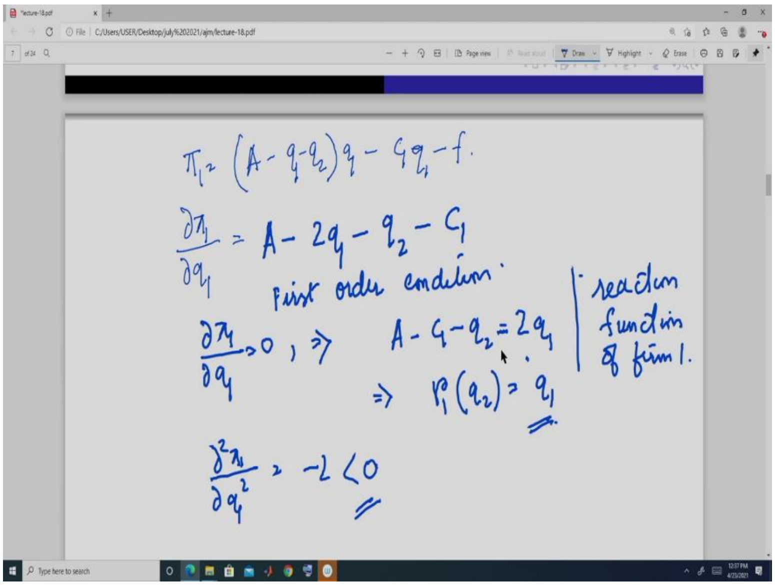

So now we solve this problem. So profit of firm 1 is this- $\pi_1 = (A - Q)q_1 - c_1q_1 - f$ so we maximize this, so this will give me if we differentiate with respect to take the partial with respect to q1 this will be this- $\frac{d\pi_1}{dq_1} = A - 2q_1 - q_2 - c_1$ now simply first order condition of maximization this gives me this- $\frac{d\pi_1}{dq_1} = 0 => A - c_1 - q_2 = 2q_1$ and this we called as reaction function of firm 1 or we can write this as it is a function of q2 in this way-$p_1(q_2) = q_1$.

So it means whatever if firm 1 takes some level of output of firm 2 as given then what is the optimal output it should produce, it is given by this function- $A - c_1 - q_2 = 2q_1$. So that is why it is called the reaction function. What it is? That if firm 1 believes that firm 2 is going to produce q2 amount then what is the optimal for firm 1 it is given by this firm. This function so this is the reaction function.

And here if you take the second derivative of this you get the second order condition of this, and it is minus 2 which is always negative so that is why this a is always, so this is always going to be the, is going to maximize the profit, okay. Second order condition is negative at this level. So we have got the reaction function of firm 1.

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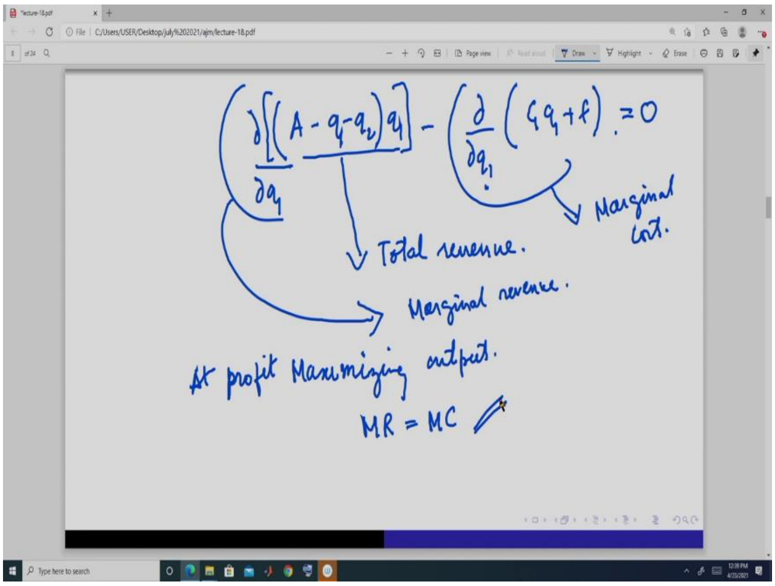

If you look at this function carefully what do we get? So this is- $\frac{\delta(A-q_1-q_2)}{\delta q_1} - \frac{\delta(c_1 q_1+f)}{\delta q_1}$, right? So this is what? This portion is total revenue, so this taking this is the marginal revenue- $\frac{\delta(A-q_1-q_2)}{\delta q_1}$. This is the total cost, so this is marginal cost-$\frac{\delta(c_1 q_1+f)}{\delta q_1}$. So at the optimal point or at profit maximizing output, we get marginal revenue should be equal to marginal cost because this is equal to 0 at the optimal point right and this condition is same as what we get in the monopoly thing.

So in the monopoly also the optimal output is decided where the marginal revenue is marginal cost so this optimization is similar to what we have done in the monopoly, only difference is here is, that we have considered some given level of output of firm 2, okay this portion- $A - 2q_1 - q_2 - c_1$.

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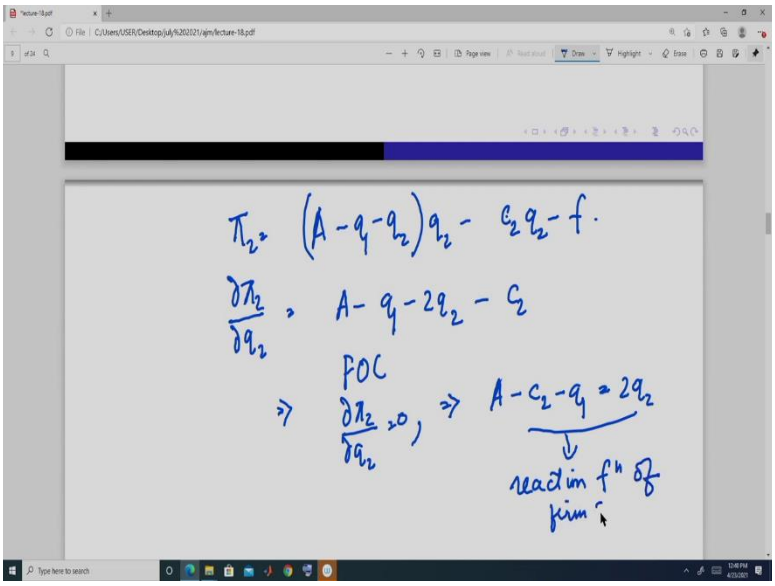

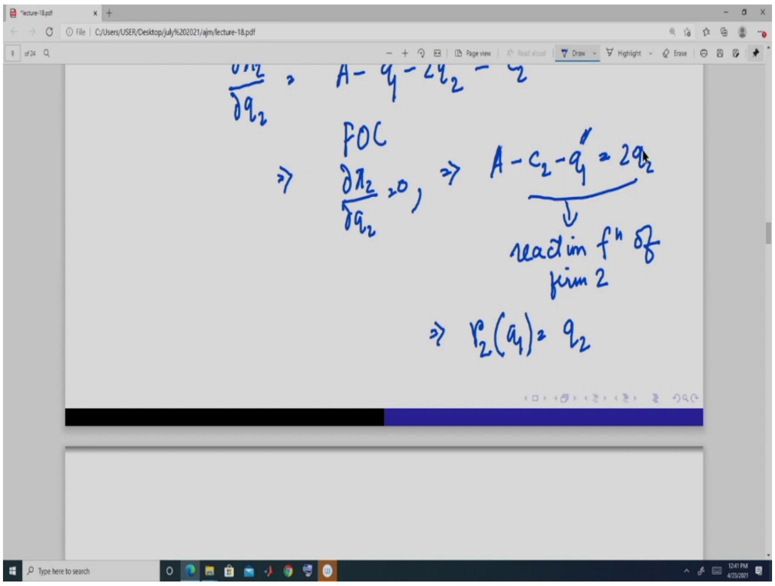

Similarly, the profit of firm 2 is this, this is again first order condition implies-$\frac{d\pi_2}{dq_2} = A - q_1 - 2q_2 - c_2$, so this implies so this is the reaction function of firm 2-$A - c_2 - q_1 = 2q_2$, which we write as function of output of firm 1. So if firm 2 believes that the output of firm 1 is this much then what is the optimal output it should produce, it should produce based on this function and this much, okay. It is given by this.

So now we know the reaction function, okay. So we know that if firm 1 believes output of firm 2 is this much then how much it is going to produce. And similarly if firm 2 believes that our firm 1 is going to produce this much then how much output it should produce.

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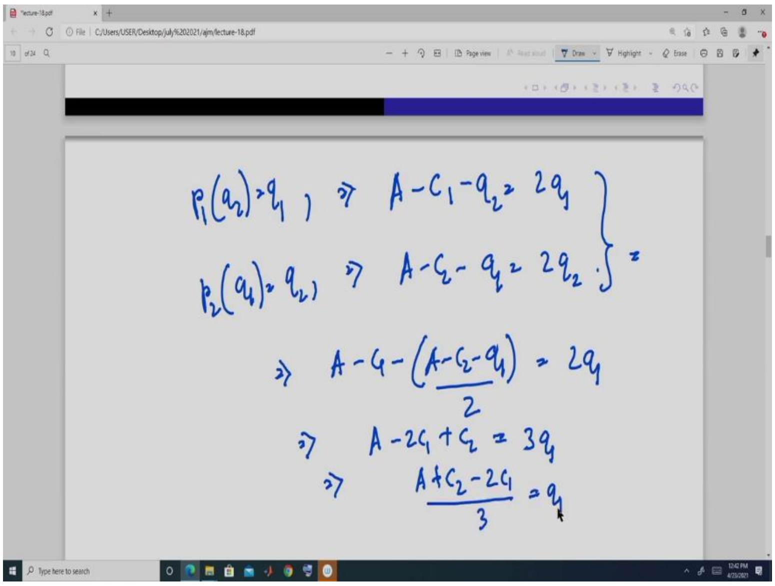

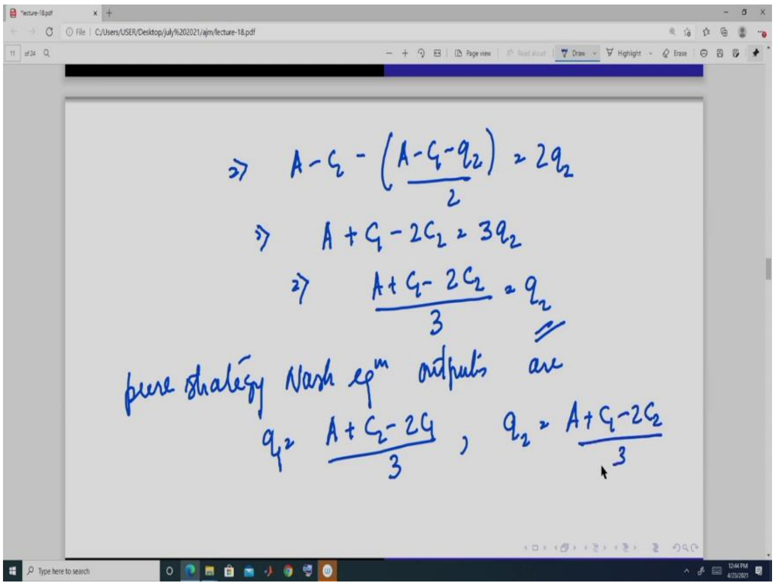

So based on these 2 reaction function so that is this- $p_1(q_2) = q_1$, we get the, this one- $A - c_1 - q_2 = 2q_1$, this- $p_2(q_1) = q_2 => A - c_2 - q_1 = 2q_2$ Now we solve these two equations. These two are linear equation and we can solve them. This is the output of firm 1- $\frac{A+c_2-2c_1}{3} =$

q_1 and for output of firm 2, we can write simply this- $\frac{A+c_1-2c_2}{3} = q_2$, so these two points is going to solve this.

So that means when firm 1 believes that the output of firm 2 is this-$\frac{A+c_1-2c_2}{3} = q_2$, then its optimal output is this- $\frac{A+c_2-2c_1}{3} = q_1$and simultaneously, firm 2 believes the output of firm 1 is this much- $\frac{A+c_2-2c_1}{3} = q_1$ so it produces this-$\frac{A+c_1-2c_2}{3} = q_2$. So that is why the Nash equilibrium here, the Nash equilibrium or you can say pure strategy Nash equilibrium outputs, are q1 is equal to this- $\frac{A+c_2-2c_1}{3}$.

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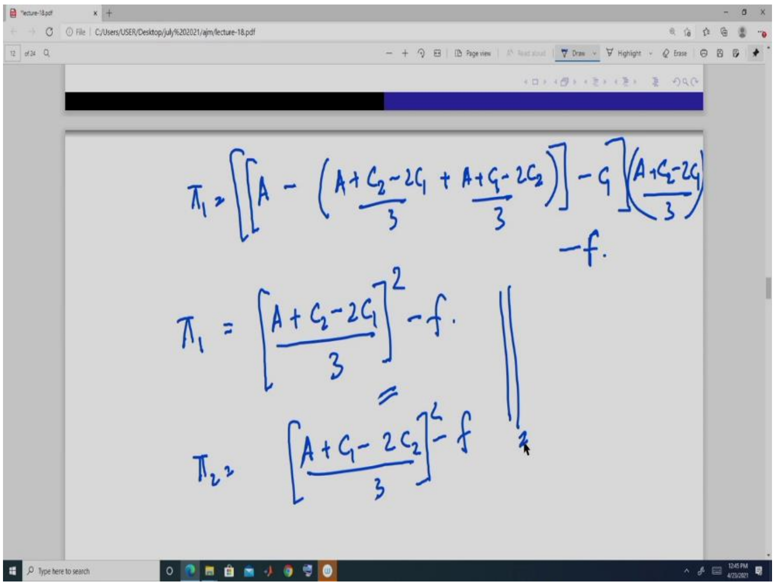

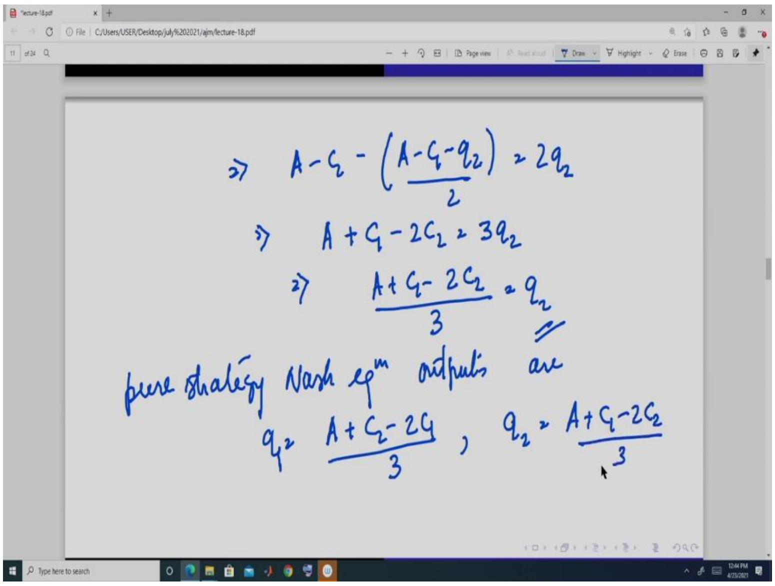

And when we plug these outputs in the profit function we get the Nash equilibrium profit. So while after solving this we get it. So this is the profit of firm 1-$\left[\frac{A+c_2-2c_1}{3}\right]^2 - \mathrm{f} = \pi$ and similarly profit of firm 2 if you solved it, is this-$\pi_2 = \left[\frac{A+c_1-2c_2}{3}\right]^2 - \mathrm{f}$. So this is a Nash equilibrium outcome, okay. Now here we have got this outcome.

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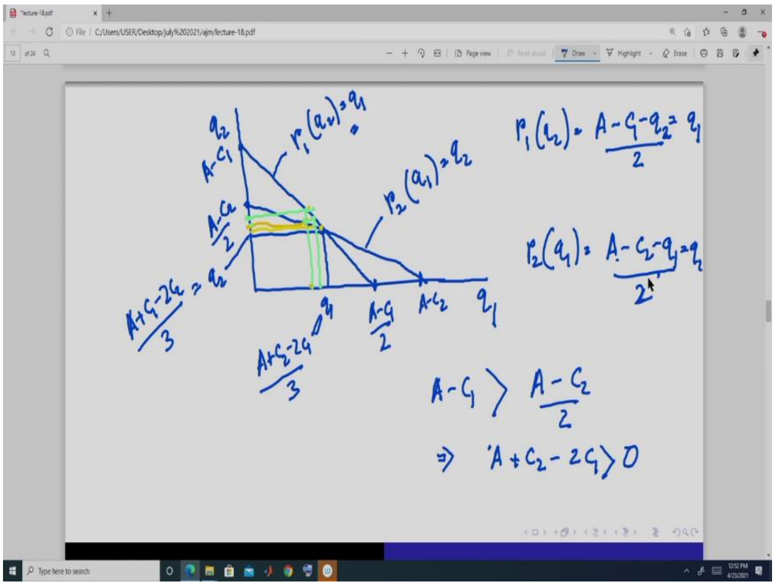

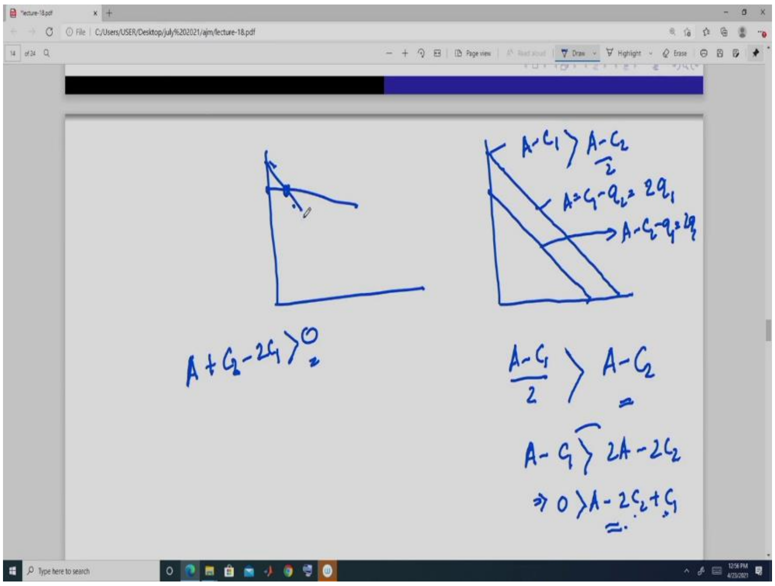

Now actually let us solve this through diagram to understand it better way. So we will use the reaction function. So reaction function of firm 1 is this-$p_1(q_2) = A - c_1 - q_2 = 2q_1$ so when output is 0 this is 0 then q2 is here it is this, right. And when q2 is 0 q1 is here this is and this is a straight line. It is this. So this is the reaction function of firm 1, okay or you take it like this, okay.

Now here reaction function of firm 2 is this-$p_2(q_1) = \frac{A - c_2 - q_1}{2} = q_2$ so if q1 is 0 it is q1 **is** 0, it is A minus c2 is equal to, divided by 2 is equal to q2. So suppose it is this. And when q2 is equal to 0 it is A minus c2 is equal to q suppose it is here, A minus C2, okay.

And this is a reaction function of firm 2. This is the point of intersection that means at this point these two equations is the, this is the solution of these two equations. So this q1 is A plus c2 minus c1 divided by 3. This q2 is equal to A plus c1 minus 2 c2 divided by 3. We have found that.

Now here this is the point where two reaction functions are intersecting and that point gives us the pure strategy Nash equilibrium. Why? Because see suppose firm 1 this is the reaction function of firm 1. If firm 1 thinks that the output of firm 2 is this, then it is going to produce, suppose firm 1 thinks the output of firm 2 is this, then the optimal output it should produce is this much from the reaction function, right?

But firm, here when firm 2 thinks that the output of firm 1 is this much, it should produce this much amount. It should produce this much amount, not this. So there is a mismatch in the belief. So that is why it is not a Nash equilibrium. Then what will happen if it thinks in this way, then firm 2, since firm 2 has thought that output is this, so it has produced this much.

Now firm 1, it will come to know that output of firm 2 is this much, so it will assume that output is this, so it will produce this so its output is going to be this much. When firm 2 thinks that the output of firm 1 is this, it is going to produce this much. So there you will see that they will slowly come to this point and this is the point Nash equilibrium point.

Now here in this situation we get this solution this under what assumption? So we have to, we have not yet specified that assumption but we are, we have implicitly assumed it. So that assumptions are that this A minus c1 should be greater than a minus c2 divided by 2, i.e $A - c_1 > \frac{A-c_2}{2}$. So this should be greater so then we will get so this implies that A minus A should be positive- $A + c_2 - 2c_1 > 0$ or so if we have only this situation so that means this is going to be lower than this.

Then since they are downward sloping from this a equation we know these two equations. So that is why what we are going to get we are going to get that actually it will be since we are assuming it is like this and it is like this, this point so it will be there. So we have a point of intersection, right? But we may can we have a situation like this. This is A is this is greater than a minus, right? like this can we have a situation like this?

So there is no intersection point so there is no solution. Can we have this situation because from here we have got this right? if we have this situation then only we will get, right. We have assumed this but then we can have this situation, if we have this situation that means what from here what do we get a minus c1 divided by 2 is actually greater than a minus 2, i.e. $\frac{A-c_1}{2} > A - c_2$, okay.

So this we can have, this will be, this is assuming what so this will imply what. This will give us this-$0 > A - 2c_2 + c_1$. Now we have assumed this- $A - c_1 > \frac{A-c_2}{2}$so we have got this- $A + c_2 - 2c_1 > 0$. Now we may have this situation so if we have this situation if this is true, i.e $A + c_2 - 2c_1 > 0$ then it does not imply that this- $0 > A - 2c_2 + c_1$is not true. This can be true, okay. If c2 is sufficiently small, right?

If c2 is sufficiently big, okay or c1 is sufficiently small, okay we can have these two situations and, in that case, we will have no intersection point in the positive orthant. So, we will require this. So, this may not happen. So, we require this condition also that 1 minus c2 is greater than, i.e $A - c_2 > \frac{A-c_1}{2}$, so this implies a minus 2 c2 plus c1 is positive- $A - 2c_2 + c_1 > 0$.

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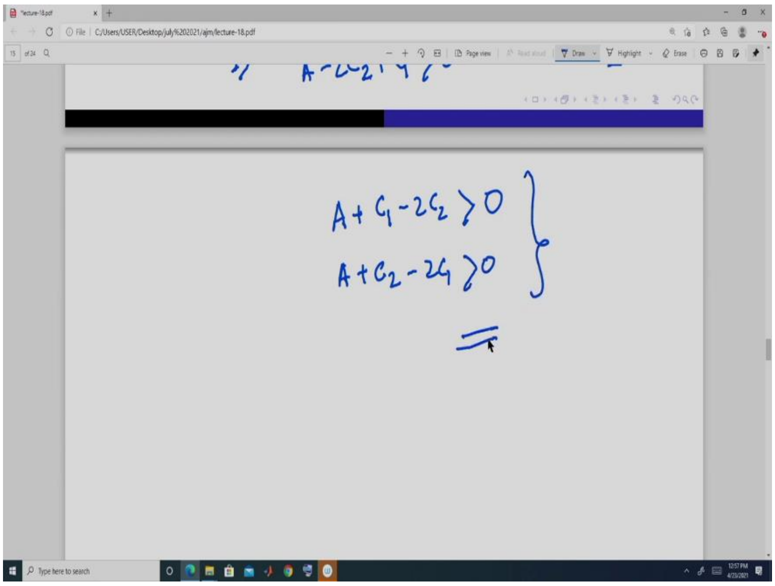

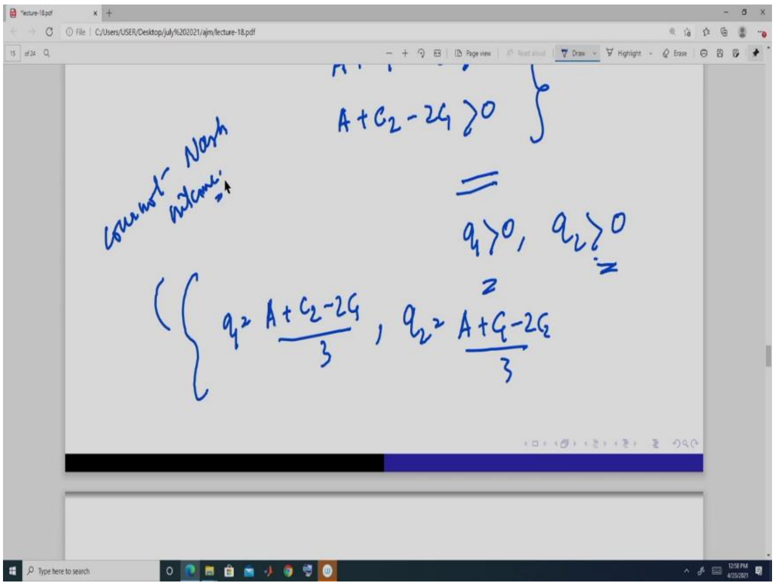

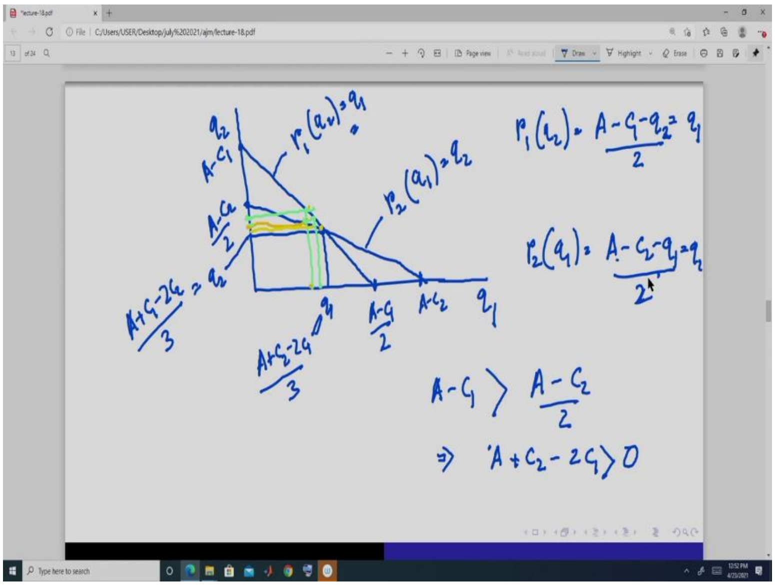

So for a solution, to exist in this case, we require these two conditions- $A + c_1 - 2c_2 > 0, A + c_2 - 2c_1 > 0$. Then only we will have a pure strategy Nash equilibrium and the situation will be such that it will be positive that is, these two conditions will ensure that Q1 is positive and Q2 is positive. If we do not have this situation, then we will not have a pure strategy Nash equilibrium where output of each firm is positive.

And this is also this outcome that is q1 is equal to $\frac{A+c_2-2c_1}{3}$ and q2 is- $\frac{A+c_1-2c_2}{3}$, this is also called Cournot Nash outcome, Why? because when Cournot proposed this model, at that time game theory was not developed. So Nash later on developed this non-cooperative game theory. And then combining these two we get this as the outcome, okay.

Now here while we are drawing this, you see at this point only believes are also match. That is when I assume that the firm 2 is going to behave in this way and in response I behave choose this, similarly firm 2 thinks my output is this and then chooses this as output. So if that is not there then we will not be in a Nash equilibrium.

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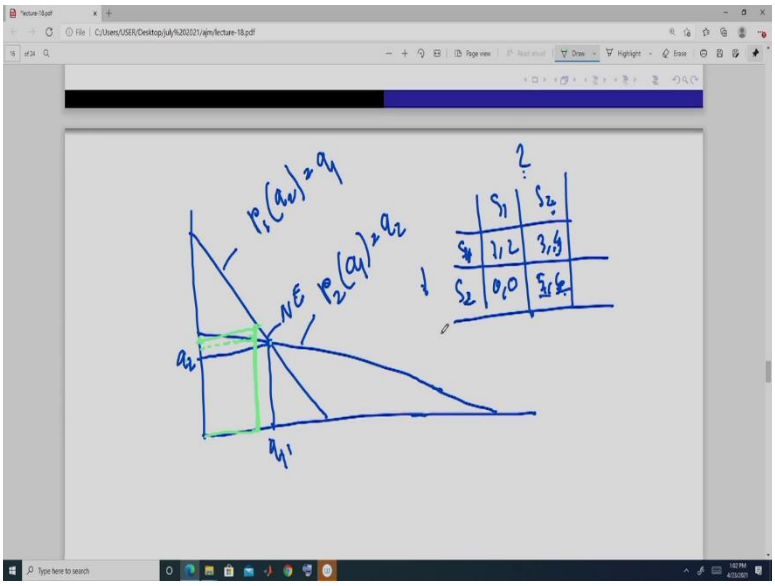

So that it is something like this that when we are, suppose consider this game how we have found the Nash equilibrium, okay. If player 1 thinks – if player 1 is choosing this, okay player 1 is choosing this then so player 2 thinks that suppose player 1 is playing this what is going to be my best is? My best is to choose this, okay.

So here you can say when firm 2 is choosing its output it should think that what is the firm 1 is going to produce, okay. And then based on that it is going to choose the optimal output so if firm 2 thinks that firm 1 is going to choose this, its best response is this, okay. And then when firm 2 is choosing this best response is to choose this. So that is why they are deviating.

It is firm 2 assume that firm 1 is producing this and then it has decided to produce this, choose this strategy. Then when firm 1 is assuming that this is the strategy of firm 2 then it is not choosing this but it is this. So that is why this (3,5) is not a Nash equilibrium so here firm 1 is choosing S2 when firm 1 is choosing S2 this firm 2 is going to choose S2, so that is why this (5,6) is a Nash equilibrium, pure strategy Nash equilibrium.

Same thing is happening here. If this is the reaction function of firm 1 and this is the reaction function of firm 2. At this point what is happening? Firm 1 believes that firm 2 is going to produce this much, so it produces this okay and firm 2 assumes that firm 1 is going to produce this and so it produces this, so they are match, so that is why this is a Nash equilibrium.

But consider any other point like this here suppose firm 1 thinks the output of firm 2 is this much then it is going to produce this much level of output. If it produces this much output so it is given by this amount then firm 2 its reaction function is this, it should produce this output. This much, not this, so that is why they are not match.

So only the solution of this reaction function is going to give you the Nash equilibrium. Because here it is deviating, right? from this output firm 1 thought that output of firm 2 is this much and then based on that it produce this but firm 2 when it assumes that the firm 1 is going to produce this much, it is not producing this but it is producing less than that. So it is deviating so that is why this is not a Nash equilibrium only the point intersection point is the Nash equilibrium, okay. So this is the solution of Cournot outcome in a 2 firm case, okay.

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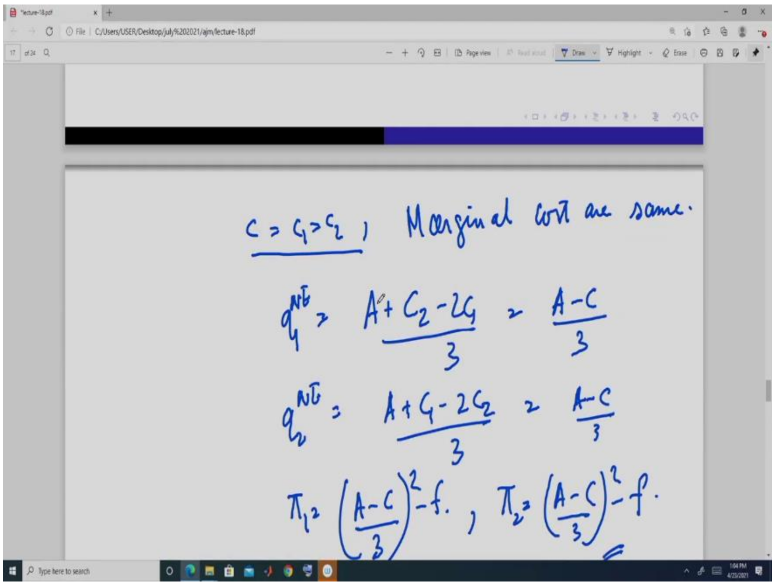

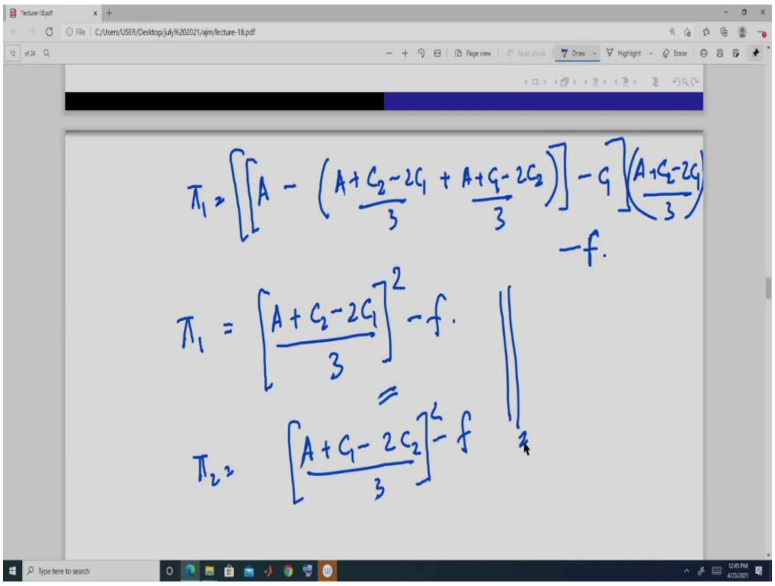

Now we are going to, we will show that this Cournot outcome is similar to something like we have done that the Prisoner's dilemma kind of thing. So to do that what we will assume for our simplicity that suppose c1 is equal to c2 that is marginal costs are same. So this Nash equilibrium output, which was this- $q_1^{NE} = \frac{A+c_2-2c_1}{3}$ it becomes- $\frac{A-c}{3}$, right?

And if you plug in these outputs in the profit, profit is going to be this $\pi_1 = \left(\frac{A-c}{3}\right)^2 - f$ and, is going to be this- $\pi_2 = \left(\frac{A-c}{3}\right)^2 - f$, right? simply in this function, you do the substitution, you will get this – this thing so the Nash equilibrium profit of firm 1 is this and it is this firm 2.

Now instead of this playing strategically choosing output simultaneously without any coordination or without any discussion among them suppose the firm 1 and firm 2 make a collusion, collusion in the sense that what they do now from here this assumption makes that these two firms are similar, right?

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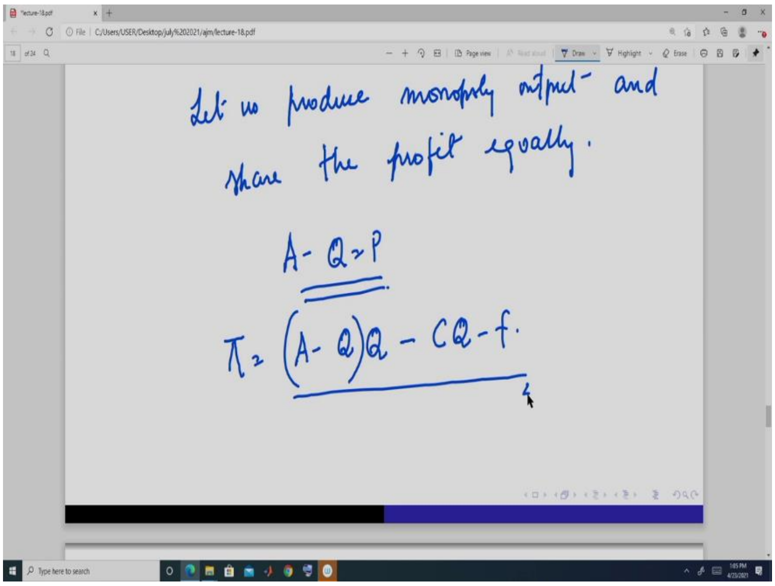

Since these two firms are similar so they decide that let us produce, let us produce monopoly output, monopoly output and share the profit equally. What does this mean? So we know the monopoly thing. So these firms are similar so it does not matter whether firm 1 produces or firm 2 produces, okay.

So market demand inverse market demand is this-A-Q=P now assume that they are going to act as a monopoly so only one firm produces. So it is you can think one firm is going to sell only. So this is the profit- $\pi = (A - Q)Q - CQ - f$.

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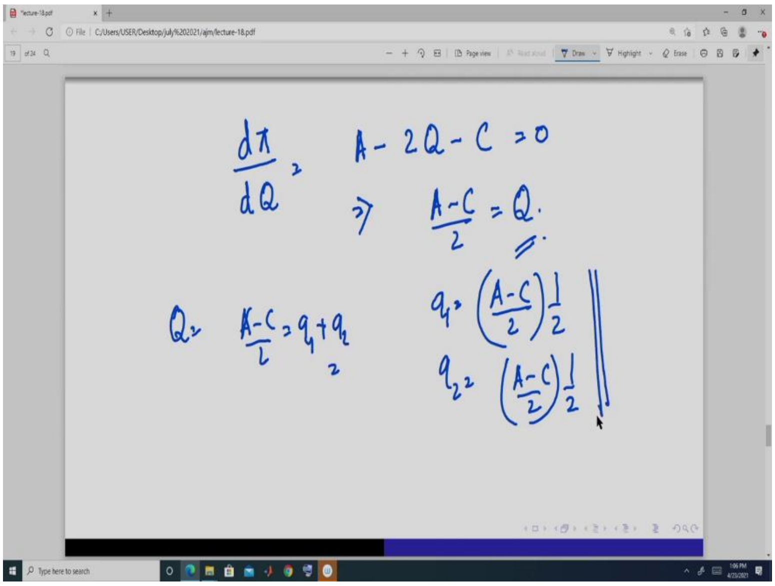

Now we, this is the monopoly output- $\frac{\text{A}-\text{C}}{2} = \text{Q}$. Now what it can do since the firms are similar, so this they can share. They say that you produce half of this and I produce half of this. So this is suppose $q_1 = \left(\frac{\text{A}-\text{C}}{2}\right).\frac{1}{2}, q_2 = \left(\frac{\text{A}-\text{C}}{2}\right).\frac{1}{2}$, okay, they are selling this and you can think is this, which is equal to q1 plus q2. Now if this is the case then what is the profit of these firms?

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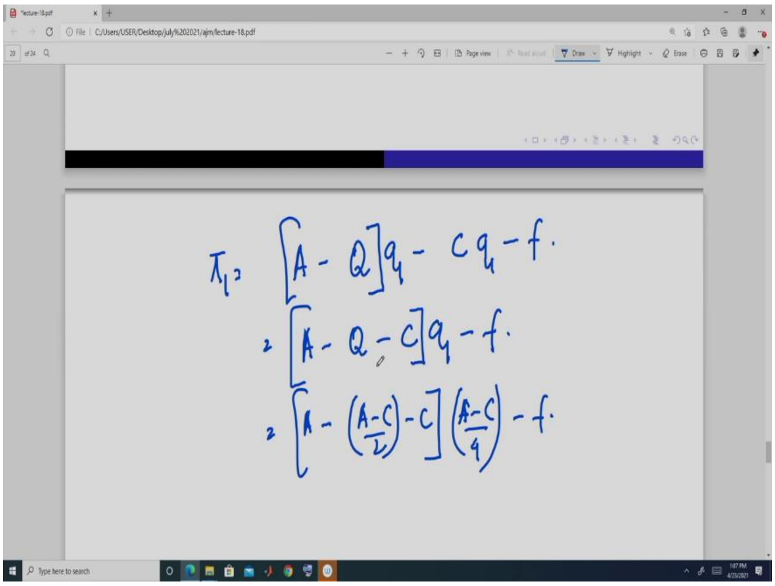

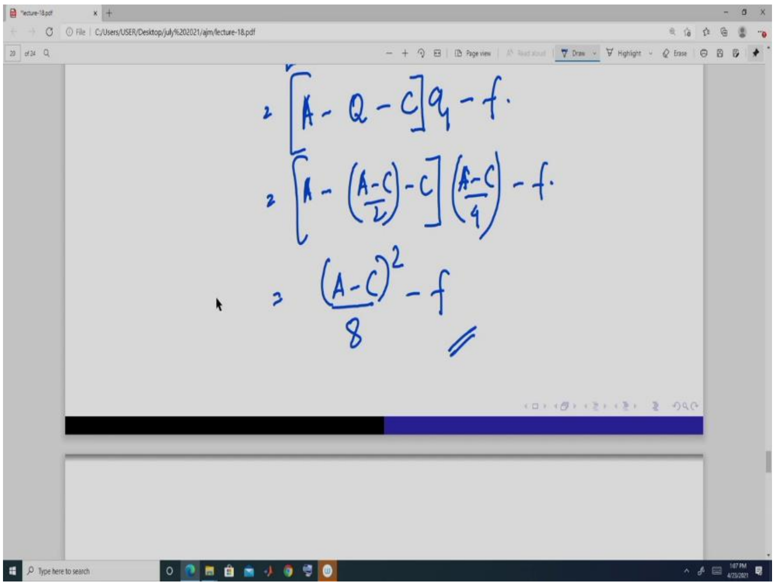

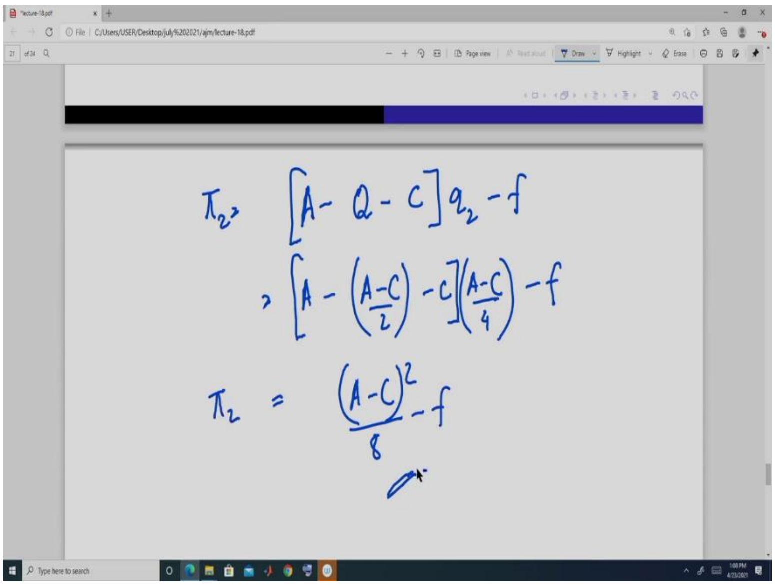

Profit of these firms are; profit of firm 1, we know this-$\pi_1 = [\mathrm{A} - \mathrm{Q} - \mathrm{C}]q_1 - \mathrm{f}$, we know this-$\pi_1 = [A - \frac{\mathrm{A-c}}{2} - \mathrm{C}].\left(\frac{\mathrm{A-c}}{4}\right) - \mathrm{f}$. It is this- $\frac{(\mathrm{A-C})^2}{8} - \mathrm{f}$ and if we follow the same method, going to be this, is this-$\pi_2 = \frac{(\mathrm{A-C})^2}{8} - \mathrm{f}$. Now the question is whether they are going to play this Cournot kind of game or they are going to share the market equally.

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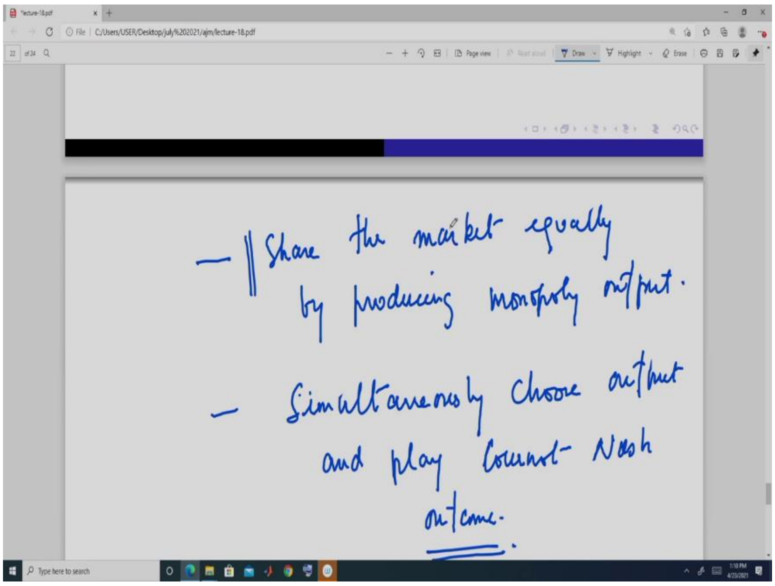

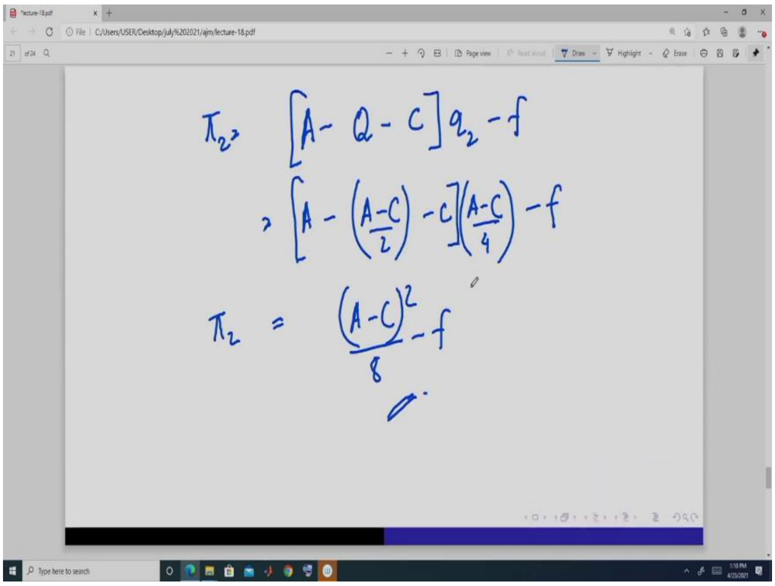

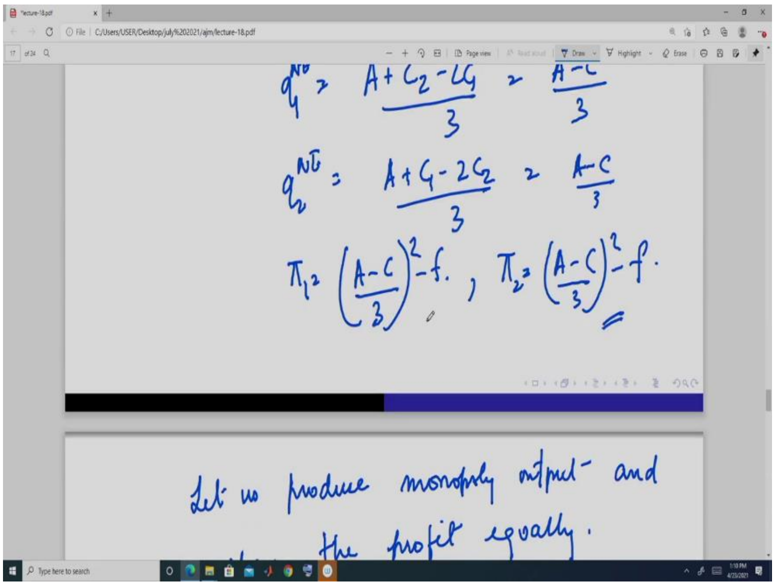

So the question is here that whether to share the market equally by producing monopoly output so this is one you can think as this is one strategy or do not bother, simultaneously choose output and play Cournot Nash outcome, okay. Suppose this.

Now here in this situation, in this situation, what are the profits, in this situation profits are this- $\frac{(A-C)^2}{8} - f$ so, when they share this, so the profit is this and in this case, the profit is what is the profit, the profit is this much-$\left(\frac{A-c}{3}\right)^2 - f$, okay. Now suppose firm 1 whether this is going to be implemented automatically.

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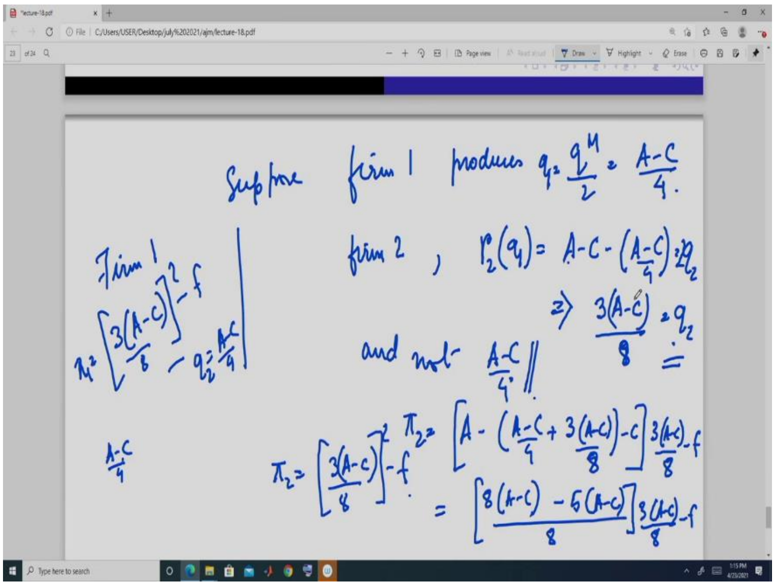

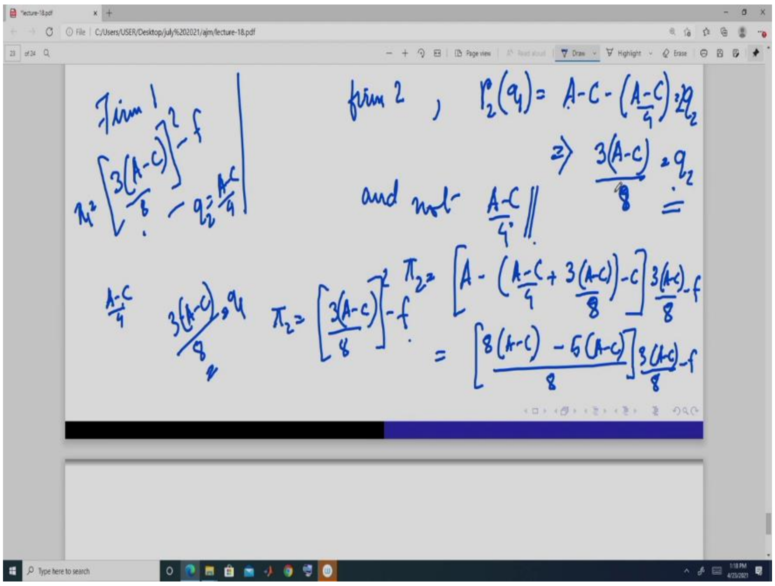

$$\pi_2 = [A - Q - C]\,q_2 - f$$

$$= \left[A - \left(\frac{A-C}{2}\right) - C\right]\left(\frac{A-C}{4}\right) - f$$

$$\pi_2 = \frac{(A-C)^2}{8} - f$$

$$\frac{(A-C)^2}{8} - f < \left(\frac{3(A-C)}{8}\right)^2 - f$$

$$\Rightarrow \quad 1 < \frac{9}{8}$$

- Share the market equally by producing monopoly output.
- Simultaneously choose output and play Cournot-Nash outcome.

$$\pi_1 = [A - Q]\,q_1 - c\,q_1 - f.$$

$$= [A - Q - C]\,q_1 - f.$$

$$= \left[A - \left(\frac{A-C}{2}\right) - C\right]\left(\frac{A-C}{4}\right) - f.$$

$$\pi_1 = \frac{(A-C)^2}{8} - f$$

Suppose firm 1 produces monopoly half of this, this- $\frac{q^M}{2} = \frac{A-C}{4}$ then firm 2 how much it should produce if we use the Nash optimization, so we get a reaction function of like this. So this is going to be output of firm 2 is this-$\frac{3(A-c)}{4} = q_2$ and not this- $\frac{A-c}{4}$, right? because this is the half of monopoly. Suppose firm 1 produces this then from the reaction function of firm 2 we get firm 2's best response is this. It is optimal and we have got this from optimizing the output of firm 2 given a output of firm 1, okay.

So if we take this combination so then the profit of firm 2 is what, this is, so the reaction function it will have are 2 here, right. So this will be 3 by 8 this will be 3 by 8 like this, so this is going to be, so it is again 8 A minus C 5 A minus C. So profit of firm 2 is going to be this- $\pi_2 = \left[\frac{3(A-c)}{8}\right]^2 - f$, right?

Similarly, if we do the same thing so we get the profit of firm 1, what it is going to be? It is going to be 3- $\left[\frac{3(A-c)}{8}\right]^2 - f$. It is going to be this one when firm 1 produces, firm 2 produces this half a monopoly. Now what you are doing? So firm 2 suppose firm 1 is producing this much amount of output- $\frac{A-c}{4}$, so firm 1 suppose produce this much amount of output$\frac{A-c}{4}$, okay.

Then if it produces this much firm 2 produces this much-$\frac{A-c}{4}$, its profit is going to be this much- $\frac{(A-C)^2}{8} - f$ right but instead if it produces this much- $\frac{3(A-c)}{8}$based on the reaction function, its profit is going to be this much$\left[\frac{3(A-c)}{8}\right]^2 - f$ so now let us compare the profit here.

This and you will see that this $\frac{(A-c)^2}{8} - f$ is always less than $\left[\frac{3(A-c)}{8}\right]^2 - f$, so this is going to be, so for firm 2 it is optimal to deviate, it is optimal to deviate instead of producing half a monopoly, it should produce more output and this. So that is why what is happening firm 2 is deviating and similarly firm 1 if we compare this profit and this we get that it is also optimal for firm 1 to deviate if firm 2 produces half of the monopoly output.

So that is why what do we get they are never going to decide that let us produce half of the monopoly. Although if you look at this profit, this profit is greater than this profit. If you simply compare, this and this you will see that this is greater than this, this $\left(\frac{A-c}{3}\right)^2 - f$ is greater than this$\frac{(A-c)^2}{8} - f$ right? because this is divided by 8 and this is going to be divided by 9, So even though the Cournot outcome is suboptimal compared to the monopoly. Why it is optimum because if they decide that let us produce half of the monopoly then the profit can be at a higher level. But what happen, they cannot decide on that, they cannot come to a conclusion because if firm 1 decided that let us produce half of the monopoly profit, firm 2 based on its reaction function is going to produce this much level of output and its profit is going to be this much, which is higher than the half of the monopoly profit.

Similarly, if firm 2 produces half of the monopoly output then firm 1 is not going to choose half of monopoly instead it is going to produce this much amount of output and so the profit is going to be this much which is greater than the half of the monopoly profit. So that is why they will not they cannot come to outcome which is better than a Cournot outcome because Cournot outcome is going to give this much profit to each but they end up having this much profit but they could have got better profit. This is half of the monopoly profit.

So we will stop at this today. We have done the Cournot duopoly thing and we have also shown that the Cournot outcome is actually similar to the Prisoner's dilemma that is there is an another outcome, which is better than these 2 outcomes but still since the firms behave in a non-cooperative way, in the sense that they behave what is best for them given the output of other firm and there is no possibility of any negotiations between them so that is why we get the outcome as same as the Prisoner's dilemma outcome, okay. And in the next class we will extend this model to n firm, okay. Thank you very much.