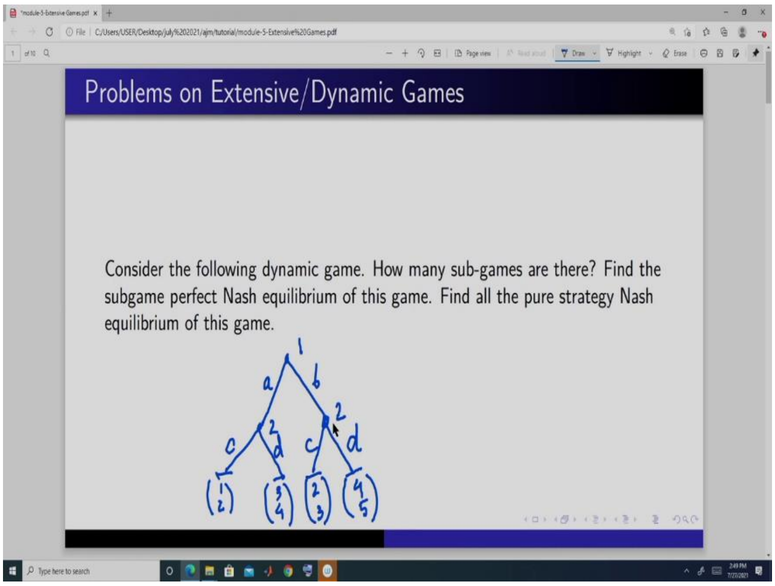
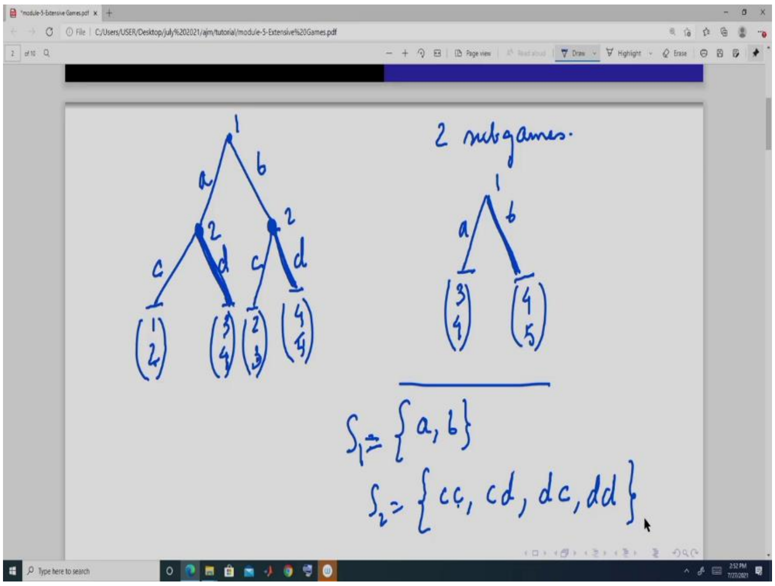
Introduction to Market Structures
Professor. Amarjyoti Mahanta
Department of Humanities & Social Sciences
Indian Institute of Technology, Guwahati
Lecture No. 24
Tutorial on Dynamic Games

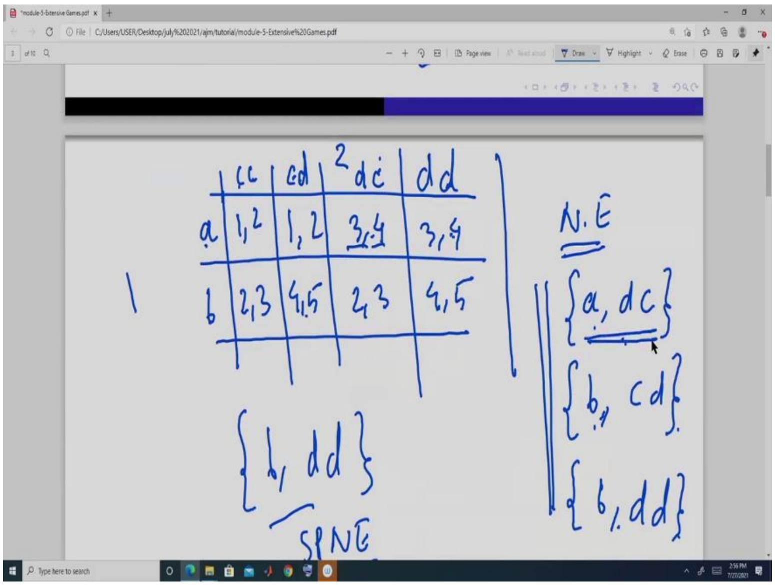
(Refer Slide Time: 0:38)

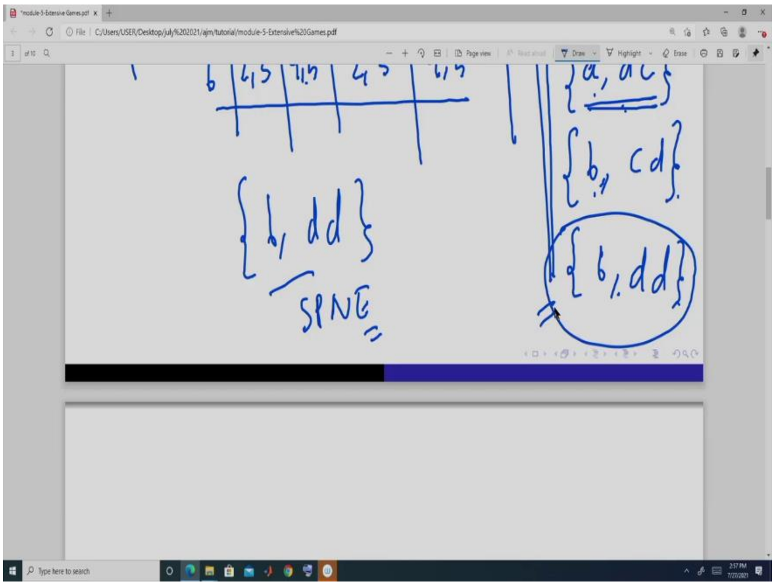


So, let us solve some problems on dynamic games, okay. We have already solved few problems, but let us do some more. Suppose let me first draw the game tree, layer one a, b, c, d, c, d, (3, 4), (2, 3), and this (4, 5), okay. So, this is a two stage game. First player 1 moves. It can choose two, it has two actions a and b, then player 2 moves and the player 2 has two actions c and d. So, we have to find out how many sub-games are there and we also have to find out the sub-game perfect Nash equilibrium, also the pure strategy Nash equilibrium of this game, okay. Now, let us do this.

(Refer Slide Time: 1:36)







So, our game is this a, b, then this 2, c, d, c, d, okay. Now, we know we have two, how many sub-games, one sub-game from here, another sub-game from here. So, we have two sub-games. And in the dynamic games, we solved it using backward induction. So, suppose player 1 has played this action, then player 2 will have to choose between c and d. If it chooses c it gets two, if it chooses d it gets 4. So, it will choose this d.

Now, suppose player 1 has chosen two, sorry, chosen b, then it will be here. 2 will be in this nod. So, when 2 is in this node, if it chooses c it gets 3, if it chooses d it gets 5. So, it will choose this d. So, this game is now reduced form is (3, 4) and (4, 5). So, player 1 will choose b, because 4 is greater than 3. So, this is the sub-game perfect Nash equilibrium. So, player 1 chooses b and player 2 when it is in this node it chooses d and when player 2 is here it chooses d, okay.

Now, here, this is the sub-game. Now, how do we find the pure strategy Nash equilibrium? Pure strategy Nash equilibrium, then we have to convert this dynamic game into normal form game. And for that we have to specify their strategies. How many, so player 1 has only one nod from which it makes a decision. So, its actions or strategy sets are same. So, S1 is suppose this- $S_1 = \{a, b\}$, the strategy same or the action set, sorry it is same.

But for player 2 it has two node, this and this. So, we have to specify an action for each node then only it will be a complete profile of action, set of actions. So, it is possible that cc, c in nod 1, c in nod 2, then it is cd, c in nod 1, d in nod 2, then it is dc, d in nod 1 and c in nod 2, then dd that is d in nod 1 and d in nod 2. So, this is the strategy shape of player 2.

Now, we specify the payoffs of this. We represent this as a normal game. So, for player 2 is in this column and player 1 is in this row. This is a, this is b, cc, cd, dc, dd, okay. So, and this here if you look at the game, you will get 1, 2, this is playing b and player 1 it is c in nod 1, c in nod 2, so it is 2, 3, it is again a and it is playing c in, then it is (4, 5), (3, 4), (2, 3), (3, 4), (4, 5), okay. So, here, we have to find that in this normal form game we have to find the Nash equilibrium. So, if player 1 plays a, player 2 is indifferent between this (dc) and this (dd), because 4, 4. But if player 2 plays this, player 1 plays this. So, this (3, 4) is one Nash equilibrium.

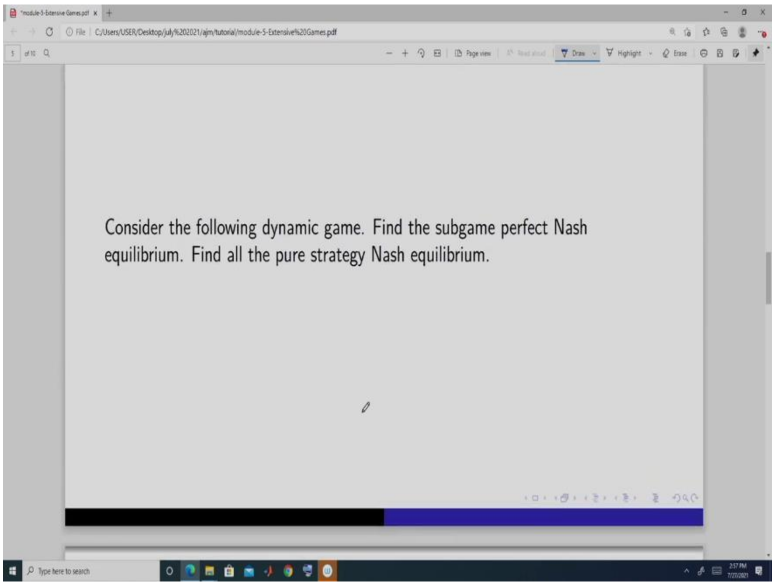
So, Nash equilibrium was the {a, d, c}, this is one. Now, if he plays this, player 2 plays this (dd) strategy, player 1 plays this (3, 4). So, this is not there. These two are not an outcome we have got. If player 1 plays b, player 2 is indifferent between this (cd) and this (dd), but if this is played he moves this b (4, 5). So, immediately b and cd is a Nash equilibrium- {b, cd}. Again here if d is played he plays b and if b is plays he is indifferent between cd and dd. So, again this {b, dd} is a weak Nash equilibrium.

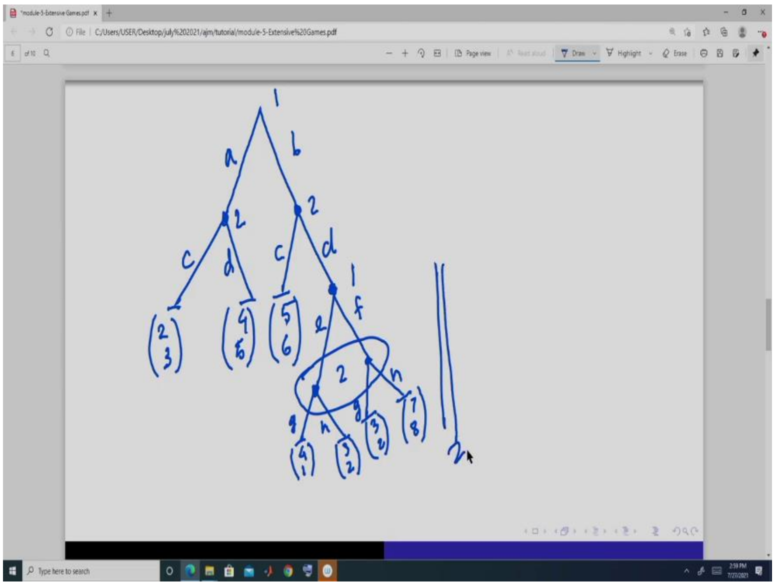
So, we get three pure strategy Nash equilibrium. And which one is sub-game from this. Sub-game we have got, sub-game is this, this and here this. So, it means when player 1, player 2 in its first node that is, it will play d and in second nod it will again play d and player 1 will play b. So, it is this. So, b, dd this is SPNE, sub-game perfect equilibrium. And these two- {a, dc}, {b, cd} are Nash equilibrium, but they are not sub-game perfect because there is no credible trait in it.

It is like this that player 1 here, if you look at this, player 1 here, it is playing a, because player 2 is playing that if you play b, I will threaten you by playing c, so better play a. So, its outcome is this, a, d, so it is a, d, because player 2 is threatening that if you play b, I will play c. But then it is not credible, because if player 1 plays b, player 2 if it is in this node, it will always play d and not c. So, that is why this is not credible.

Again look at this, here it is c and d. So, here player 2 is saying that if you play a here, then I will play c. So, you play b so that I will play d. But then here again this is not credible. Why, because if player 1 plays a, then player 2 will not play c, but instead it is going to play d, because 4 is greater than 2. So, that is why only this {b, dd} has that credibility. So, that is why this is a sub-game perfect Nash equilibrium and other two Nash equilibrium, pure strategy Nash equilibrium are not sub-game perfect Nash equilibrium, okay.

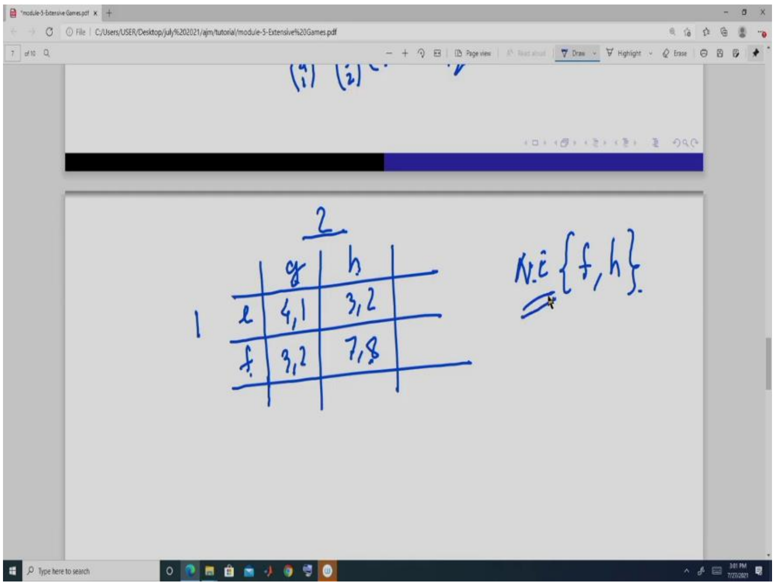
(Refer Slide Time: 9:43)

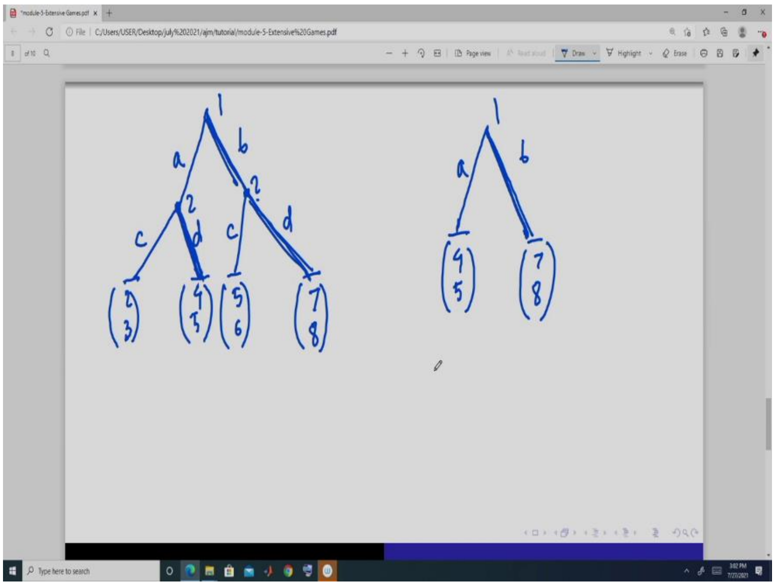


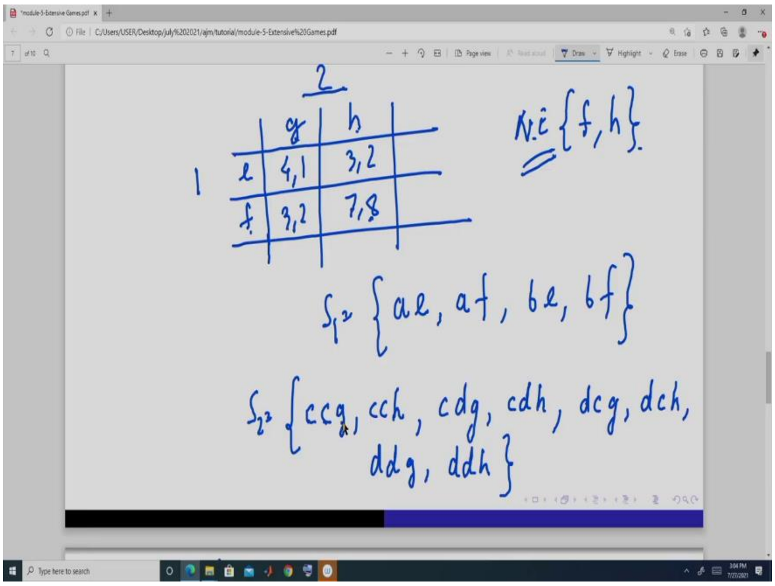


Now, let us solve another problem and this is an imperfect game, okay. Let me first draw the game. So, this game tree is slightly, okay. So, this game is an information but imperfect information, because here if this node is rich then the player 1 and player 2 simultaneously choose their action, okay or even if they do it sequentially but the actions are not observable. So, that is why the information set of player 2 here it has two nodes, okay. So, this we will first specify this as a normal game.

(Refer Slide Time: 11:43)







e	4,1	3,2
f	3,2	7,8

$S_1 = \{ae, af, be, bf\}$

$S_2 = \{ccg, cch, cdg, cdh, dcg, dch, ddg, ddh\}$

2

1	ccg	cch	cdg	cdh	dcg	dch	ddg	ddh
ae	2,3	2,3	2,3	2,3	4,5	4,5	4,5	4,5
af	2,3	2,3	2,3	2,3	4,5	4,5	4,5	4,5
be	5,6	5,6	4,1	3,2	5,6	5,6	4,1	3,2
bf	5,6	5,6	3,2	7,8	5,6	5,6	3,2	7,8

N.E : {be, dcg}, {be, dch}, {af, ddg}

N.E : {be, dcg}, {be, dch}, {af, ddg}

{bf, ddh} , {bf, cdh}

SPNE

And so this game becomes like if you take player 2 in the column and player 1 in the row then player 1 has two action and those are e and f, player 2 has two actions g and h, and so, this is the game that they play when it is starting from this node, right? Now, this is the simultaneous move game, single shot simultaneous move game.

So, if we look at this game, player 1 if it plays e, best response for player 2 is to play h. If player 2 plays h best response for player 1 is to play f. And if player 1 plays f, best response is to play h. So, here Nash equilibrium is {f, h}, okay. This is the node. We plug in this outcome here. So, the ever game now it is, the outcome here is 7, 8, this. So, suppose again player 1 plays a, then player 2 starting here is going to play d, because 4 is greater than 3. So, this is the outcome.

If player 1 place b, then player 2 starting from this node, it will compare 6 and 8 it will play d (7, 8). So, the game reduced form is now (4, 5), (7, 8). So, player 1 will choose this- b(7, 8). It will be this. So, this is the outcome and this is the sub-game perfect Nash equilibrium. Now, we have to present this as a normal form game. So, if we look at this as a normal form game, then we have to specify the actions.

Now, specify, the actions are already being specified. So, we have to specify the strategies. Player 1 is making decision two nodes in this node and in this node. Player 2 is making decision in three positions, you can say this, this and in this information set. It has two nodes, but this is single information set. This is one information set, this is one information set, this is one information set. So, it is taking at three positions.

So, action set, strategy set of player 1 say it is S1= {ae, af, be, bf}, because here in this node it takes, it can take only two actions e and f and here it has two a and b. So, that is why combinations we are getting this. So, these are each point, each element here is a strategy of player 1. For player 2, we have to specify like c in node one or information set one, c in information set two and then you choose g and h. So, it is g in the third information set and then again you can have this, then again we can have this. So, these are the strategies of player 2- $S_2 = \{ccg, cch, cdg, cdh, dcg, dch, ddg, ddh\}$.

So, this is one strategy where information set one it play c, information say two it plays c, again it plays g in the third information set when it is making a decision here in this, right? So, like that we will, can construct the, these are the strategy of player 1 and player 2, i.e. S1= {ae, af, be, bf}and $S_2 = \{ccg, cch, cdg, cdh, dcg, dch, ddg, ddh\}$. Now, let us represent this as a normal form. Now, normal form of game it will be very big game actually. So, it will be ae, af, be, bf, it will be like this and we write the payoffs. So, these are the payoffs, right? So, this is the normal form game where this is the player 2 which is in the column, player 1 which is in the row.

If player 1 plays this al, player 2 is indifferent between these four strategies- dcg, dch, ddg and ddh, okay. And if it is this is played player 2, 1 is indifferent between these two be, bf. If this is played, is again indifferent between this, this, this. So, this is one Nash equilibrium- dcg and dch. So, here if I write the Nash equilibrium set, then I get one be and dcg. If this- dch is played again he is going to indifferent between this- be (5,6) and this- bf (5, 6). If this is played, he is indifferent between this, this, this, this.

So, again this is be, dch. And if this is played he is, player 1 is indifferent between this, this and this. And if this is played he shifts here. So, this is not an outcome. If this is player 1 plays here this is indifferent between this, this. So, this is one Nash equilibrium, i.e ddg (4, 5). If this (ddh) is played then player 1 is going to play this (2, 8) and if this bf is played he is going to play this (2, 8). So, another Nash equilibrium is bf and it is ddh, ddh. And if suppose this bf is played, because he is going to be indifferent between this and this player 2 and if this ddh is played, he is going, so this is one. If this is played by player 2, then he is going to, if this is he is going to choose this, if this is so this (7,8) is also another Nash equilibrium that is bf and it is cdh.

Now, if this is played, I know we have already discussed, he is going to be indifferent between these and we have found out all the Nash equilibrium in this region. If this is played, then he is indifferent between these four, player 2 is indifferent between these four strategies and we have find out. So, these are the 1, 2, 3, 4, 5 are the pure strategy Nash equilibrium in this game, i.e {bl, dcg}, {bl, dch}, {af, ddg} {bf, ddh}, {bf, cdh}.

And what to do with the sub-game perfect Nash equilibrium. Sub-game perfect Nash equilibrium we have seen b and player is f. So, here bf, so these have bf, then ddh and here cdh. Ddh, so it here d, d and in this game it is h. So, this is the sub-game SPNE and rest are not sub-game perfect Nash equilibrium. These are the pure strategy Nash equilibrium, but they are not sub-game perfect, because in one of their nod**es**, we will always find that it is not credible enough, okay. So, that is why it is not a sub-game perfect Nash equilibrium and only we have a unique sub-game perfect. Thank you.