Introduction to Market Structures
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Lecture No. 23
Subgame Perfect Nash Equilibrium

Hello, everyone. Welcome to my course Introduction to Market Structures. So, we were doing dynamic games that is extensive games. So, today we will conclude that portion.

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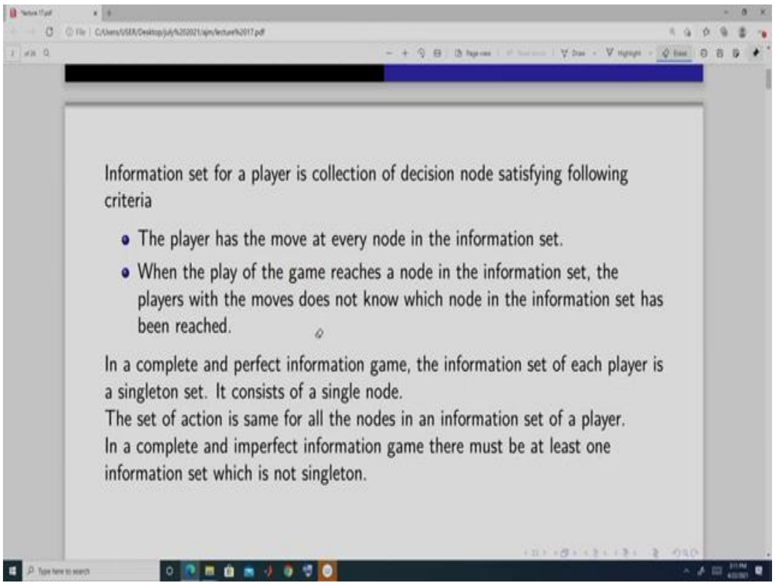

So, we have already done what is information set. Information set is a collection of decision nodes of a player and it can be singleton set or it may have many decision nodes. If it has many decision nodes that is more than one decision node, then the player does not know in which decision node he is taking the decision. So, he only knows that in which node, in which information set it belongs to, but it does not know exactly which node it is in, okay.

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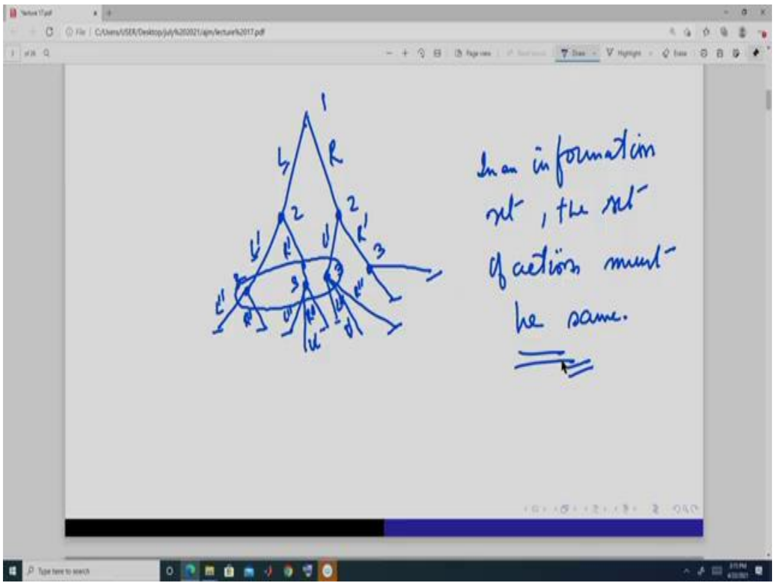

So, to make, so if suppose a player is already they have played a game like this suppose. Suppose they have already played a game of this nature. This is player 1, L, R and this is player 2, this is L dash, R dash, L dash, R dash, and then suppose again player 3, okay. So, this is player 3, player 3, 3, 3. Now, suppose I have a situation like this, okay. And here I have some payoffs. Payoffs I will not specify now.

Now, here player 3, in this situation player 3 is it is moving it does not know whether it is in this node or in this node, or this node, because player 3 has not observed these movements of or these actions of player 1 and player 2. So, that is why it is imperfect information. But it is complete information because all the payoffs are known, okay. So, here what is happening, in this situation player 3 does not know which node it is.

But suppose we have a situation where it is L double dash, R double dash, L double dash, R double dash, L double dash, R double dash, but it has one more strategy and that is suppose u, one more action that is u and this has one more action and that is suppose d dash. So, then see each node has a different sets of action. In this node it has L double dash, R double dash. In this node it has L double dash, R double dash and u dash. In this node it has L double dash, R double dash and d dash.

So, if player 1 has chosen this action and player 2 has chosen this action then the game is here and player 3 has not observed this action. But if player 3 only can choose from these two actions, from these two, only from these two actions it has to choose then it knows it is in this node. So, for this reason what happens that the all the action set must be seen in a information set. So, in an information set the set of actions must be same, okay.

Otherwise, the player 1, it will no more be an imperfect information, because player 3 even though it has not observed actions, but from the set from which it has to choose an action, it will, it can decipher or it can infer what are the past actions, okay. So, that is why the action must be, set of action must be same, okay.

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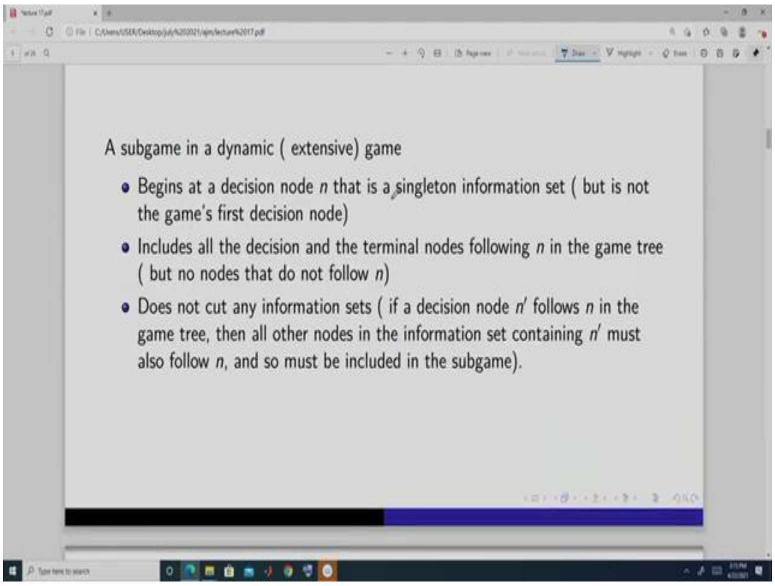

And then we have also defined what is subgame. Subgame, it begins from any node which is other than the first node and subgame, it follows all the decision nodes from any specific decision node that is includes all decision and terminal nodes following n in the game tree, and does not cut any information set. So, if, a whole information set should belong to only one subgame. Information set cannot be partitioned into two different subgames, okay. So, this is.

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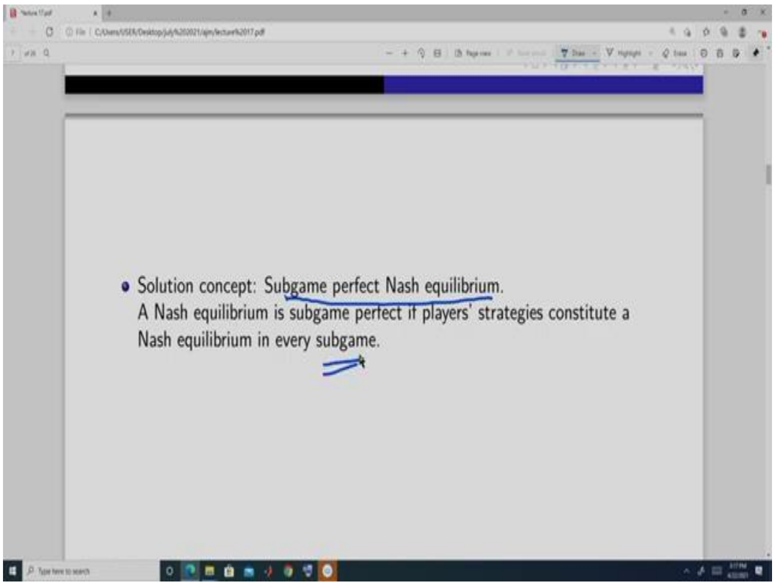

Now, we will discuss some solution concept. So, we have done backward induction. Backward induction is a method of solving the game. But what solution, while using backward induction what we were doing? We were moving from the last subgame to and then we have moved backwards and we have tried to find the Nash equilibrium at each stage, okay. So, a specific solution concept for this type of dynamic game is subgame perfect Nash equilibrium and this was proposed by Selten, okay.

So, it says a Nash equilibrium, it is a Nash equilibrium is subgame perfect if players' strategies constitute a Nash equilibrium in every subgame. So, let us take, in every subgame.

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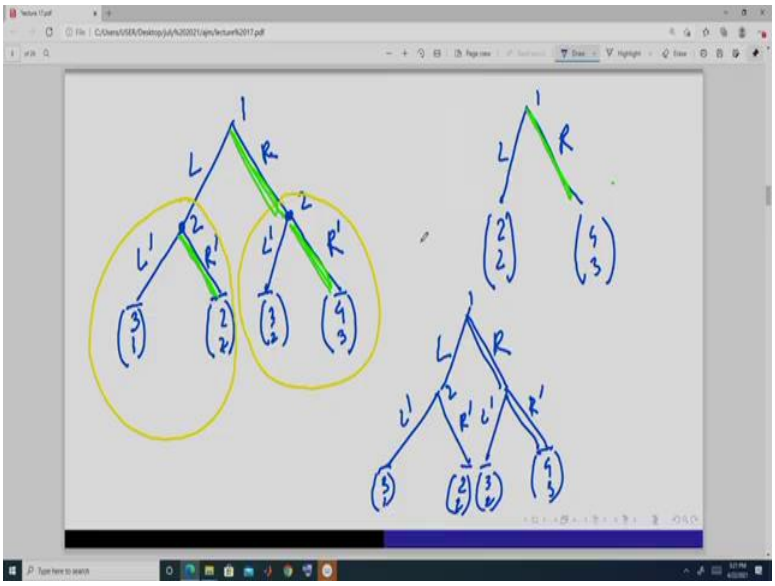

So, let us take an example, okay. Player 1, its action is L and R, player 2 its action is L dash, R dash, L dash, R dash, player 2, so it is a complete information game, okay complete information and also perfect, okay. This is the subgame. Now, here, it has two subgames. This is starting from this and this is a node. We have this- $L' = (3,2), R = (4,3)$ as one subgame and this- $L' = (3,1), R' = (2,2)$ is another subgame, okay. So, in this subgame, so, how do we solve? We use the same the backward induction only, but here we specify that it has to be a Nash equilibrium in each subgame. So, it is a Nash equilibrium in this subgame, it has to be a Nash equilibrium in this subgame and also in the whole game, okay.

So, this, if player 2 is here, then we know it is optimal to choose this R dash- $R' = (2,2)$, because 2 is greater. So, in this game, this-$R' = (2,2)$ is the action that player 2 is going to choose. In this game, when player 2 is choosing it is again going to choose this- $R = (4,3)$ 3 is greater than 2, here 2 is greater, right? And then, so this game, now, it is what. So, player 1, since we are using backward induction, so it knows that if it chooses L player 2 is in this node and player 2 is going to choose R. And if player 1 chooses R, player 2 is going to be in this node and it is going to choose R. So, player 1 knows this by, means player 1 can infer this based on this game, because it is a, information is complete.

So, player 1, what it will do? It will compare these two payoffs. So, for player 1, it is going to be like- L= (2, 2), R= (4, 3), so player 1 is going to choose this action- R= (4, 3). So, it is going to be this here. So, in this game subgame perfect Nash equilibrium is, in this (L) game it is this- $R' = (2,2)$, and in this (R) game it is this- R= (4, 3), and then it is chooses this one. So, this, but while we were doing backward induction outcome and we are trying to find the optimal outcome in the backward induction in the same game, it is here, it is this, here it is this and then from this it is here.

So, we do not need to, because we do not need to specify this. So, backward induction outcome is only this-$R' = (4, 3)$, but subgame perfect Nash equilibrium outcome is this. We have to specify the Nash equilibrium in each subgame. In backward induction outcome we do not need to. So, this is the difference. So, it is a complete profile of this, all this has to be provided, okay.

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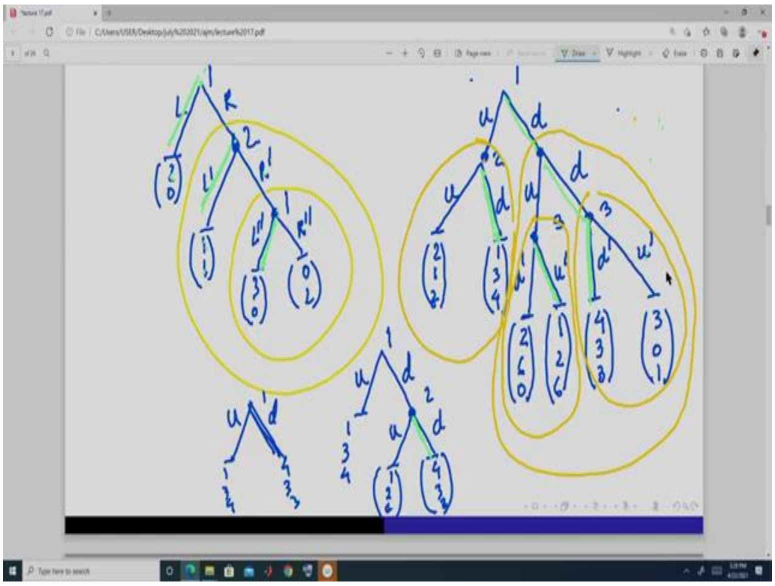

So, let us do another example. Suppose player 1, player 2, again, player 1, okay this is L, R, L dash, R dash, L double dash, R double dash, payoffs are- (2,0), (1,1). (3,0), (0,2). So, the subgames are, this is one subgame- L'', R'', this is another subgame- L'', R'', L', R', okay. Now, here we have to find the Nash equilibrium. In this subgame-L'', R'' player 1 is always going to choose this-$L''(3,0)$. So, player 2, it has this if it chooses L dash 1, and if it chooses R dash it will, player 1 is going to choose L dash, so it is going to get 0.

So, player 2 chooses this-$L'(1,1)$. And player 1 if it chooses L it is going to get 2, if it chooses R then it is going, player 2 is going to choose L dash, so it is going to get 1. So, it is going to choose L. So, this is the subgame perfect Nash equilibrium outcome. So, in this starting the games this is the initial node, the action is this- L (1, 1). In this, action is this. In this L'', action is this one- (3, 0), okay.

Let us take another example. So, this example is slightly big game. It is played between three players. It is u, d, this is again u, d, it is again u, d, okay. Suppose this is the game tree. There are three players and game tree is of this nature. So, the subgames are, it has many subgames.

This is one subgame-$d'(4, 3, 3), u'(3, 0, 1)$, this is another subgame-$d'(2, 6, 0), u'(1, 2, 6)$, this is another subgame- u(2, 1, 2), d(1, 3, 4) and this whole is also a subgame-$d'(4, 3, 3), u'(3, 0, 1), d'(2, 6, 0), u'(1, 2, 6)$. So, it has 1, 2, 3, 4, 4 subgames in this whole game, okay.

So, let us start from this. In this game, player 2, it will compare 1 and 3. So, player 1 is always going to choose d here in this subgame we know. Then let us start from this subgame. This subgame, player 3, d dash, u dash, if d dash gives 0, u dash gives 6 so player 3 is going to choose this. In this subgame, player 3 is choosing d dash, u dash, d dash 3, u dash 1. So, it is this. And here this now you can see that this game has become something like this. So, player 2 is compared 2 and 3. So, player 2 is going to choose this and here this is going to be, and this is (4, 3, 3), so player 1 is going to choose this.

So, in the whole game you can look at it in this way, it is this and it is this. So, in this subgame, the subgames are, Nash equilibrium are this, this and here it is this, in this it is this, so then finally subgames are represented by this green lines in this game tree, okay. Here again by the green lines in this game tree, okay.

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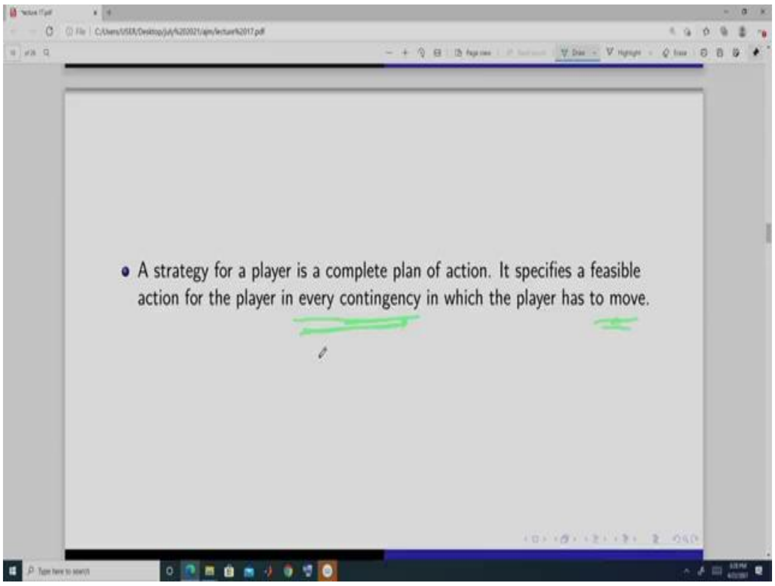

Now, we are going to define what is the strategy. We have talked the actions here. In this game, what is the action, in this game, subgame what is the action, in this subgame what are the actions like that. Now, we will specify strategies. What is a strategy of a player? Strategy of for a player is a complete plan of action. It specifies a feasible action, feasible action for the player in every contingency in which the player has to move, okay. So, keep this in mind, in every contingency in which the player has to move, okay.

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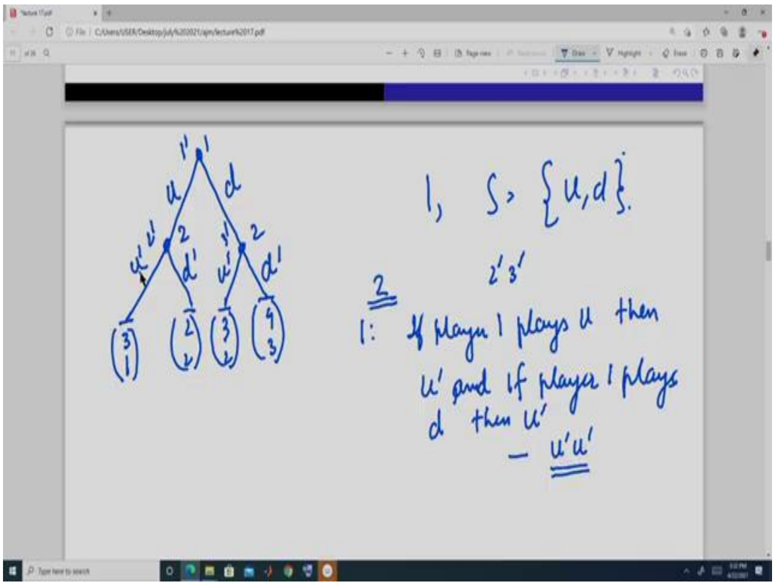

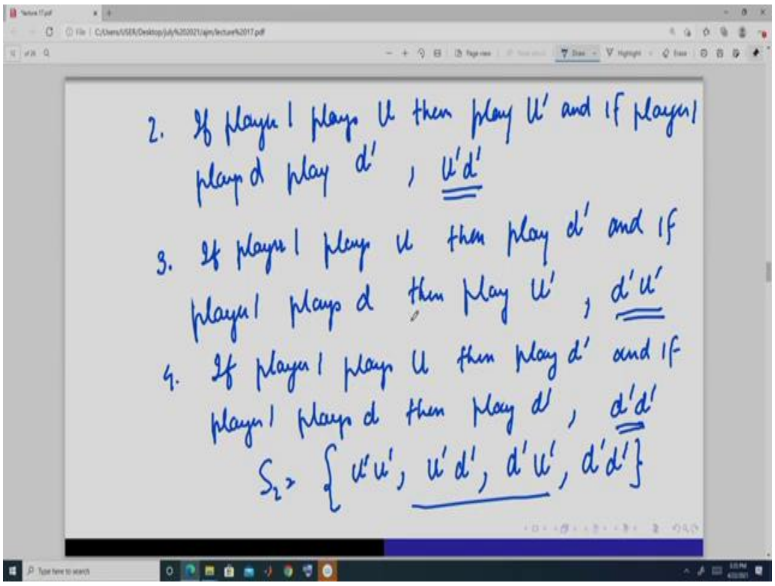

So, now let us define it for a very simple game, okay. So, let us take this initial game u, d, u dash, d dash, u dash, d dash, okay. Let us take this game. We have solved this game. We have no, we know what is the subgame perfect Nash equilibrium of this game, right? Now, player 2 is taking decision in two contingency. It has two contingency, in this node and this node.

So, contingency means mainly in how many nodes it is taking a decision. It is not or you can say in how many information set, okay. Best is going to be one information set implies one contingency. For player 1 it is one only, it is one decision node. So, the set of actions, so for player 1 set of action or the strategy sets are same and it is u and d, okay. But for player 2, it has two. So, let us take, name this. This is 1 dash and this is 2 dash. These are the name of or you can say, sorry, this is 3 dash, okay. So, these are the name of the nodes. So, its contingency is 2 dash and 3 dash, okay. So, how do we, we have to define the complete plan of action. So, we will suppose strategy 1 of, these are for player 2, okay strategy 1.

So, if player 1 plays u then u specify u dash and if player 1 plays d, then u dash. So, this is u dash, u dash. This you can say it is one strategy of player 1, because it has two contingencies or it has two information set on in which it is going to take the decision. So, it is u dash, here also it is taking u dash, and here also it is taking u dash. So, this is suppose strategy 1.

Then we can specify strategy 2, second strategy. Second strategy is, if player 1 plays u then play u dash and if player 1 plays d play d dash. So, this is u dash, d dash strategy. This is strategy 2. This is for node 1, node 2 dash, this is for node 3 dash. Third strategy, if player 1 plays u then play d dash and if player 1 plays d, then play u dash. So, this is d dash, u dash strategy. This is the third strategy.

And we have a fourth strategy and that is if player 1 plays u then play d dash and if player 1 plays d then play d dash. So, it is d dash, d dash strategy. So, the whole strategy set of player 2 you can say it is u dash, u dash, u dash, d dash, d dash, u dash, d dash, d dash, i.e $S_2 = \{u'u', u'd', d'u', d'd'\}$. So, we have four strategies. So, it is, here it will play this (3,1), this (3,2) one strategy, this (3,1), this (4,3) the second strategy, this (2,2) and this (3,2) third strategy, this (2,2) and this (4,3) it is going to be the fourth strategy, okay. So, these are the strategies. But for player 1, it is only this- S={u,d}.

Now, so this is the way we are going to define the strategies. Now, once we know the strategies, so then we can specify this dynamic game as a normal form or a static game. And there how do we do, I will show you just now.

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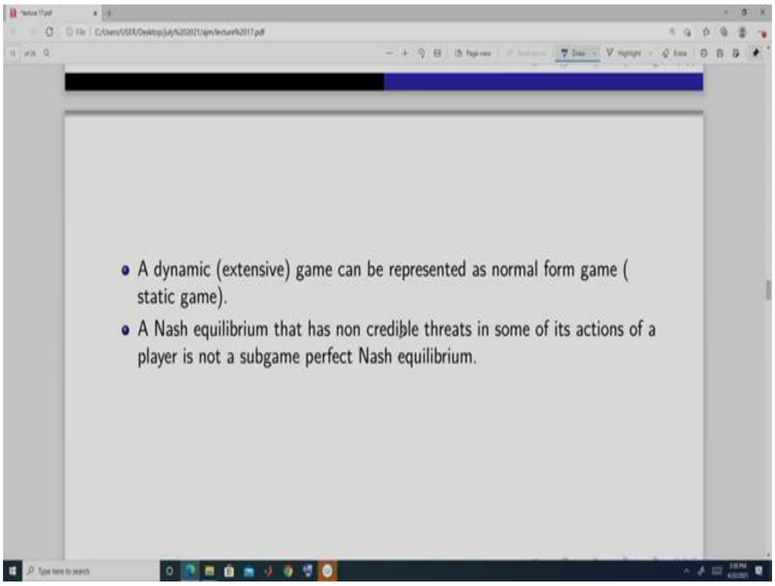

And then we know if we have a static game or a normal form game, then we know the how to find the Nash equilibrium and then from all the set of Nash equilibrium of that game how to find the sub game perfect Nash equilibrium only those which do not have any tradeable threat. If there are some tradable threat, then it is subgame perfect Nash equilibrium and if the Nash equilibrium and there are some actions which are not credible enough, then we say it is not subject subgame perfect Nash equilibrium. So, it is a Nash equilibrium, but it is not a subgame perfect. So, I will discuss that now, okay.

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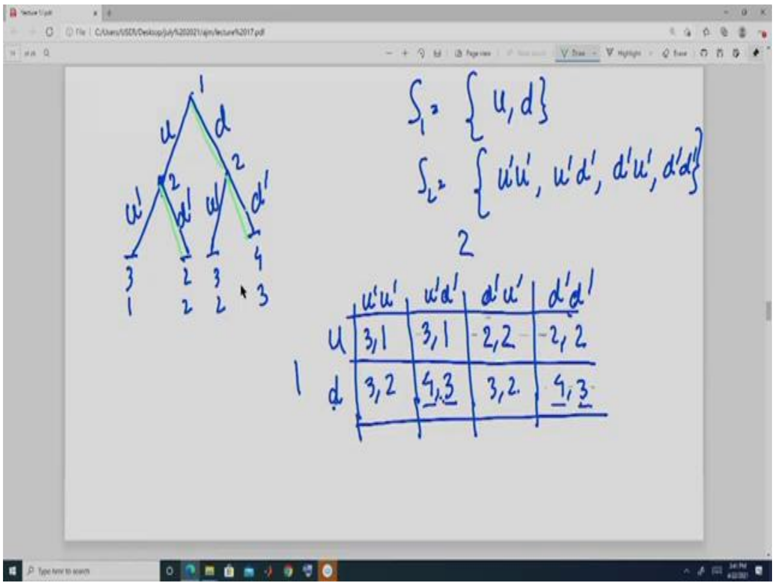

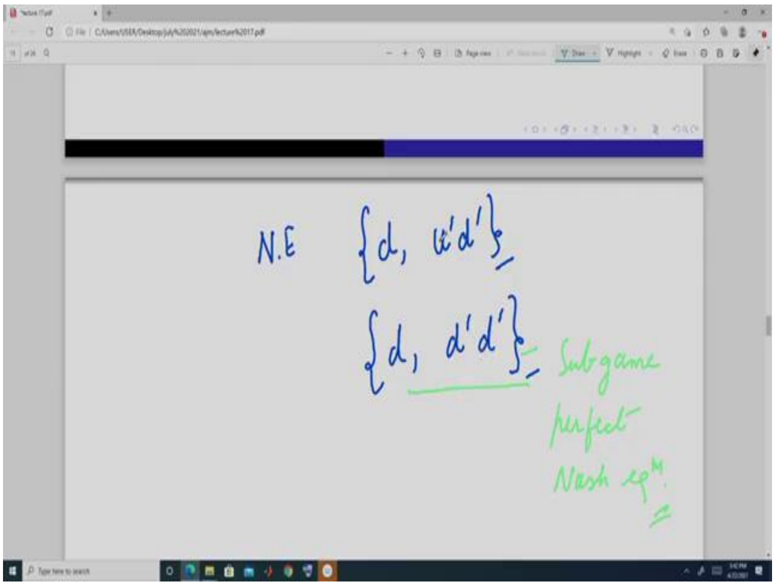

So, let us take this game. So, this game, if you look at this game, what are the, player 1, u, d, player 2, u dash, d dash, u dash, d dash (3, 1), (2, 2), (3, 2), (4, 3), set of strategies for player 1 it is u and d, i.e. $S_1 = \{u, d\}$, set of strategies for player 2 it is u dash, u dash, u dash, d dash, d dash, u dash, d dash, d dash- $S_2 = \{u'u', u'd', d'u', d'd'\}$, right? Now, normal form game can be u dash, u dash, u dash, d dash, d dash, u dash, d dash, d dash. This is for player 2. U, d and this is for player 1, okay.

Now, if it is, player 1 has suppose played u so here and it is, so this, payoff in this block it is going to be (3, 1). And here it is going to play this and this, but player 1 has played this, so it is going to be (3, 2), okay. Player 1 is playing u and it is choosing u dash, okay. And here, and player 2 is choosing d, it is doing d dash, so it is. And player 1 is playing u it is choosing d. When player 1 is choosing u it is choosing d dash, so it is again. Here, player 1 is choosing d and it is choosing u dash. Here it is choosing d dash. So, this is how we will get the payoff metrics of a normal form game.

And in this normal form game, if you want to find the pure strategy Nash equilibrium, what are these pure strategy, if it plays u, so player 2 is indifferent between these two strategies- $d'u'$ and $d'd'$ and if it plays this player 1 is going to follow this- $d'd' = (2,2)$. So, this is not a Nash equilibrium, right? But if it plays again this player 1 is going to choose this. So, again this- $d'u' = (2,2)$ is not a Nash equilibrium. But if player 1 chooses d, player 2 is indifferent between this- $u'd' = (4,3)$ and this- $d'd' = (4,3)$.

And here if player 2 chooses this, so this (4,3) is one Nash equilibrium. It is a weak Nash equilibrium, pure strategy, weak pure strategy, but it is a Nash. Here again if player 2 chooses this (3,1), player 1 is going to choose d and if it chooses d, it is indifferent. So, this (4,3) is again a Nash equilibrium. So, the pure strategy Nash equilibrium, are d and u dash, d dash and again d and d dash, d dash, right? Now, we have two pure strategy Nash equilibrium.

Now, we have to find out which is subgame perfect. If you look at this here, we know here it is going to choose this- $d' = (2,2)$ and here it is going to choose this- $d' = (4,3)$ and player 1 finally chooses this, so d, d dash, d dash,- $\{d, d'd'\}$. So, this is subgame perfect Nash equilibrium. This- $\{d, u'd'\}$ is not a subgame perfect. Why, because see, what it is saying. If player 1 plays u, this is u dash, d dash. If player 1 plays u, if it is in this node, then player 2 is going to choose this. And because of that, because it is giving a threat that if you are, if you choose this, I will choose u dash.

So, better choose this-d, then I am going to choose this- d'. But then player 1 knows this that if player 1 chooses this, when player 2 is here, it is never going to choose this-u', instead it is always going to choose d dash because this is 2 is greater than 1. So, that is why this strategy u dash in node 1, node 2 dash and first node here and d dash the second node of player 2, so this and this it is not credible. This is credible, but this is not.

So, that is why this is not a subgame perfect Nash equilibrium. But this, it is always going to choose this- d' here in this subgame and this- d' in this subgame so that is why this is a subgame. So, we have a Nash equilibrium in each subgame here in this strategies-$\{d, d'd'\}$. But here this u dash is not a subgame if we take this, okay.

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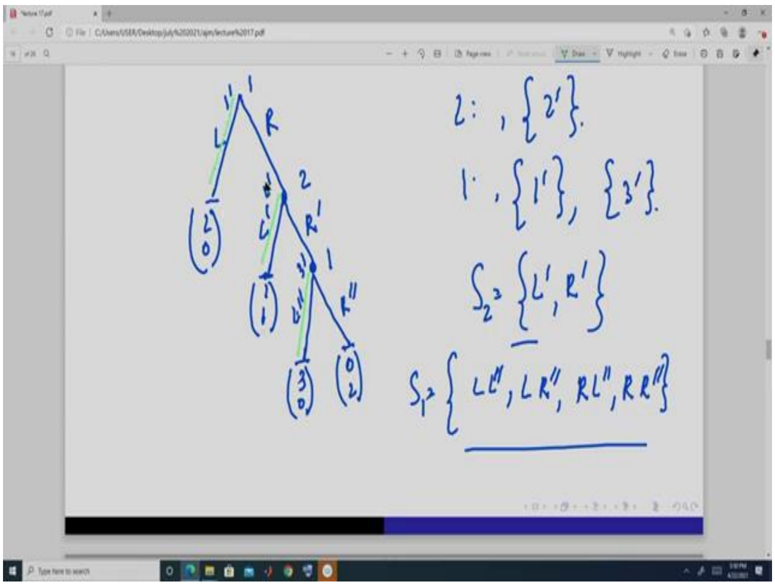

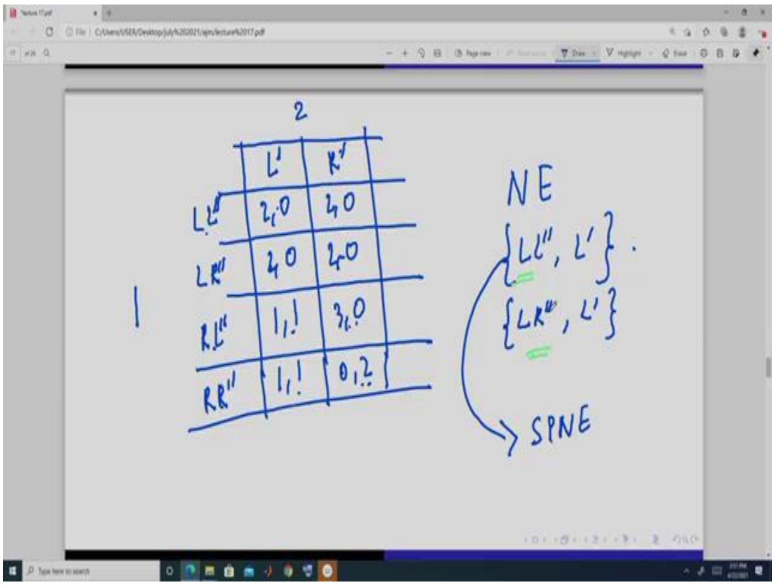

So, now let us do another example, okay, okay. So, this game has this as one subgame, this is another subgame, okay. Now, here, player 2 has only one information set. So, suppose these nodes are 1 dash, 2 dash, 3 dash and player 2's information set is only this 2 dash, but player 1's information set it has two information set that is 1 dash and 3 dash. So, it has two contingencies, player 2 has only one contingency. So, the action strategy set of player 2 is L dash and R dash- $S_2 = \{L', R'\}$. But for player 1 it is this L and next it is L dash, then again L and R dash this is for the second contingency.

We have to discuss all the possible plan, right? So, we have to specify the actions for each nodes or each information set, R, L dash, R, R dash- $S_1 = \{LL', LR', RL', RR'\}$. But here if player 1 plays L, then it is never going to choose this here, because the game ends here, but still we have to specify it. So, these are the action strategies of player 1 and these are the strategies of player 2.

So, the normal form game, it is for player 1 this is, sorry, player 2 L dash, R dash. This is L, L double dash, L R double dash, R double dash, it is this. Now, if player 1 plays L, player 2 does not get any chance to play. The game ends here. So, this for this thing if player 1, so this is for player 1, if it plays L it is going to be 2, 0, it is going to be 2, 0, because since it has played L so player 2 it is same. So, for all these contingencies we have get the same payoffs, okay.

But suppose player 1 has played R and player 2 has played L dash, L dash then it is going to be this one, 1, 1, and here player 2 is not getting any chance, because player 1 is playing R so the game has moved to the second node and player 2 is making a decision that is L dash. So, the game ends here. So, that is why the payoff, in this case, it is also going to be (1, 1) and then we will get this, okay.

Now, here if you look at this game and you have to find out suppose the pure strategy Nash equilibrium, then if player 2 plays this strategy L dash, player 1 is indifferent between these two. If it plays this he is indifferent. So, this is one Nash equilibrium- $\{LL'', L'\}$. And here if it plays this- $\{L\,R''\}$ he is indifferent and if it plays this he is indifferent between these two. So, this is also a Nash equilibrium- $\{LR'', L'\}$. These are weak Nash equilibrium.

If it plays this-R' , player 1 moves here- (3,0). If it plays this- $\{RL''\}$, it will, player 2 is going to switch here. So, this (3,0) is not a Nash. Again this is not a Nash equilibrium. If it plays this-RR'' , player 2 is going to choose this-(0,2). But if it choose R dash, player 1 is going to choose R L double dash, so this is. So, these are the two pure strategy Nash equilibrium. Out of this, we will, we have to find the subgame, which is subgame. So, this is double dash, okay. Otherwise, it will be confusing, right? Make it double dash.

So, this, in this, it is this. Here it is, if we plug in this value here it is 3, 0. So, player 2 is going to choose here. It is 1, 1. So, player 1 is, so these are the, we have already shown the subgame, Nash equilibrium of each subgames. So, here it is going to choose. In this here it is going to choose this L double dash, not L, R double dash. So, that is why this $\{L\,R''\}$ is not a Nash equilibrium. This $LL''$is a Nash equilibrium. This is a Nash equilibrium, but this is not a subgame perfect Nash equilibrium. So, this is you can say SPNE, subgame perfect Nash equilibrium, this is not, because here L double dash is not credible enough.

If you play this strategy and player 2 plays this strategy, sorry if you play this action, player 2 plays this action I will be here, player 1 is going to be again here, then it is not going to play this. This is not credible. This there, it is going to be this, because 3 is always greater than 0. So, that is why in this strategy L, L double dash, it is credible. But L, R double dash, it is not credible. So, in that sense it is a subgame perfect Nash equilibrium.

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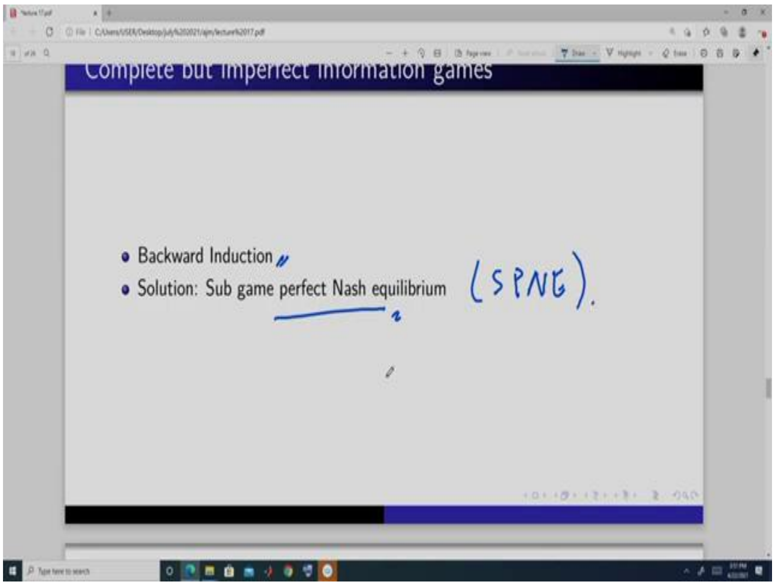

Now, let us, we have already done or talk, discuss about perfect information case, now we will do imperfect information. Now, the solution concept, we will again use the backward induction that is we will start from the lowest or the last subgame and then we will move backward and the solution concept is same as subgame perfect Nash equilibrium or SPNE. Let us do some example to understand this.

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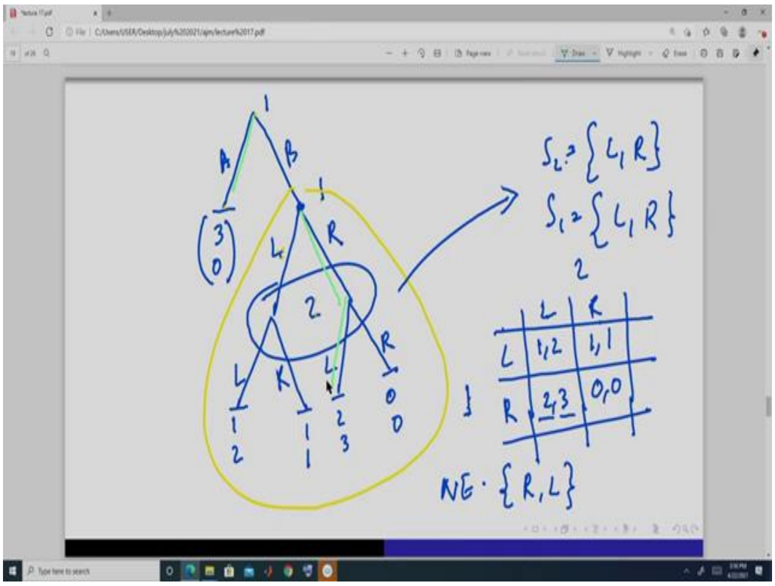

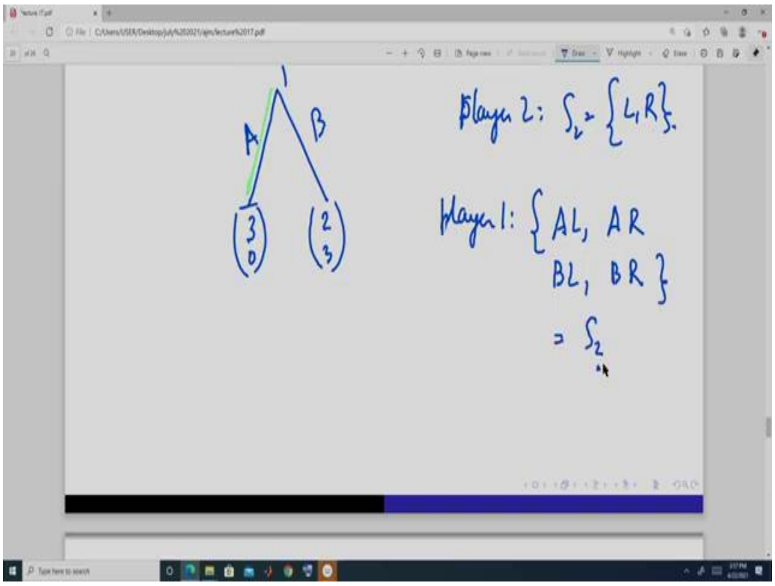

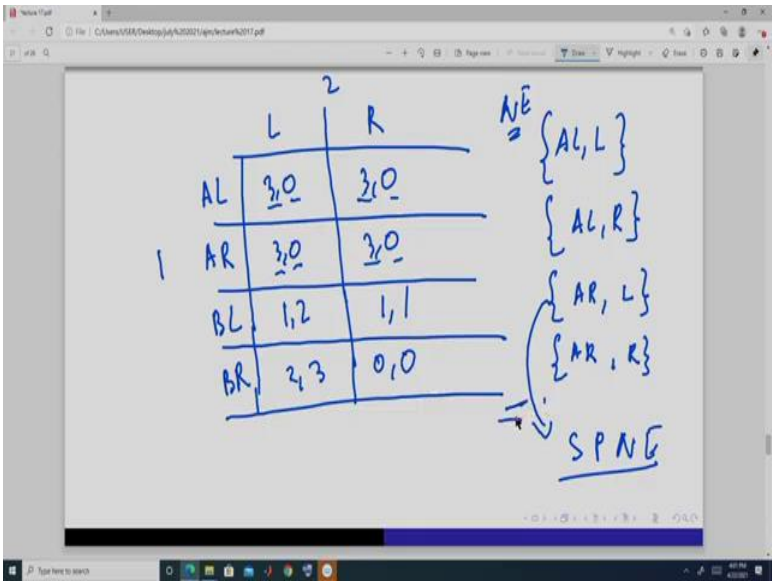

So, it is again a two player game and suppose it is player 1, it is A, it is B, again player 1, okay L, R. So, this game, this player 2 is not going to observe the action of player this one and also it is not going to observe this, okay on this is not known. Player 2 when it is moving it will know that player 1 has chosen B, otherwise it will not get the chance to move at all, okay. So, in that sense player 2 knows that player 1 has chosen this. But player 2 is not going to observe the action of, this action L R. And similarly player 1 is not going to observe the action of R, okay.

So, now, how do we solve this? So, we will start with this subgame, okay. So, this subgame is you can think this subgame as a simultaneous move game or you can say a normal game, where since this is the information set of player 2 and it has two nodes, so it is not sure. So, action set is same here. So, the here strategy set of player 2 is simply L, R, strategy set of player 1 in this subgame is again L and R. So, this game can be represented as a normal form game and it is in this way. This is for player 1, this is for player 2 actions L and R, so player 1 L, player 2 L, L, L this combination 1, 2. L, that is L and then R, this is R, 1, 1, this R, R then player 2 L, R, L, R, R, R, R. So, this is the normal form game, okay.

Now, so we play this game and we find the Nash equilibrium of this game. So, in this the Nash equilibrium is, so it is obvious that for player 2 L is a dominant strategy or R is dominated by L. So, it is going to be choosing this and so player 1 will choose this (2,3), so this is a Nash equilibrium. So, here in this game the player 1's action or a, so the Nash equilibrium is R, L, okay. Then what do we do? Again for player 1 A, B, so it is (3, 0) and this is (2, 3), so player 1 is going to choose this- A= (3,0).

So, here you can represent the Nash R and L, this, okay. But how do we specify them strategies of players. This is strategies are for only this subgame. For the whole game, the strategies are like this. For player 2 it is, a strategy set is same. It is, it has only one information set here and then it has two nodes in it and they have same. So, it is L and R. But for player 1 it is taking decision at two nodes here and here, right? So, it is going to be AL, AR, BL, BR. This is the strategy set of player 2, okay whole game.

Now, we can also represent this game as a normal form game and because we know the strategies, so you can say this is L and this is R, AL, AR, BL, BR, right? AL, AR, if it plays A then it is game ends here. So, in all these A so the payoff is same. It does not matter. Player 2 does not get any chance or player 1 does not get the second chance or second move. Here it is B and then player is choosing L and then again L, so this- (1,2). So, now here we will place this game here, payoffs here. So, it is 1, 2, okay.

Now, we can find all the Nash equilibrium on this game. What are the Nash equilibrium If it plays L, it can choose this- AL= (3,0) or this- AR= (3,0) indifferent between, but it will be here it is going to be indifferent between this, this, so this is a Nash equilibrium, again these are the Nash equilibrium. So, you can say, all these- {AL, L}, {AL, R}, {AR, L}, {AR, R} are Nash equilibrium. But are all of them subgame perfect, because here if you look at this game it is L, R, so it is L, R, only this, because L, L, L, L, L, L it is not, it is dominated by, if you play this, by this, right? this, so R.

Now, so that is why in this subgame, the Nash equilibrium of this subgame is not part of this strategy profile. Here again, this is L, R, this is also not A. So, this is all, that is why this is not a subgame, but this is R, L, R, L, R, L. So, in this subgame, it is a Nash equilibrium is L R, L. So, this is subgame. This is R, R, this R, R, it is not a Nash equilibrium in this subgame. So, that is why only this is SPNE. But in this normal form game we have four pure strategy Nash equilibrium this- {AR, L}, {AR, R}.

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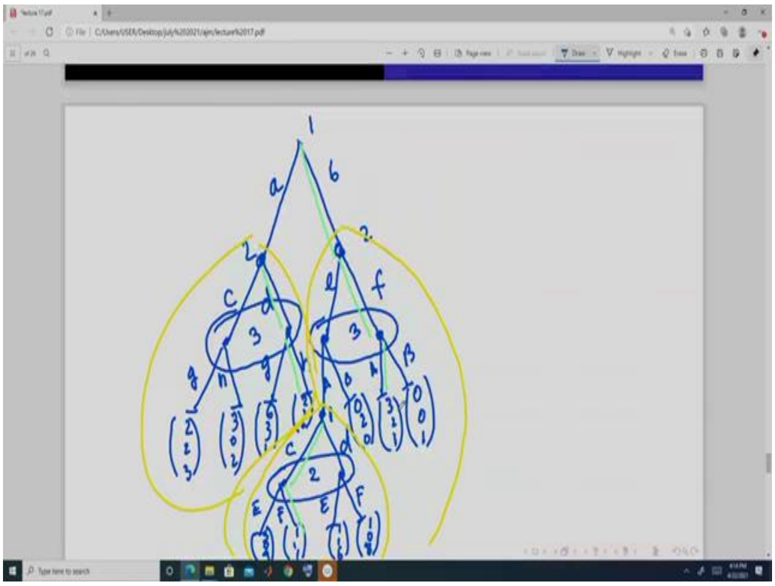

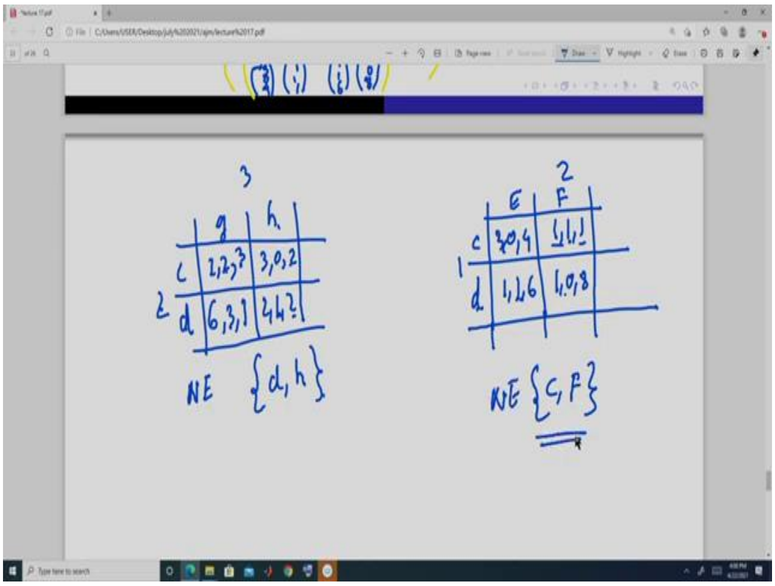

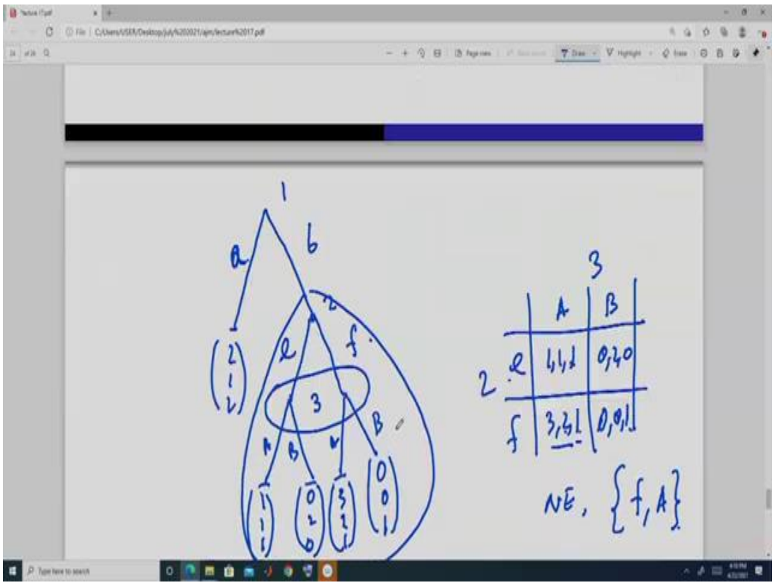

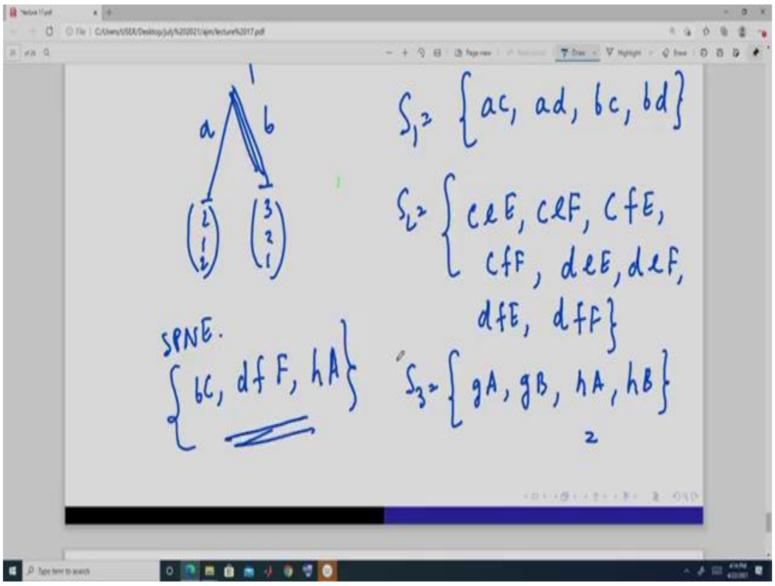

Let us do another example and this is going to be our last example and with this we will end the game theory portion and start industrial organization, okay. Suppose is this, okay so imperfect game between player 2 and 3, A, B, A, B, C, D, E, F, E, F, (3, 0, 4), (1, 1, 1), (3, 0, 4), okay this is (1, 1, 6), this is (1, 0, 8), this is (0, 2, 0), this is (3, 2, 1), this is (0, 0, 1), again this, suppose this here, this payoff here is (2, 1, 2), payoff here is (6, 3, 1), payoff here is (3, 0, 2), here is (2, 2, 3). So, in this game, if you look at the subgame, one subgame is starting from here, another subgame is starting from here and this whole, another subgame is this, okay. So, it has three subgames.

So, let us first solve this subgame. This is between player 2 and player 3. Now, this is an imperfect, because player 3 does not know that it is, what is the action chosen by the player 2, okay. So, we can, so it is c, d and g, h, payoffs are c, g, so it is (2, 2, 3), then it is (3, 0, 2), (6, 3, 1), (2, 1, 2), okay. So, it is played between player 3 and player 2. And the payoffs are, we have to compare 2, 3, this 0, 2, 3, 1, 1, 2.

So, here a player 2 plays c, player 3 is going to play h, 3 is greater than this and if player 3 plays g it is going to play d because 3 is greater than 2 and if it plays this- h(2,1,2) it is going to choose this one-2, because 2 is greater than 1 and if it plays h it is going to choose d, because 1 is greater than a, because d is actually a dominant strategy for player 2 here if you look at this. C is dominated by d. So, in this the Nash equilibrium is d, h. So, in this subgame Nash equilibrium is d, h.

Now, let us look at this subgame. This is played between player 1 and player 2. Player 2 is E, F, player 1 it is c, d, okay and the payoffs are, you can look at the payoffs from here. It will be like this. And here if you play, player 1 plays c, player 2 is going to play F, because 1 is greater than 0. If it plays F player 1 is going to be indifferent between these two. But if it plays d, player 2 is going to play E and when it plays E it is going to, so this- F (1,1,1) is a Nash equilibrium. It is a weak Nash, sorry. This (C, F) is a pure strategy Nash equilibrium.

Now, from this we can get, so the first this game becomes here for this payoff is (2, 1, 2), right? d and h and then here for player 2 if it plays, so here the outcome is c, f. So, this is (1, 1, 1) and rest we know. So, we now play this subgame, this. Play this subgame. So, here again we have two player, player 2 and player 3, player 3 A, B, player 2 it is e and f. So, if it plays e and A so it is (1, 1, 1), e and B, so it is (0, 2, 0), e, f and it is A (3, 2, 1), (0, 0, 1).

So, we have to compare the last two digits. So, for player 2, if it plays e best response for player 3 is A, because 1 is greater than 0. If it plays with A, player 2 is going to choose f, because 2 is greater than 1. If it plays here f, it is indifferent okay between 1 and 1. But if it plays this, player 2 is going to choose e, not this. So, this is a Nash equilibrium. So, here Nash equilibrium is f and A, okay.

So, this game now, a, b, a is (2, 1, 2), and this is (3, 2, 1). So, player 1 is going to choose this. So, in this game, if we want to find the path which shows the game this and then it is c and f and here it is d and h, d and h, it is d and h and then finally here it is f and A, it is f and A. So, we get the subgames in this way. In this subgame, this is the Nash equilibrium. In this subgame this is the Nash equilibrium. Then in this subgame, this is and these are the Nash equilibrium. So, finally this.

So, but here if we try to present this game as a normal form game then it will take a huge space, so we will not do that. Instead we will simply specify the strategies of each player, suppose strategy says of the whole game of each player. Player 1 has two nodes, right? this and this. So, its action set is or its ac, ad, bc, bd, i.e. $S_1 = \{ac, ad, bc, bd\}$, right? two contingencies. For player 2 it is taking a decision here, it is taking a decision here and it is taking a decision here. Here each information set is singleton, this is again singleton, but this is not a singleton. So, its strategy profile it has three nodes or three different information sets, so three contingencies.

So, these- $S_2 = \{ceE, ceF, cfE, cfF, deE, deF, dfE, dfF\}$ are the complete set of action plan that player 2 can have. For player 3 we have 2a, one is here and another is here. So, for player 3 it is the gA, gB, hA, hB, i.e. $S_3 = \{gA, gB, hA, hB\}$. So, from this we know what is going to be the SPNE from this A, it is bc for player 1, dfF, hA so this- {bc, dfF, hA} is actually subgame perfect. Why bc, because player 1 when it is choosing it is choosing here b and here it is choosing c. For player 2 you see in this it is choosing d here, it is choosing e here and it is choosing F here. So, it is df and F, dfF. So, this is this one, dfF. Then here it is hA for player 3. Player 3 here it is choosing h and here it is choosing A. So, this is a subgame perfect Nash equilibrium, okay.

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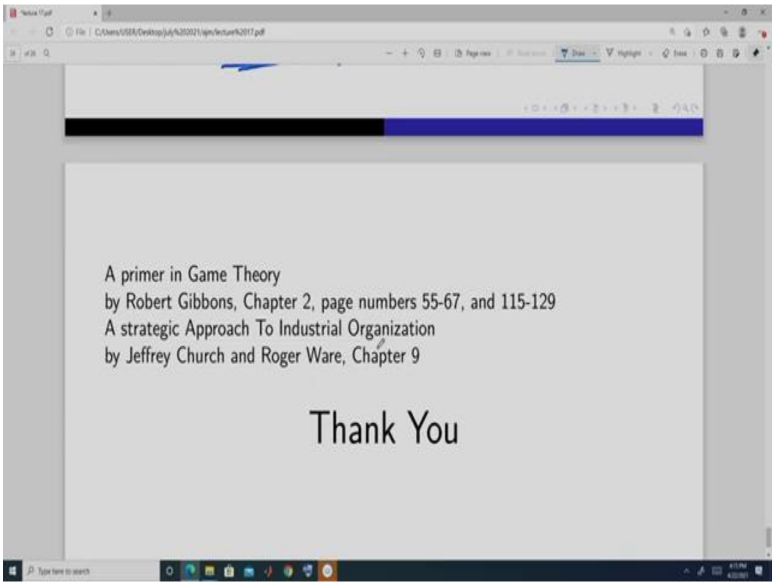

So, thank you very much and this is the, from where you can read this portion, okay Chapter 2 of Gibbons and Chapter 9 of Church and Ware, okay. So, these are the specific page numbers you can look at. Thank you again.