Introduction to Market Structures

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Module 5
Lecture 21
Tutorial on Normal Form Games

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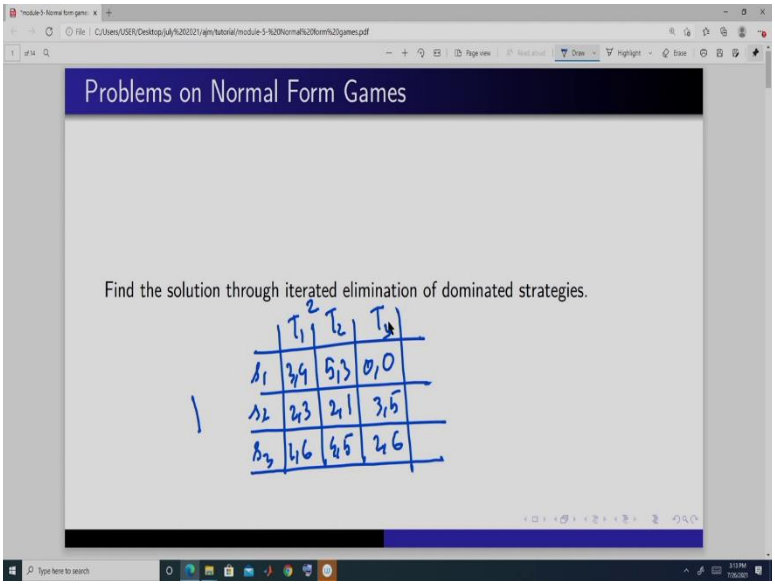

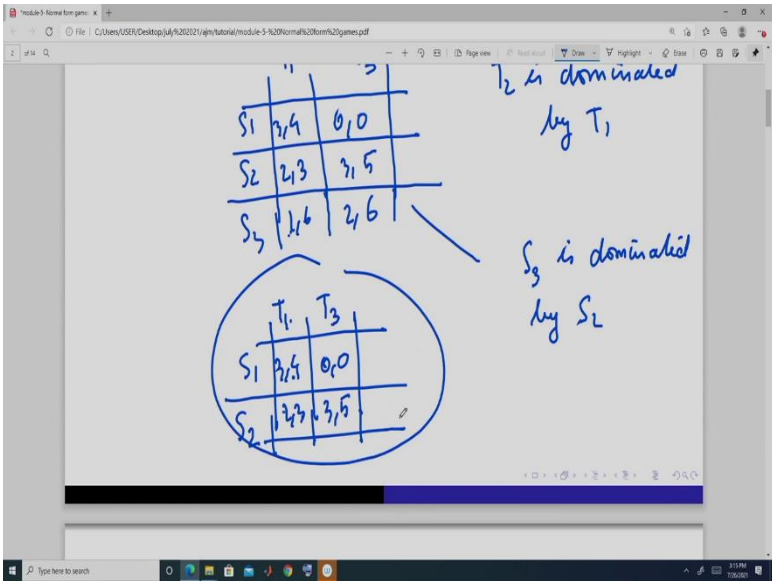

Week 5, module 5. It is Normal Form Game. So, let us solve some problem on Normal Form Game, okay. So, let us take this example S1, S2, S3, T1, T2, T3, this is player 1, this is player 2, payoffs are like this- (3,4), (5,3), (0,0), (2,3), (2,1), (3,5), (1,6), (4.5), (2,6), okay. So, this is a Normal Form Game, okay, player 1 it has 3 action that is S1, S2, S3 and player 2 has 3 action T1, T2, T3, okay. Now, we want to solve this, find the solution through iterated elimination of dominated strategies.

So, if we look at this game you will see that this 2, 2, 3, okay. 3, 4, 3, 5, 0, 3 but here it is this… in this case (2, 1), (2, 4) so that is… but if you compare T1, T2 we will say 4, 3, 3 so this can be eliminated. So, T2 is dominated by T1 so we remove T2 and the game is now like this. So, this is (3, 4), (0, 0), (2, 3), (3, 5), (1, 6), (2, 6), okay. So, here….so again here S2 is dominated by S2, okay. So, S2 actually dominates S3, S3 is dominated by S2.

So, we are left with T1, T3, S1, S2, (3, 4), (0, 0), (2, 3), (3, 5), okay. Now, here 4 is greater than 0 but 5 is greater than 3, so neither T1 dominates T3 nor T3 dominates T1 as it is (3, 2), it is (3, 2) so as here and it is (0, 3). So neither S1 dominates S2 nor S2 dominates S1. So, we are left with this, so if we use iterated elimination of dominated strategy, we will end up here only, we cannot move further, okay. So, that is why we use something like a pure strategy Nash Equilibrium to find out. And let us solve one problem related to Nash Equilibrium, okay.

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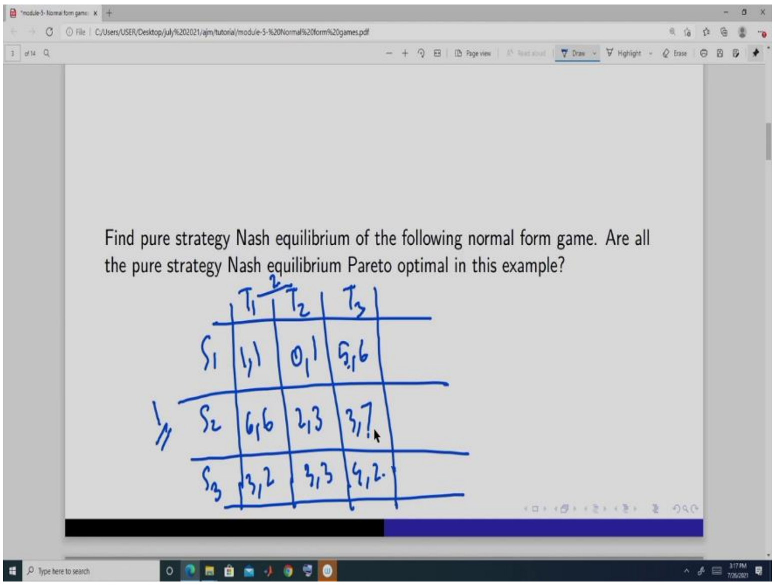

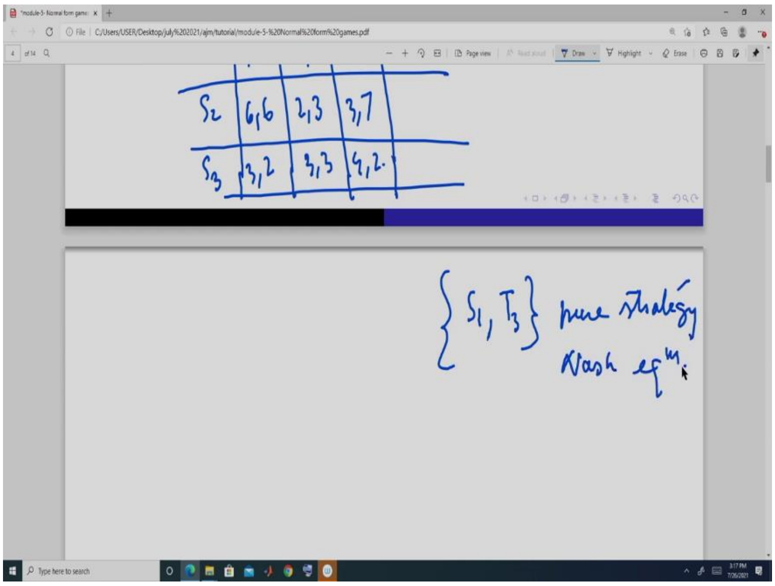

So, suppose the game is of this form T1, T2, T3, S1, S2, S3, okay? Payoffs are of this nature- (1,1), (0,1), (5,6), (6,6), (2,3), (3,7), (3,2), (3,3), (4,4), okay now if we look at this game let us S1, suppose player 1 plays S1, then you will see T3 gives the highest, right? So, if T3 is played then S1 is again best response because 4, 3, 5 if it plays S2 it is 3 and if it plays S3 it is 4. So, {S1, T3} is a pure strategy Nash Equilibrium, okay.

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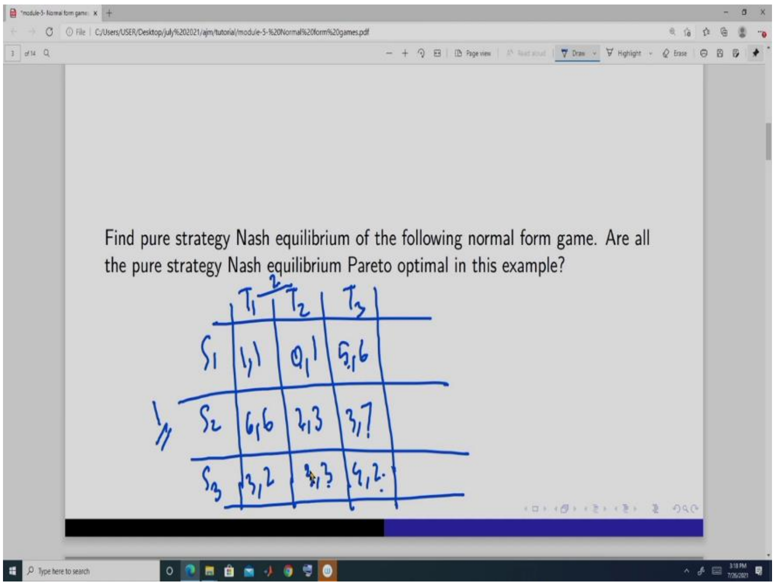

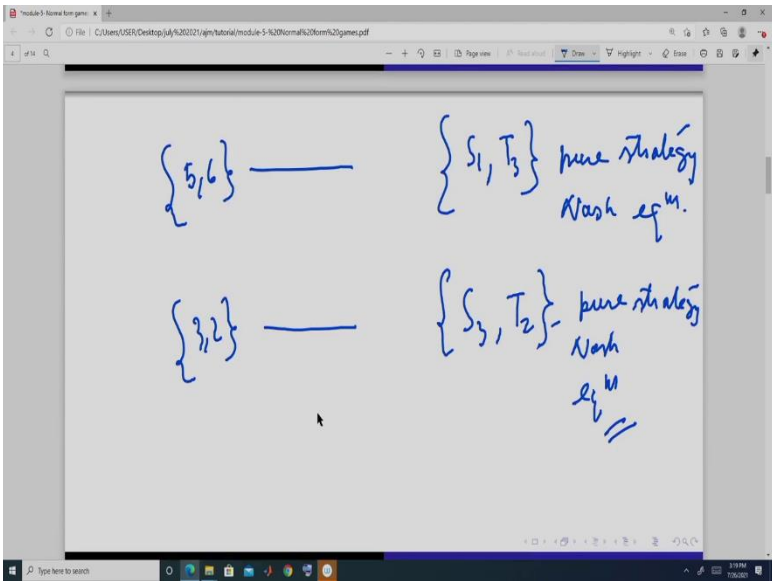

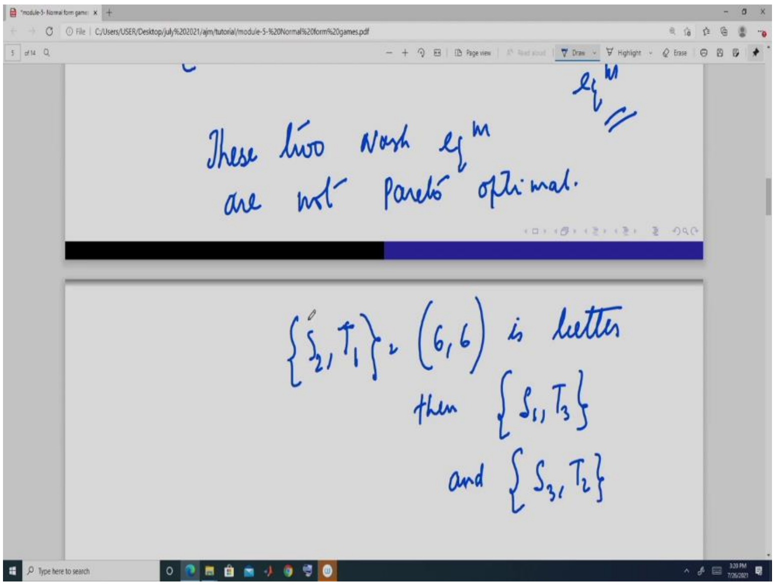

Now, let us play S2, if S2 is played then player 2 because player 2 is player 1, okay? Player 2 is going to choose this T3, if T3 is played we know it is going to choose S1 so there is no Nash Equilibrium where S2 is a strategy or action of player 1. Now, let us play S3, if S3 is played then the best response is T2 because 2, 3, 2 and if T2 is played 0, 2, 3, so S3 is going to… So, again another is {S3, T2}, this is again a pure strategy Nash Equilibrium, okay.

So, here we have two pure strategy Nash Equilibrium. Now, the next question is are all the pure strategy Nash Equilibrium Pareto Optimal in this case? So, what are the Nash Equilibrium out payoffs? One is (5, 6) when we have this $\{S_1, T_3\}$as a Nash Equilibrium another is (3, 2) when we have this $\{S_3, T_2\}$ as a Nash equilibrium. But we have a payoff like this (6, 6) we also have a payoff like (3, 7) but here this is not…. but this (6,6) is a Pareto optimal. If we move from here to here from (5, 6) to (6, 6) what we are?

We are keeping the payoff of player 2 same but we can improve the payoff of player 1. Here this is 1 so we can improve the payoff of both the player if we move from this (3,3) to this (6,6). So, that is why these two, these, these two Nash Equilibrium are not Pareto Optimal, why? Because there exists another payoff that is this (6,6) combination, combination of payoff which gives which is better than both these two Nash Equilibrium outcome. Because this S1, sorry {S2, T1} which is equal to (6,6) is better than {S1, T3}. And because these two are the Nash Equilibrium {S3, T2} and {S1, T3}, this, so that is why it is…

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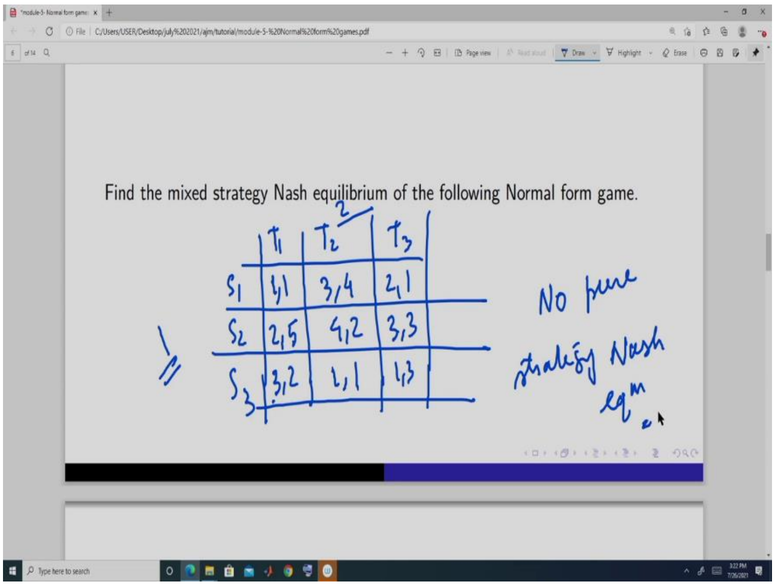

Now, let us solve one, find the mixed strategy Nash Equilibrium. Suppose the game matrix form or the normal form game is of this nature (1, 1), (3, 4), (2, 1), (2, 5), (4, 2), (3, 3), (3, 2), (1, 1), (1, 3), okay. Now, if you look at this if S1 is played then this is best response T2, T2 is played S2 is the best response, if S2 is played T1 is the best response, if T1 is played S3 is the best response, if S3 is played T3 is the best response, if T3 is played S2 is the best response, if again S2 is played then T1 is the… so we see there is no pure strategy, this is for player 2, this is for player 1, okay. No pure strategy maximum, so let us find the mixed strategy now.

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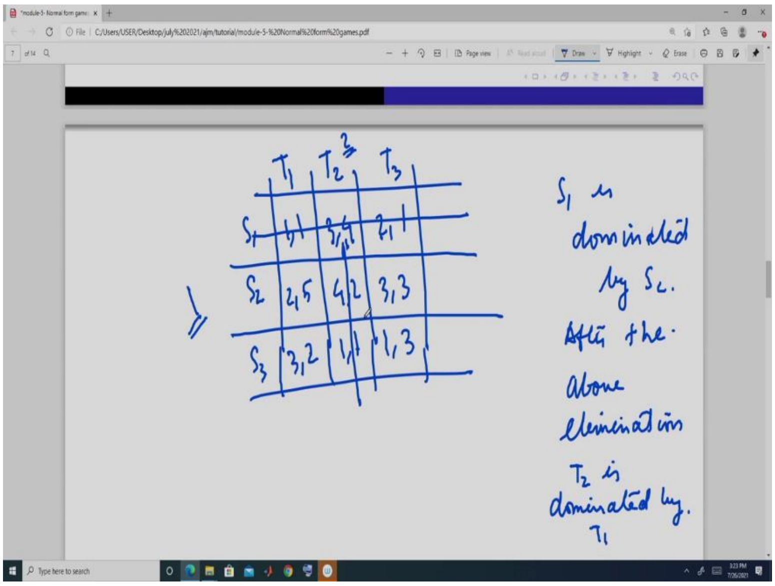

Now, here so we have found that there is no pure strategy Nash Equilibrium, okay. Now, here if you compare 1, 2, 3, 4, 2, 3 so this dominates… so S1 is dominated by S2 so remove this eliminate this, now we are left with this here you will see 5, 2, 2, 1. Again this is (3, 2), (1, 3), so again T2 is dominated by T1 we are left with this.

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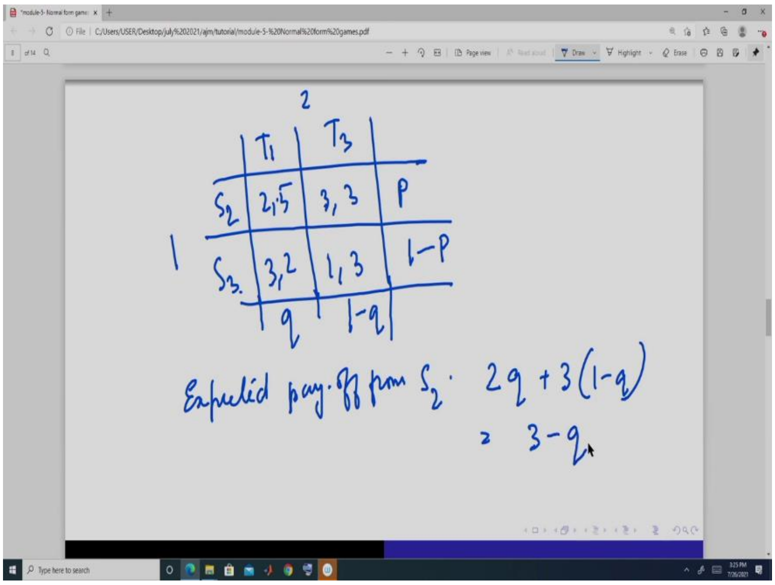

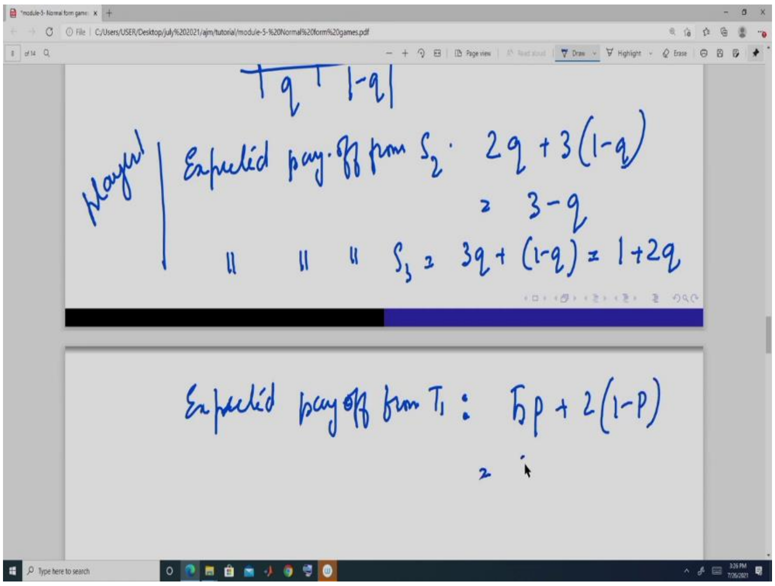

So, what is the game we are left with, we have T1… now here if S2 is played then T1, if T1 is played the best response is to play S3, if S3 is played best response is to play T3, if T3 is played best response is S2, so again no pure strategy. So, suppose player 1 attaches probability P to this S2, action and 1 minus P to this action- S3, and player 2 attaches Q to T1 and 1 minus Q2 T3, okay. So, the expected payoff from S2 is 2 Q plus, so this is equal to 3 this and again expected payoff from S3 is 3Q plus this- $S_3 = 3q + (1 - q)$. So, this is- 1+2q this…

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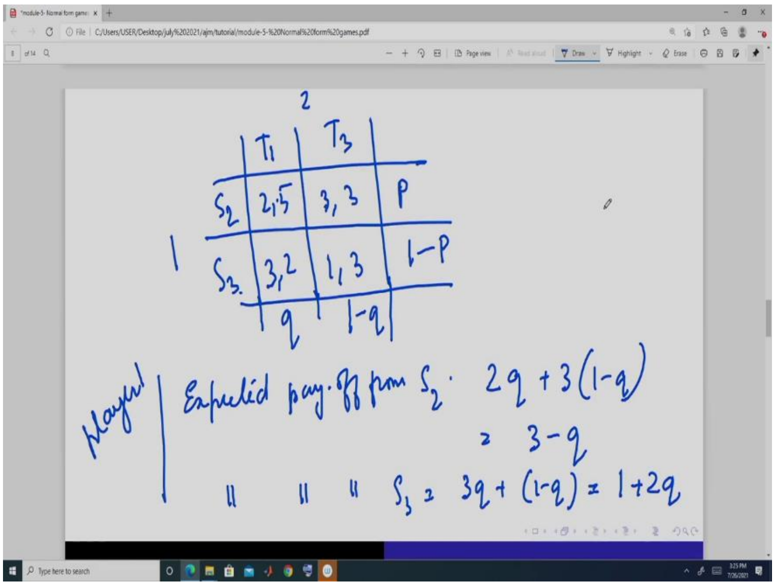

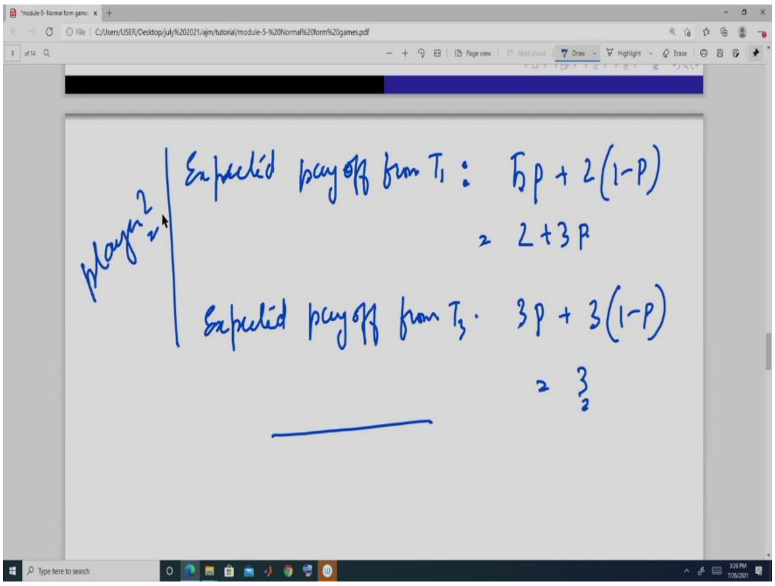

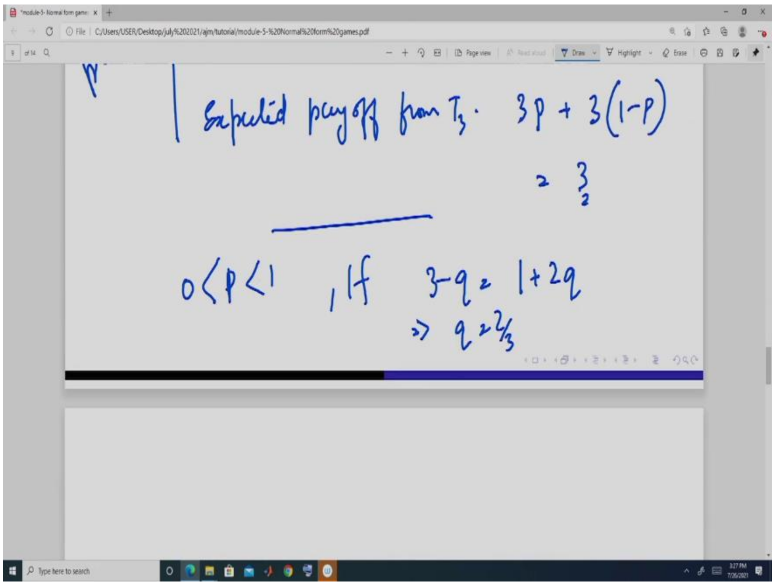

Now, if you look at this expected payoff this is for player 1, expected payoff from T1 is, T1 is, this-$T_1 = 5p + 2(1-p) = 2 + 3p$. Again, expected payoff from T3 is T3 is (3,3) it is same 3P plus T1 minus so it is 3- $T_3 = 3p + 3(1-p) = 3$. Now, we know player 1, this is for player 2. Now, we know player 1 will attach some positive probability that is P will be some positive value that is P will lie between 0 and 1, if these two are equal, i.e $3 - q = 1 + 2q$ only when… it is this… so this is equal to Q is equal. When Q is equal to 2 by 3 then P takes a value lying between 0 and 1, okay.

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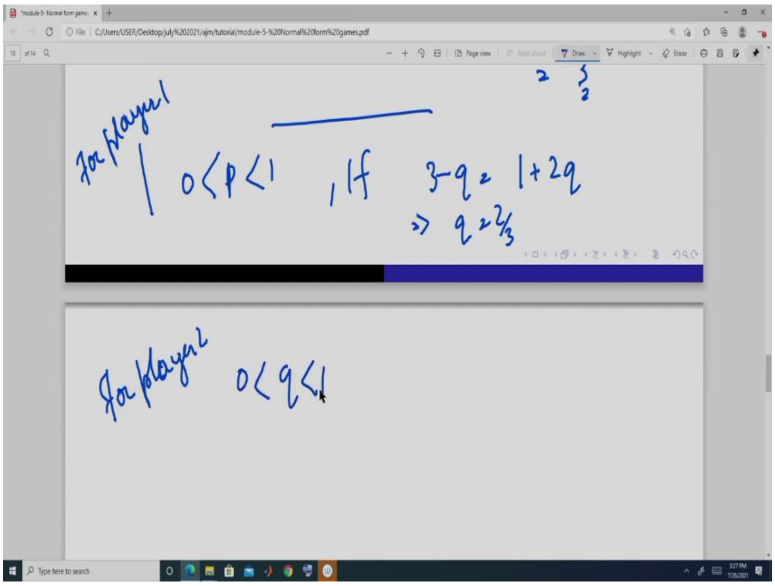

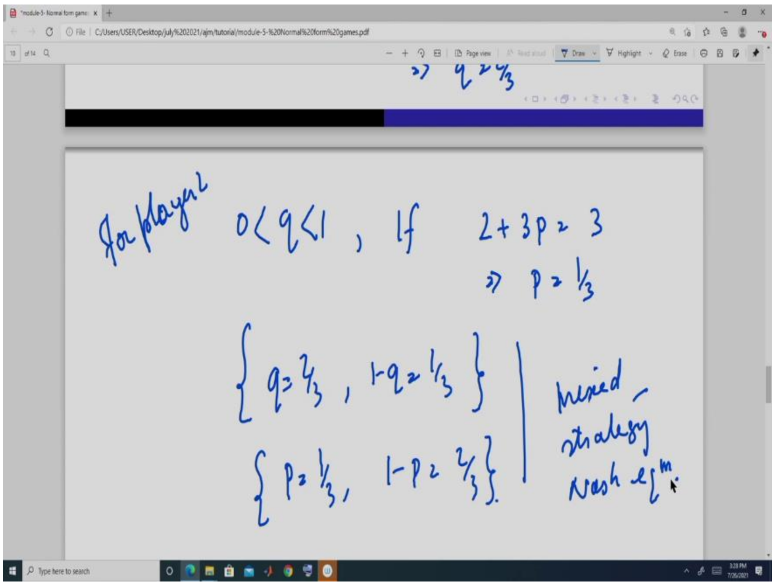

Now, when Q, this is for player 1, for player 2, Q takes a value this- 0<q<1 if these 2 are equal- 2+3p=3, so that means 2 plus 3 P is equal to 3. So, this means P is equal to 1 by 3. So, Q lies between here, so this implies that Q is equal to 2 by 3, 1 minus Q is 1 by 3 this- $\{q = \frac{2}{3}, 1 - q = \frac{1}{3}\}$ when P is equal to 1 by 3 and 1 by P, i.e $\{p = \frac{1}{3}, 1 - p = \frac{2}{3}\}$, okay. And this is the mix strategy Nash Equilibrium, okay. Thank you!