Introduction to Market Structures
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Module 05: Game Theory
Lecture 18
Pure Strategy Nash Equilibrium

Hello. Welcome to my course Introduction to Market Structures, and we were doing Game Theory and we have done the first solution concept that is iterated elimination of dominated strategies.

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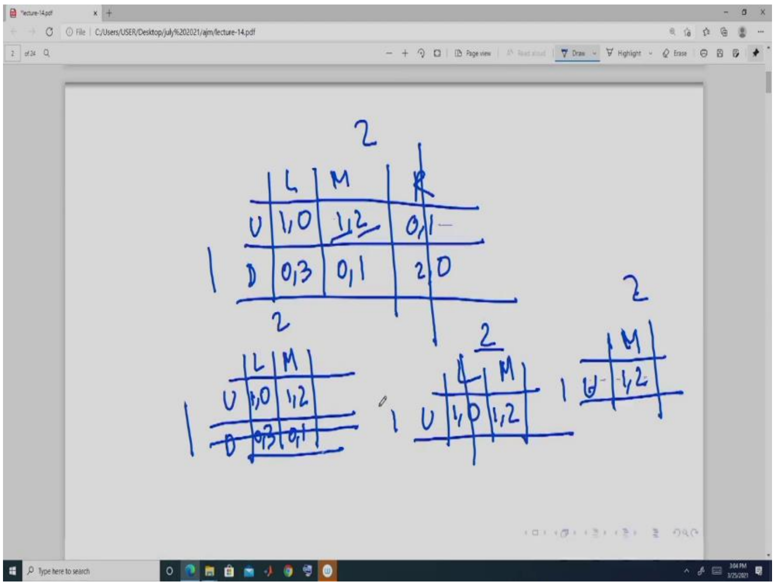

And let us recap through one example and then we will start the new thing today. So, suppose we have a two-player game. Player 1 is in the row. It has two actions or two strategies U and D. Player 2 is the column player and it has 3 strategies or action that is L, M, R. And payoffs are like this, okay. Now, what do we do in a iterated elimination of dominated strategies? From here we see that M dominates R or R is dominated by M for player 2. Why? 2 is greater than 1.1 is greater than 0. So, we can remove this, because player 2 will never going to choose R when M is available, okay. So, our game is now of this form.

Now, if look at this 0, 2, 3, 1. So, neither M dominates L nor L dominates M. But if we look at U and D; (1, 0); (1, 0). So, U dominates D or D is dominated by U. So, we can remove this. So, we are eliminating this action of player 1 that is D. When U and D are available player 1 is always going to choose U because for each action of player 2 U gives greater payoff than D.

So, the game now is, is this. And if we look at this here L is dominated by M because 0 is less than 2. So, we leave this. So, we… so through this process of elimination we come to the conclusion that player 1 is always going to choose U and player 2 is always going to choose M. So, this is going to be the outcome of this game. And this whole process of elimination is going to be taking place in the minds of this player. And when the game is played only once and simultaneously each of them are going to choose.

Player 1 is going to choose U. Player 2 is going to choose M, because player 2 will do this elimination of dominated strategies, and similarly player 1 is also going to do this, okay. Now, here and we have done weakly dominated strategy also in the last class.

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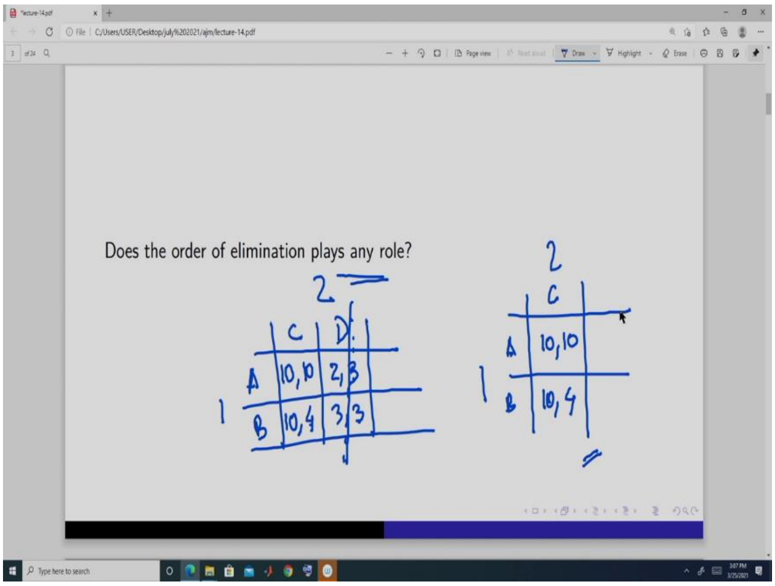

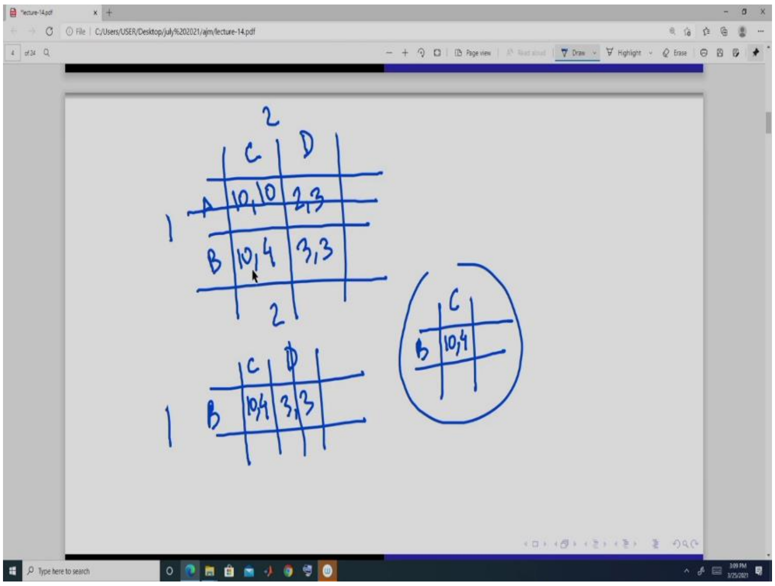

Now, the question is does the order of elimination plays any role? Means if we start the elimination from the strategies of player 1 then do we get the same outcome as when we start the process of elimination from the strategy set of player 2. So, let us do one example and then see what happens. Suppose we take a game of this form. Player 2 is again in the column and its action is C and D. And player 1 is in the row and its actions are A and B. Or you can say these are the strategies, okay. And payoffs are suppose (10, 10); (2, 3); (10, 4); (3, 3). Now, here if you look at this A. Now, let us start the game from, in this way. Suppose player 2 we eliminate, so we know that C; D is dominated by C because 10 is greater than 3 and 4 is greater than 3. So, we can eliminate this, this action or strategies of player 2.

So, the game is now; if we eliminate this… so now only one strategy for player 2 but there are two actions and we cannot eliminate because it is weakly dominated. They are same. A is not dominated by B and neither B is dominated. So, we end here. So, we do not get any precise result, okay. So, the outcome is that if we start from the player 2, if we start the elimination process from the actions of the player 2 then we end up here. Now, let us take the same example and let us start the process of elimination from the action set of player 1, okay. Now, here if you look at this player 2 and player 1. So, for player 1 strategy B weakly dominates A or A is weakly dominated by B. So, we can remove this. So, we are left with this.

Now, we can see from here for player 2, D is dominated by C so we can remove this and we end up in this outcome, okay. So, if we start from the elimination of the strategies of player 1 then we get a precise outcome and that is player 1 chooses B and player 2 chooses C. But if we start the elimination process from the strategy sets or action sets of player 2 we remove this first and then we end up in this situation. And then we cannot remove. So, we, this is an imprecise here.

So, from here we can say that order of elimination from how we start the elimination process, from whose strategy set or action set; it actually determines the outcome that we are going to end up. And it is true only in the case of weakly dominated strategies. So, if we have weakly dominated strategies then we will face this problem, otherwise we will not face this problem, okay.

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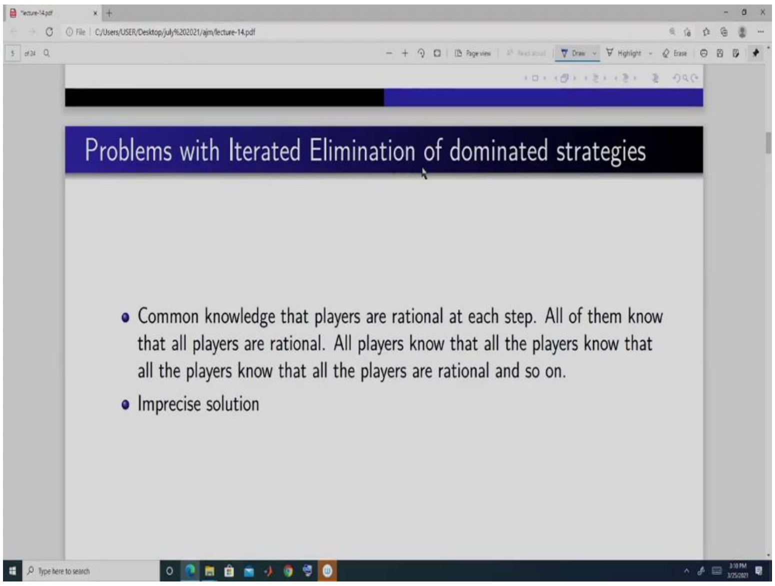

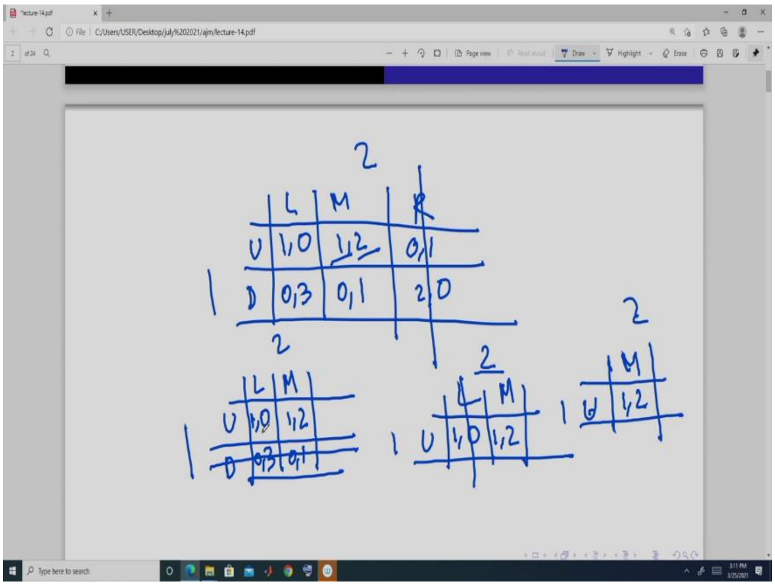

Now, the next important thing here is what is the problem in the, this solution concept that is iterated elimination of dominated strategies? So, first while doing this elimination, when we have done this example, so what we have done here? We have done that suppose player 2 eliminates this because R is dominated by M. Then player 2 eliminates… if suppose player 2 while thinking about which strategy to play, first it will eliminate this. Then it will eliminate this and then it will eliminate this. And then we will be at this.

Similarly player 1 while choosing U will first eliminate this, then eliminate this and then eliminate this and then end up in this sequence. So, what is happening? So, first player 2 is assuming that player 1 knows that player 2 is going to remove this. Then player 2 is again making an assumption that player 1 is rational and that it knows that player 1 knows that 2 is rational, so that is why it is removing this.

And again, player 2 again in the next step it is assuming that the player 1 again going to behaving in a rational way and so player 2 again knows that player 1 knows that it is going to behave in a rational way, because player 1 again knows that player 2 is rational, so player 2 is also going to behave in a rational way so player 2 is going to remove this. So, in this way we have to assume rationality in a sequence of rationality, okay.

So, this is a very strong assumption. And so this is, and also we call it as a common knowledge that; common knowledge is that the all players behave in a rational way, and in each step they are behaving in a rational way. So, this is actually a very strong assumption which we have implicitly made in this.

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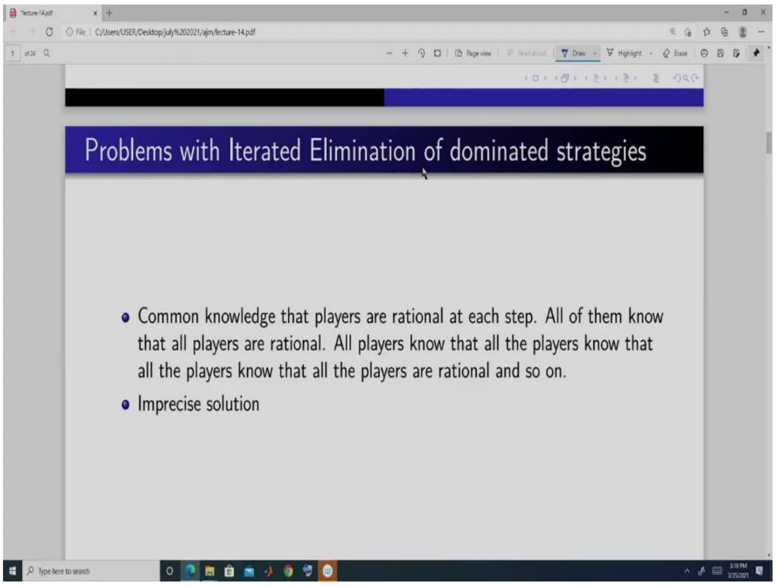

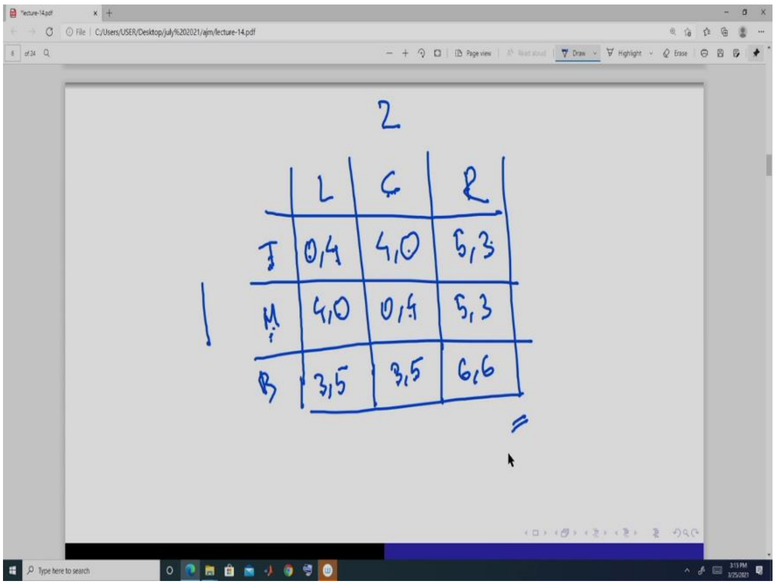

Another problem is that we always, we may get an imprecise result, that we may not be able to eliminate any strategies. Now, let us take an example of this. And the example is this. Suppose there are two players; player 2 which plays the column, player 1 which is a row player, okay. And suppose the payoffs are, for each combination of… So, these are the payoffs for each combinations of outcome. So, in T, L 0 to player 1, 4 to player 2 like this we will have all these 8 outcome, 9 outcomes and all these payoffs are defined.

Now, if we look at this game player 1 has 3 strategy or 3 action T, M, B. Player 2 has 3 strategies or action L, R, N. Now, if you compare this R and C, (0, 3); (4, 3); (5, 6). This does not dominate this neither this dominates this, okay, because 4 is greater than 3. 5 is less than 6 and 0 is less than 3. Again if you compare this (4, 0); (0, 4). So, we cannot say L is dominated by C or C is dominated by C. Now, compare L and R. 4, 3 greater than this. This is less than this. So, we cannot eliminate any of the strategies of player 2 in this here.

Now, look at the player (1; 0), 4; (4, 0). So, we cannot say T is dominated by M or M is dominated by M. Now, compare this (4, 0; 3). So, neither M dominates by B nor B dominates M. Now, compare T, B; (0, 3); (4, 3). So, neither B dominates T nor T dominates B. So, here if we use this solution concept that is iterated elimination of nominated strategies we cannot choose any action or any strategy. So, player 1 and player 2 is going to stuck here. They cannot eliminate any strategies. So, they will not be able to choose any actions. So, we need to improve our solution concept.

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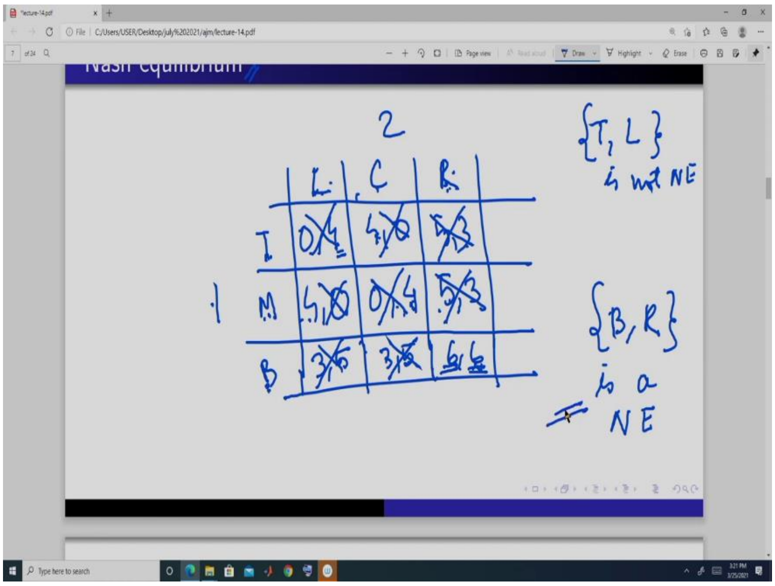

So, here comes this solution concept, and that is Nash equilibrium, okay. And we will find the Nash equilibrium of this game, okay. So, let us take this game again. L,C, R for player 2; this is T, M and this is B. (0,4); (4, 0), (5,3); (4, 0), (0,4); (5, 3); (3, 5); (3, 5); (6, 6), okay. Now, here suppose player; how do we play the game if we are using Nash equilibrium concept? So, player 1 thinks that suppose I choose T. Then what player 2 is going to do? So, player 2 will compare; if he plays L it will get 4; if he plays C it will get 0; if he plays R it will get 3. So, the best response for this T is to play this, that is L because 4 is greater than 0 and 3.

Now, suppose player 2; player 1 things that if I played T then player 2 is going to choose L. Now, if player 2 is going to choose L then what is my best response? So, best response is not T but it is M. So, therefore this T and L is not Nash equilibrium, okay because if player 2 plays

L I am not going to choose T; instead I am going to choose M. So, this cannot be a Nash equilibrium, okay.

Now, so we can also strike off this and this outcome because whenever player 1 plays T player 2 chooses L not C nor M. So, these are not an outcome, right. Now, here if you look at this, if player 2 plays this L, player 1 chooses M. So, we can also remove this. So, this is not a Nash equilibrium. This outcome is also not a Nash equilibrium. This is not a Nash equilibrium. This is not a Nash equilibrium, right.

Now, suppose player 1 chooses M, suppose. Then player 2, the best response, what is the action that gives player 2 the maximum payoff? It is 0, 4, 3; out of this it will always choose C because 4 is highest. So, player 1 when it chooses M, best response for player 2 is to choose C.

Now, when player 2 chooses C best response for player 1 is to choose T because T gives him maximum; 4 is greater than 0 and 4 is greater than 3, this. So, this (0,4) is not a Nash equilibrium, okay. And we know this is already not a Nash equilibrium; because when player 1 chooses T, the best response of player 2 is to choose L. What is the best response? The strategy or action which gives him or her the maximum payoff.

Now, suppose player 1 chooses B. If he chooses B; 5 from playing L, 5 from playing C and 6 from playing R. So, player 2 chooses R. And if player 2 chooses R; player 1 if it plays T it will get 5, if it plays M it will get 5, if it plays B it will get 6. So, this is maximum. So, it is this. So, when player 1 chooses B, player 2 chooses R. And when player 2 chooses R player 1 chooses B. So, this outcome, this is Nash equilibrium, okay.

Now, here you know when player 2 is playing R player 1 is choosing B not M. So, this is not a Nash equilibrium. When player 1 is choosing B player 2 is choosing R not C. So, this is not a Nash equilibrium. And we also know that this is not a Nash equilibrium because when player 1 chooses M best response is to play C for player 2 because …

So, in all these outcomes we see that there is a tendency for any one of the player to deviate. So, if… but here we see that there is no tendency to deviate. So, if player 1 chooses by B, player 1 is going to choose R because it gives him the maximum payoff. And if player 2 chooses R player 1 is going to choose B because it gives … So, that is why this is a Nash equilibrium. So, this is the idea of Nash equilibrium, okay.

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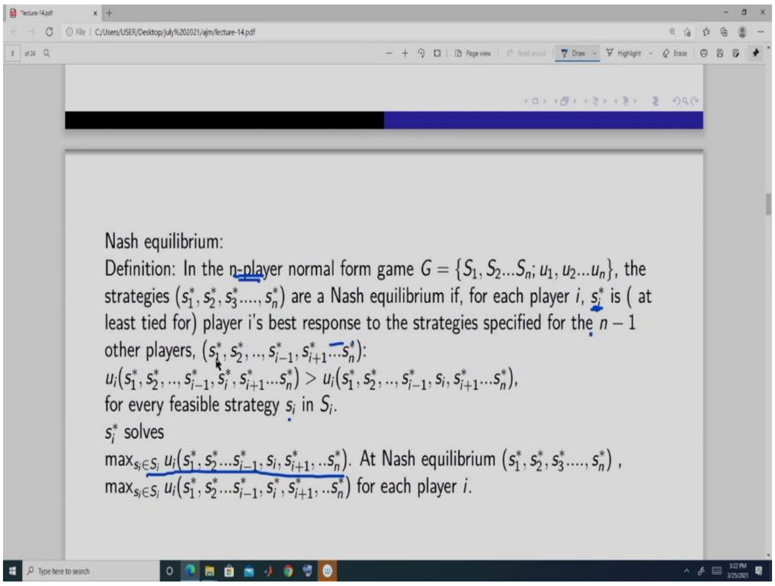

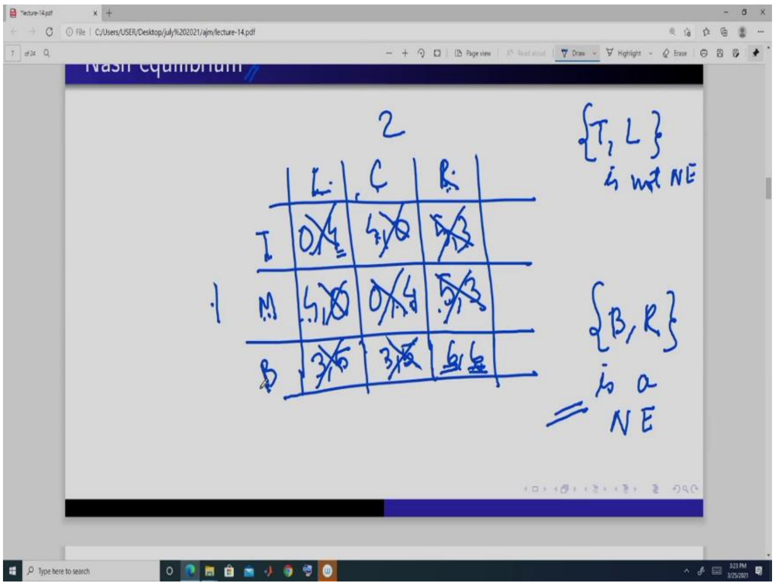

So, formally we define Nash equilibrium in this way. In a normal form game where there are n players, so the game is represented in this way- $G = (S_1, S_2, ... S_n. u_1, u_2 ... u_n)$; so there are n players, each player has a strategy space or action space given by S, capital S, and there is a payoff defined for each of this outcomes u1 for player 1, u2 for player 2 like this.

So, the strategy this- $(S_1^*, S_2^*, ... S_n^*)$, so there are many strategies like each for one player, so there are strategies; are a Nash equilibrium, it is Nash equilibrium if for each player i, si that is here is at least tied for, is player i's best response to the strategy specified for the n minus 1 other players. So, here if you look at these strategies, so these are the strategies of all other players except for the i.

Now, here if all others are playing this then player i should play this strategy. So, it means that the payoff from paying this si star is more than any other element that is there in S other than star, okay. If we plug here si star then it should be equal. But for any other it should be greater, okay for every feasible strategy si in this. So, it means that si solves this problem, that it maximizes the utility given all others are playing this strategy, okay. And it should be true for all players.

So, when player 1 is choosing so si star, si star should maximize the payoff of player 1 when all other players like player 2, player 3, player 4 they are choosing… like player 2 is choosing s2 star, player 3 is choosing s3 star, player 4 is choosing s4 star, like this, okay. So, it is actually a maximization problem for each player, okay.

So, as we have done here like when this is played by player 1 what is the action of player 2 that gives him the maximum payoff? It is R. And when player 2 chooses R it is B which is… So, we have fixed this and we have found the best strategy of player 2 and it is R. Now, we have fixed R and then we have seen whether what is the best strategy for player 1 and it is B. So, that is why this is a Nash equilibrium, okay. So, in a Nash equilibrium we see that there should not be any tendency to deviate, okay.

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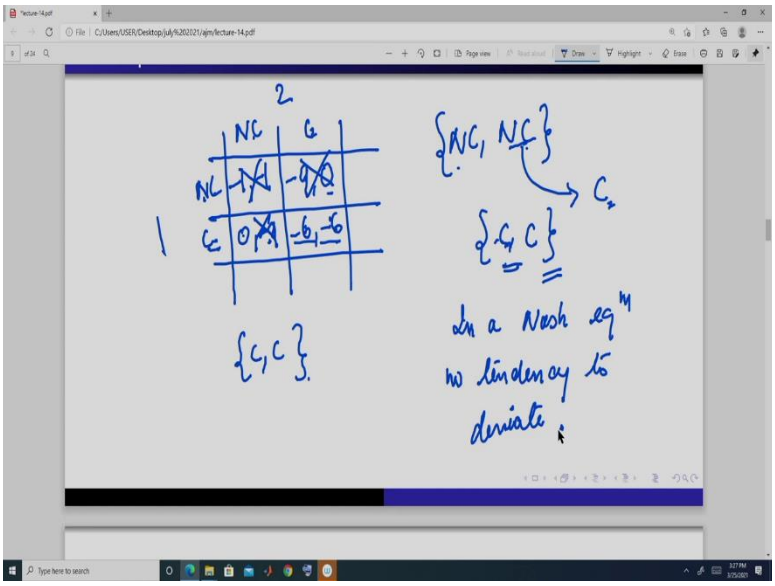

So, now let us do some example. So, let us do the Prisoner's Dilemma example. So, Prisoner's Dilemma we have already done. This is for player 1 and this is for player 2. This, now here, if player 1 chooses NC that is not confess then player 2 will compare minus 1 and 0. So, 0 is greater. So, it is going to choose this C. When player 2 chooses C player 1 will compare minus 9 and minus 6, because if he chooses NC it will get minus 1 and if it chooses C it will get minus 6. So, it will choose this.

So, this is a Nash equilibrium because if we choose, player 1 chooses this player 2 is going to choose minus, compare minus 9 and minus 6 and it will get this. So, one Nash equilibrium is this. Now, from here it is obvious that this is not a Nash equilibrium and here it is obvious because if player 2 chooses C it deviates here. This is also not a Nash equilibrium because if player 1 chooses C player 2 chooses here, C. And this (-1, -1) is not a Nash equilibrium. Why? Because when player 1 chooses NC player 2 chooses C, so not this.

So, what do we see? That if we take this outcome {NC, NC}, suppose. So, player 1 is choosing NC player 2 has a tendency to deviate from NC. And it will deviate to C. But here if you look at this, when player, and when player 2 is choosing NC, is choosing C, player 2 is suppose choosing C here then player 1 is not going to choose this but it is going to choose this {C, C}, okay. So, we get this {C, C} outcome. So, this is a Nash equilibrium.

But if you look at this here when player 1 is choosing C, this; player 2 if it is choosing C it has no tendency to switch from C to NC. And when player 2 is choosing C and player 1 is choosing C, player 1 has no tendency to switch from C to NC. So, here we see that in a Nash equilibrium no tendency to deviate, okay. So, none of the player has any tendency to deviate from it, okay. So, that is one another criteria to check whether outcome is Nash equilibrium or …

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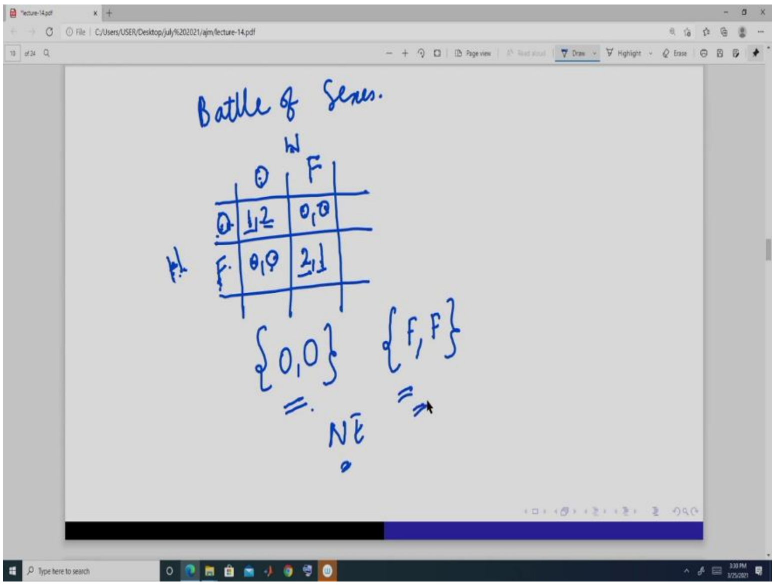

Let us do another game and that is, this is very which is called Battle of Sexes. Suppose there are two, there is a couple, husband and wife. So, this is husband and this is wife. They have to choose from two actions that is either they can go to watch an Opera or they can go and watch a Fight, Opera and Fight, okay. So, husband prefers Fight and woman prefers Opera. So, the payoffs are of this nature.

So, if both of them are in Opera Opera, woman the wife gets 2 and husband gets 1. If husband is in Opera and wife is in Fight since they want, prefer to be together so they get (0, 0) payoff. If the husband is in Fight and women is in Opera they again R not in the same place together. So, they get (0, 0). But if both of them are in Fight husband gets more payoff than wife.

Now, here you will see that if player 1 plays, if husband plays Opera best response for the wife is to choose Opera, and if wife, because 2 is greater than 1. If wife chooses Opera best response for husband is to choose Opera because 1 is greater than 0. So, one Nash equilibrium is this, this {0,0}, okay.

Next, so this is one Nash equilibrium. Now, if husband wife, or husband chooses F, best response for the wife is to choose Fight because 1 is greater than 0. If wife chooses Fight best response for husband is to choose Fight because 0 is less than 2. So, again this is an… So, we have two Nash equilibrium in this game, in this Battle of Sexes game, okay.

And here you can see that if we are in this outcome player 1 choosing Opera and wife also choosing Opera then there is none of the player has any tendency to deviate because player 1, if wife has chosen Opera and husband has also chosen Opera. Husband does not have any incentive to choose, shift from Opera to Fight because 1 is greater than 0. If you are looking at this, this outcome, if player, if husband chooses Fight; player 2, that is wife has is also choosing Fight so the outcome is this, and it has no tendency to shift to O because 1 is greater than 0. So, this, so we can see that at Nash equilibrium, so we will see no tendency to deviation, of deviation.

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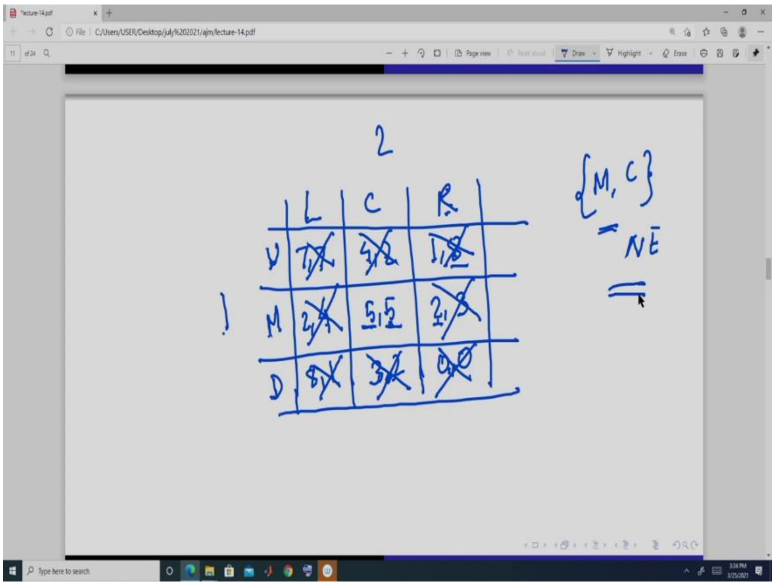

So, let us do another example. So, let us take a slightly bigger game. So, there is player 2 and its actions are L, C and R. Player 1, its action is U, M and D, okay. Now, we have to find the Nash equilibrium. How do we do? Suppose player 1 is playing U, okay. So, this is again a simultaneous move single shot game. So, each player is going to choose only once and they are going to choose their actions simultaneously and only once, okay.

So, player 1 while choosing which action to choose, it will do this mental calculation. So, player 1, if chooses U, best response for player 2 is to play R because 7, 2, 8; 8 gives the maximum. So, it is choosing R. Now, if player 2 chooses R best response for 1 is to choose M not U. So, this is not a Nash equilibrium. So, it is going to choose this, okay. Now, if player 1 chooses M, player 2 best response is to choose C because 4, 5, 3; out of that 5, C gives the maximum. So, it will choose 5.

And if player 1; so if player 2 chooses C, best response for player 1 it will compare 4, 5 and 2. So, that is, U gives 4, M gives 5, D gives 3. So, player 1 is going to choose M. So, this is a Nash equilibrium; {M, C}. Because when player 1 chooses M best response for player 2 is to choose C. When player 2 chooses C best response for player 1 is to choose M. So, this is a Nash equilibrium.

Now, we have to check whether there exist any other Nash equilibrium or not, okay. So, from here we know that this {4,2}is not Nash and this {3,2} is not Nash, okay. Again, we know already this is not; because if player 2 chooses M it chooses this. So, this {2,4} is also not a Nash equilibrium. If player 1 chooses D, best response is to choose this {8,1} for player 2. So, this is not a Nash equilibrium. If player 1 chooses U we know already it is choosing this. So, this is not a Nash equilibrium because U, L there is a tendency for player 1 to deviate, for player 2 to deviate from L to R if player 1 chooses U.

Here this is again not a Nash equilibrium. Why, because if player 1 chooses D player 2 has a tendency to deviate from this outcome and it will choose C. So, there is unique Nash equilibrium and that is M, C. Player 1 is always going to choose M and player 2 is going to choose C, okay. So, these are some of the examples that we are doing try to understand he Nash equilibrium, okay.

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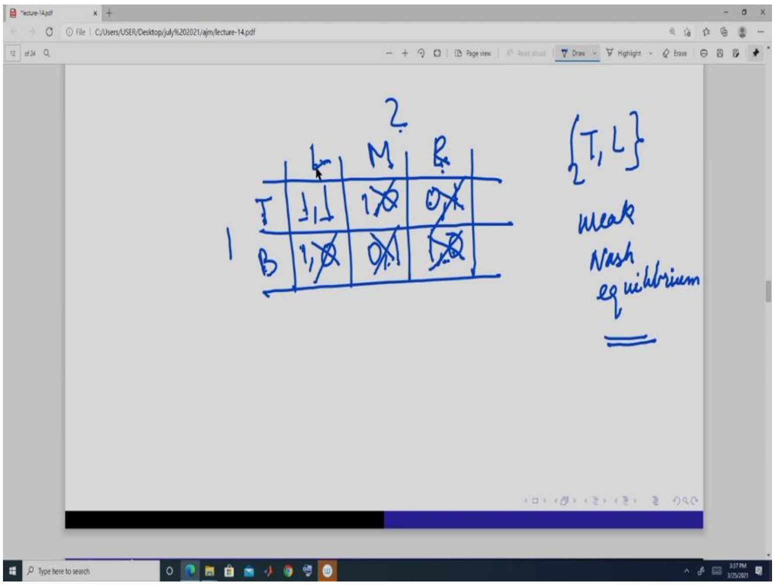

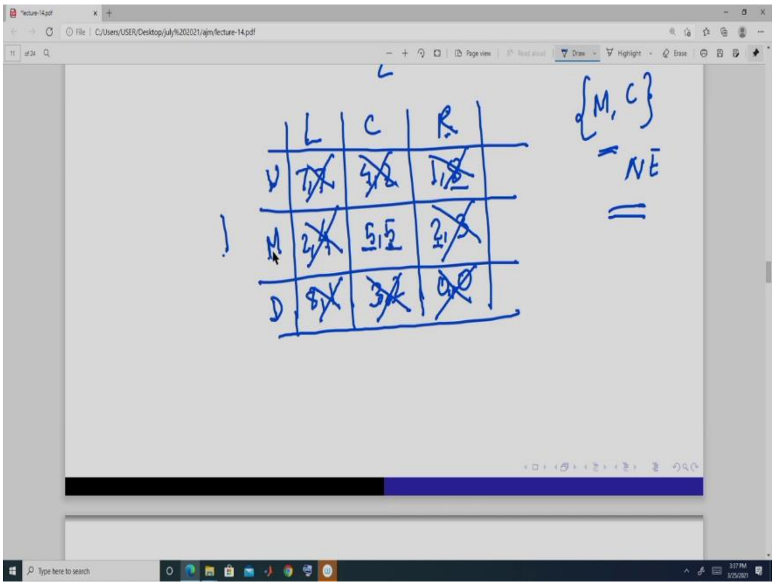

Now, let us do one more example and then we will … So, this is for player 1. This is for player 2. L, M, R and this T and B. Now, here notice if player 1 plays T best response for player 2 is to L 1, R 1. So, player 2 is indifferent between L and R. It is greater than M but it is same, okay. Now, suppose player 2 chooses L, then player 1 is again indifferent between T and B because {1, 1}, okay.

So, this {T, L} you can say is a weak Nash equilibrium, okay; because when player 1 plays T, player 2 is indifferent between L and R. When player 2 plays L, player 1 is indifferent between T and B. But it is in same payoff. But if player 1 plays B player 2 does not play L, it deviates to M. So, this is not a Nash equilibrium, okay. And player 2 plays M player 1 switches to T not … So, this is not a Nash equilibrium. If player 1 plays B this, player 2 is going to switch to

M because 1 is greater than, so this {1,0} is again not a Nash equilibrium. If player 1 is choosing, so this we know this is not a Nash equilibrium.

Again here if player 1 is choosing R, player 1 is going to choose B. So, this is not a Nash equilibrium. So, again the unique Nash equilibrium here is T, L and it is a weak Nash equilibrium, okay. So, we can have both strict Nash equilibrium where this here it is weak because T and B is giving same payoff. And again L and R is giving the same payoff. So, that's why in that sense it is a weak.

But here if you look at this game here M, C here it is strict because if it plays for M; 4, 5, 3, so 5 is strictly greater than 4 and 3. So, that is why it is a strict here. And if player 2 plays C; 4, 5 and 3, again this 5 is a strictly greater than 4 and 3. So, that is why it is a strict Nash equilibrium, okay. So, we can have both strict Nash equilibrium and also we can have weak Nash equilibrium, okay.

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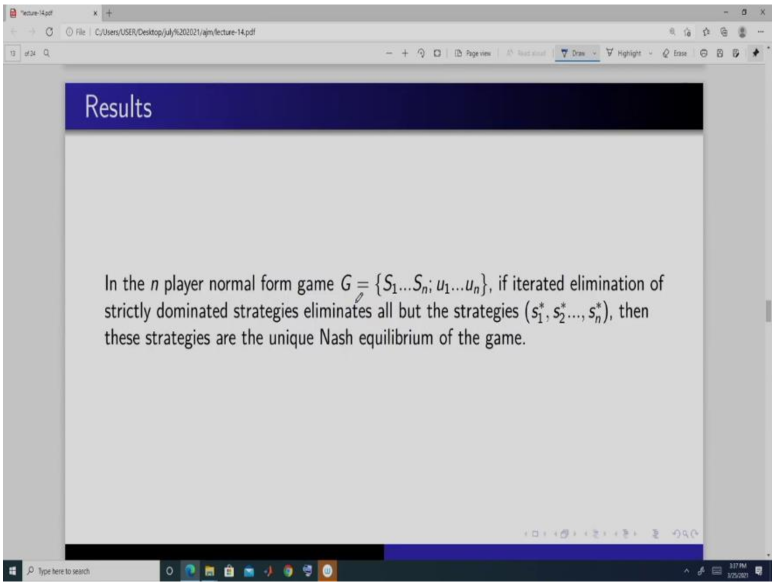

Now, we discuss one result. That is in n player game which is a normal form game where it has this strategy sets S1, S2, Sn, i.e $G = (S_1, S_2, ... S_n. u_1, u_2 ... u_n)$; and the payoff function u1, u2, un. If iterated elimination of strictly dominated strategies eliminates all but the strategies this, so we are left with only one outcome this then these strategies are the unique Nash equilibrium of the game, okay. So, we will not prove this result because this course is an introductory course. So, but we will do a more or less general example to show this result, okay.

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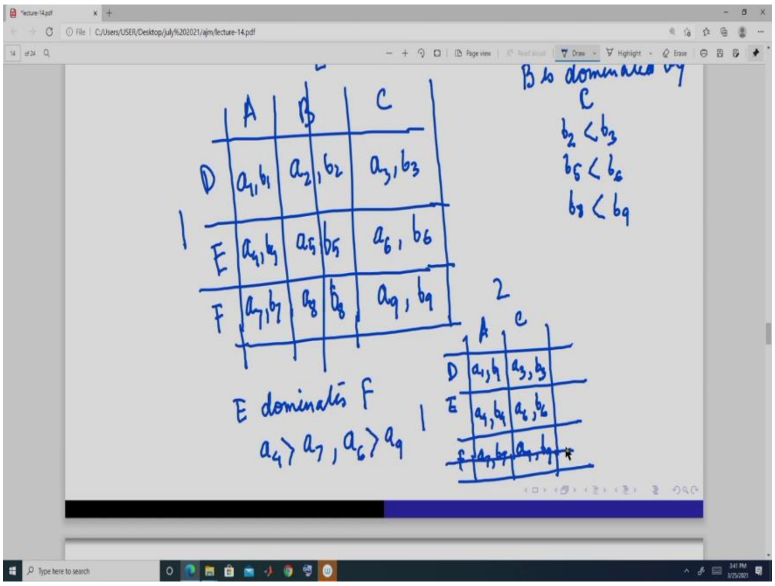

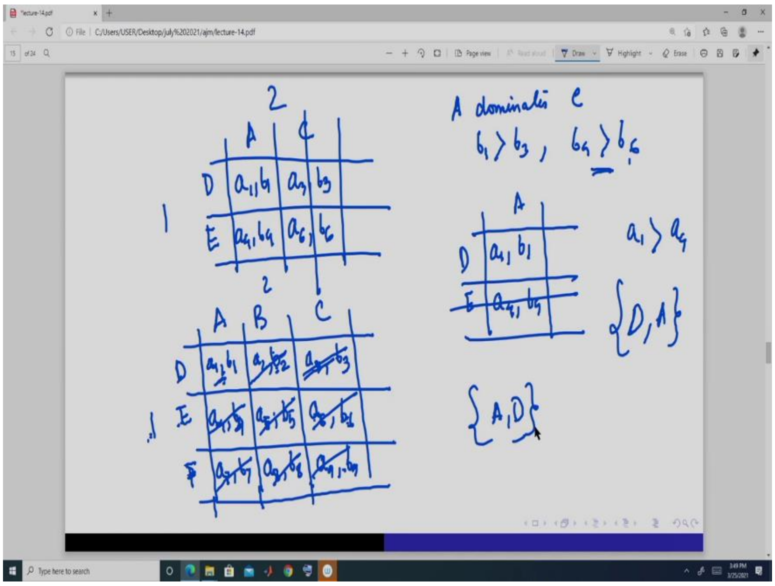

Now, let us take a game where there are two players; player 2 and player 1, and suppose the strategy is A, B and C. And here strategies of player 2 is D, E and F. And the payoffs are (a, b1); (a2, b2); (a3, b3); (a4, b4); (a5, b5); (a6, b6); (a7, b7); (a8, b8); (a9, b9), okay. Suppose C is dominated, or suppose for A, B is dominated by C. So, if B is strictly suppose dominated by C then it means what? b2 is less than b3; b5 is less than b6; b8 is less than b9. So, that is why we remove this, okay.

And next the reduced form is A and C; D; (a1, b1); (a, b3), (a4, b4); (a6,b6); (a7, b7); (a9, b9), okay. And suppose this E dominates, dominates and suppose here E dominates F. So, this means that a4 is greater than a7 and a6 is greater than a9. So, we remove this. So, we end up having this, okay, and player two and suppose A dominates C. So, this means b1 is greater than B3 and b4 is greater than b6. So, we are now…

So, A, D and E; a1, b1; and a4…. (a1, b1); (a4, b4). And suppose a1 is greater than a4. So, this B dominates E. So, we are left with this. So, the outcome of iterated elimination of dominated strategy is D and A and we have eliminated them in this process. That is first we have eliminated B, then we have eliminated F, and then we have eliminated C, and after that we have eliminated E and we end up here. And in this elimination process we have… why it is possible? Because of this X, because of this condition- $\mathrm{b}_2 < \mathrm{b}_3, \mathrm{b}_5 < \mathrm{b}_6, \mathrm{b}_8 < \mathrm{b}_9$; this condition- $\mathrm{a}_4 > \mathrm{a}_7, \mathrm{a}_6 > \mathrm{a}_9$, this condition- $\mathrm{b}_1 < \mathrm{b}_3, b_4 > b_6$ and this condition- $\mathrm{a}_1 > \mathrm{a}_4$, okay.

Now, we find the Nash equilibrium of this game using these conditions, okay. We are in this. Now, we know from this first A that b3 is greater than b2. So, if player 1 plays D, b3 is greater than b2. So, it is not going to choose this. So, this is not. B3 is greater than b2. So, it is not going to choose this. So, we are here. Again we know that b1 is greater than b3 so it is going to choose this if player 1 chooses D.

Now, if player 2 chooses A, from this we know a4 is greater than a7; a4 is greater than a7. So, this is not a Nash equilibrium. And again we know a1 is greater than a4. So, this is not again… so this is a Nash equilibrium from here because if player 2 chooses A best response is to choose D because a1 is greater than a4 and a7. If player 1 chooses D it is best response is to choose A because b1 is greater than b2 and b3, okay. Now, we can remove so that these are also not a Nash equilibrium. Why? Because if player 1 is choosing E since this is A, so b4, b5 is less than b7 so this is not a…. Now, here it is choosing this.

Now, if player 1 is choosing E and the player 2 is choosing C, this because, suppose it is choosing this. Then we know if we compare this; a6 is greater than a, so this is greater than this, okay. So, it is going to choose this. Now, here we have to compare from this a6 and a3, okay. Now, if player 1 chooses E, player 2 is not going to stick with this. So, this, it will shift to this. Again if we compare b4 and b6, from here we know that b4 is more than b6, so it will choose this. And from here we know that this is, a1 is greater than or not. So, this is not a Nash equilibrium.

Now, here if player 1 chooses F we know b9 is greater than b8 so this is not here. So, if it chooses this, from here we know that a6 is greater than a9. So, this it will be deviate to this. And again from here we know this is not a Nash equilibrium. So, this has a unique Nash equilibrium and this outcome is {A, D}.

So, these two things matches if we look at… So, this result actually holds for a general two player and three strategies. And if you look, if you do, we can actually prove it for a general case for taking k Strategies and n player but that is going to be more involved so we are not doing it here. But this is a simple proof.

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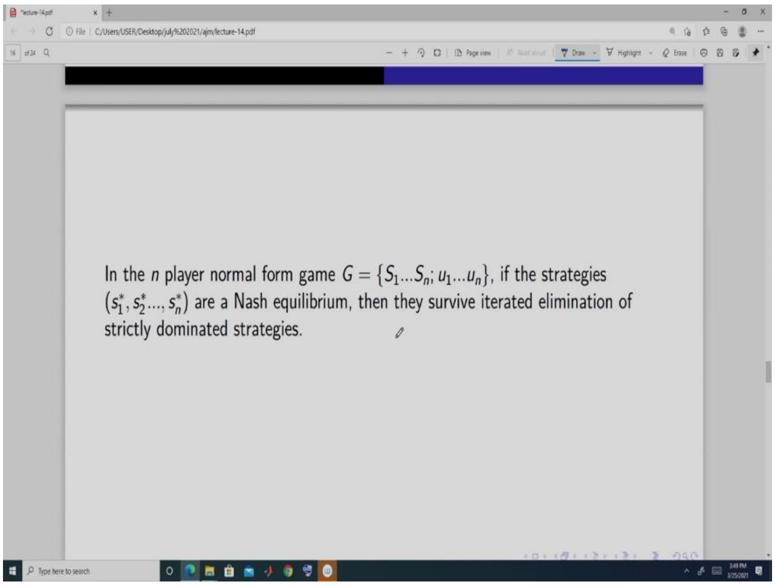

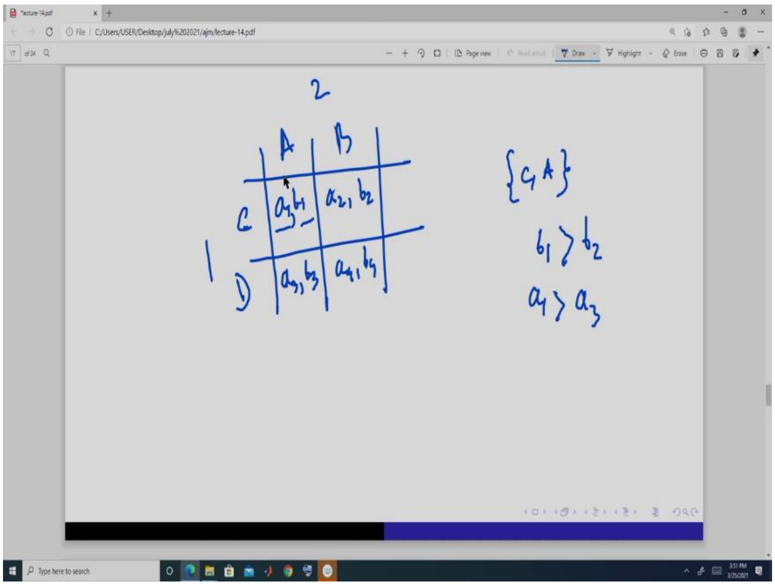

Now, we can have a converse of this also. So, if we have a Nash equilibrium, then of a game, if we have a n player game and this is the game and if there is a Nash equilibrium this, then this will always survive the iterated elimination of strictly dominated strategies. So, one example, simply take this example suppose A, B, C, D; this is for player 1; this is for player 2; take like this.

Now, suppose this {C, A} is a Nash equilibrium. So, if this is in Nash equilibrium it means what? b is greater than b2, and a1 is greater than a3. Now, if b1 is greater than b2, then this in this A this is greater than this. So, we cannot remove, eliminate A because b1 is greater than b2. And you again cannot eliminate C because a1 is greater than a3. So, therefore this survives the iterative elimination of strictly dominated strategies. So, that is why this is an example of this result, okay.

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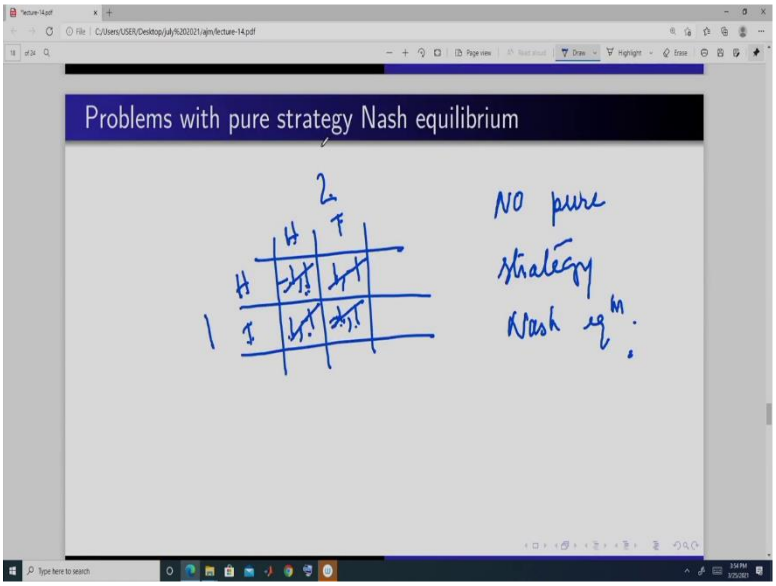

Now, we want to see what is the… So, can we find the Nash equilibrium in all the situations? So, let us give you an example. Let me give you an example where we do not find a pure strategy Nash equilibrium, okay. And this is the famous Matching Penny example that we have done in the first class on the Game Theory.

Suppose this is player 1 this is player 2; and they have two actions that is head and tail, okay. So, these are the players. So, if player 1 plays head and player 2 plays head; player 1 gets minus 1, player 2 gets 1. Head and tail player 1 gets 1, player 2 gets minus 1. If player 1 plays tail and player 2 plays head player 1 gets 1 and player 2 gets minus 1. If player 1 plays tail and player 2 also plays tail, player 1 gets minus 1 and player 2 gets 1, okay. So, this is the game.

Now, here we will see that there is no pure strategy Nash equilibrium So, if player 1 plays H then the best response for player 2 is to play H, 1 and minus 1. When player 2 play H, best response for player 1 is to play tail because minus 1 and 1, okay. So, it will deviate. So, this H, H is not a Nash equilibrium.

Now, if player 1 plays tail best response for player 2 is to choose, compare between these two head and tail and it is tail rather than head, so this (1, -1) is not a Nash equilibrium. And if player 2 play tail best response for player 1 is to choose head; 1 is greater than minus 1. So, this (-1, 1) is not a Nash equilibrium. And if player 1 plays head best response for player 2 is to choose head; because 1 is greater than this, so this is… So, we do not have pure strategy. No pure strategy Nash equilibrium. Pure strategy here means choosing its action like head or tail or again head or tail, okay.

So, we will now define strategies and we will attach probabilities to it. And if the probabilities are 1 and 0, 0 then it is a pure strategy. And if there is some probability to some of the strategies then it is a something called a mixed strategy. So, here in this game we are going to find the mixed strategy Nash equilibrium.

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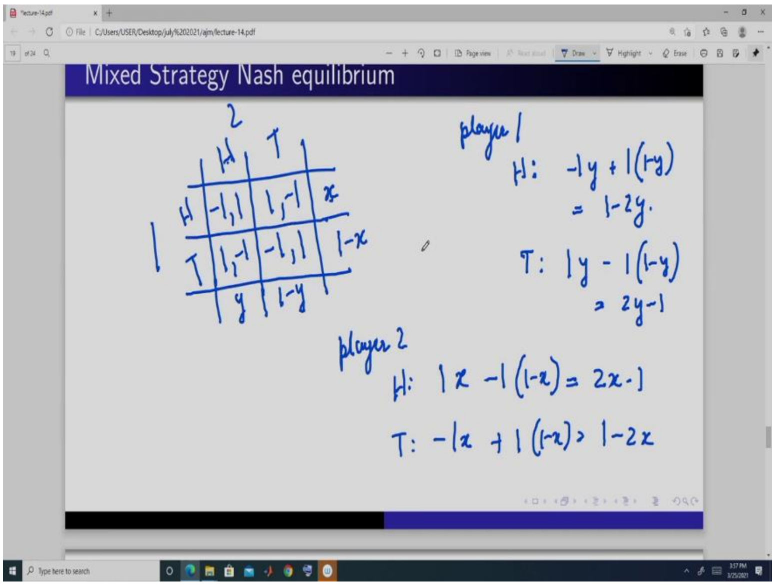

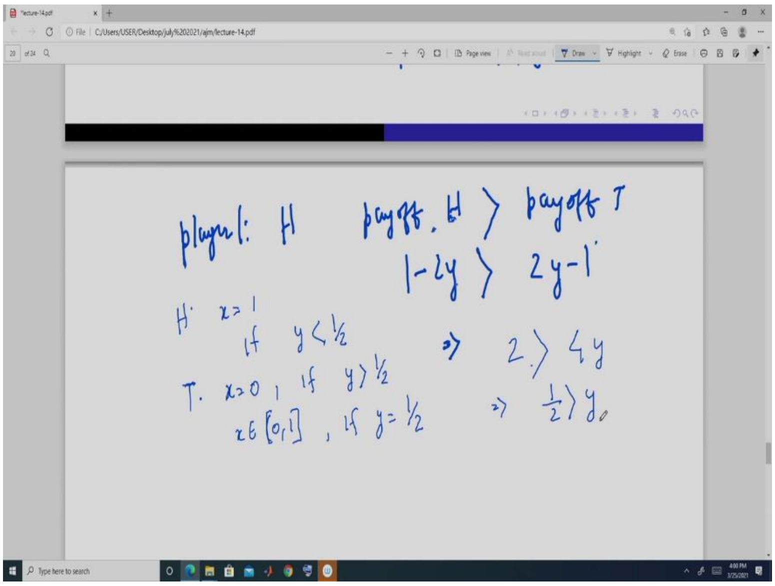

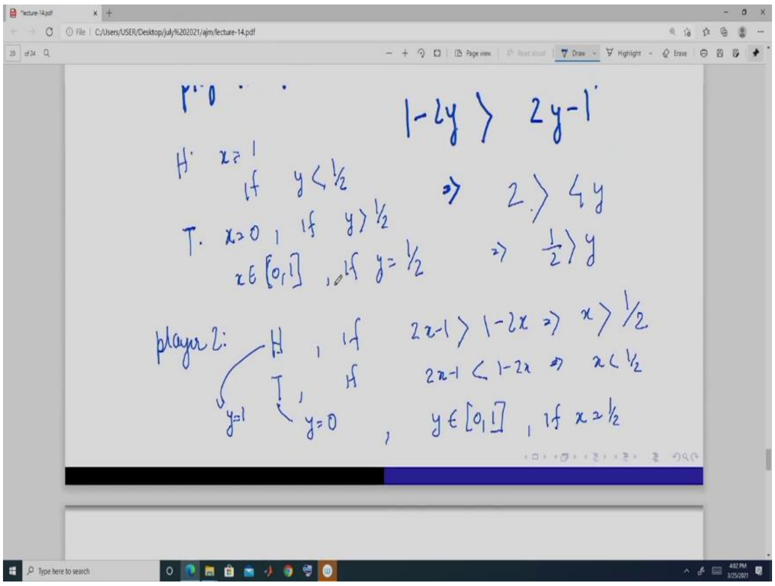

So, in mixed strategy what happens? Each player attaches some probability to their actions or their strategies. Suppose player 1 attaches probability x to this action each and it attaches 1 minus x to this. And player 2 attaches y to this strategy H and 1 minus y to the strategy, okay. Now, if 1 player 1 plays head; for player 1, okay so head, if it plays head then it will get minus 1 with probability y and it will get 1 with probability 1 minus x. So, the payoff is 1 minus 2y if it gets head. And if it plays tail it will get 1 with probability y and minus 1 with probability 1 minus y. So, the payoff is 2y minus 1, okay. So, this is the payoff for player 1.

Now, let us calculate the payoff of player 2. Player 2 if it plays head, if player 2 plays head it will get 1 with probability x with probability 1 probability and minus 1 if with probability 1 minus x. So, this payoff is 2x minus 1, i.e H: 1x-1(1-x)= 2x-1. And if it plays tail it will get minus 1 with probability x and plus 1 with probability 1 minus x. So, the payoff is 1 minus 2x, i.e T: $-1x+1(1-x)= 1-2x$.

Now, when a player is going to attach probabilities? Now, if, for let us look here if player 1 is going to play head; player 1 is going to play head if payoff from head is greater than payoff from tail; that means if 1 minus 2 Y is greater than 2y minus 1. This means that whenever, whenever 2 is greater than 4 y. So, that means whenever y is less than half, player 2 is going to choose each for sure because his payoff is greater than… And whenever…

So, this is going to play, player 1 is going to play head always. So, it is going to play head with probability x is equal to 1, right, if y is less than half. And it is going to; from here it is going to play tail that is x is equal to 0 if y is greater than half. And it is going to attach some probability that is x is going to lie between 0 and 1 if y is going to be equal to half, okay.

So, we can derive this because whenever this is greater than this so it is going to always play head. So, x is equal to 1. x equals to 1 means it is always going to be a head. If y is greater than half then this is just opposite sign, right. So, tail is giving the player 1 more. So, that is why player 1 is always going to play tail. So, x, means it is going to attach 0 probability to this. So, it is this.

Now, if y is equal to half so these two are equal. So, it will attach any probability here, okay. So, we get this, okay. So, in this situation we get that whenever y is equal to half then only x takes some positive probabilities here. Otherwise x will either take 1 or it will take 0, okay. Next we look at the player 2. Player 2 plays head if this- $2x-1$ is greater than this- $1-2x$. So, player 2 plays head if 2x minus 1 is greater than 1 minus 2x.

So, this means x is greater than half. And it plays tail if x is less than half. So, it means it will always play head. So, in this case what is happening? Here y is always equal to 1 and here it means y is always equal to 0. And y will be taking value, any value between, any value between 0 and 1 if x is equal to half, okay. So, if you look at these two this- [H: $x=1$, if $y<1/2$, T: $x=0$, if $y>1/2$, $x \varepsilon [0,1]$, if $y= ½$] is something called the reaction function of firm 1. And this- [H: $y=1$, if $2x-1>1-2x$, T: $y=0$, if $2x-1<1-2x$] is called the reaction function of firm 2, okay.

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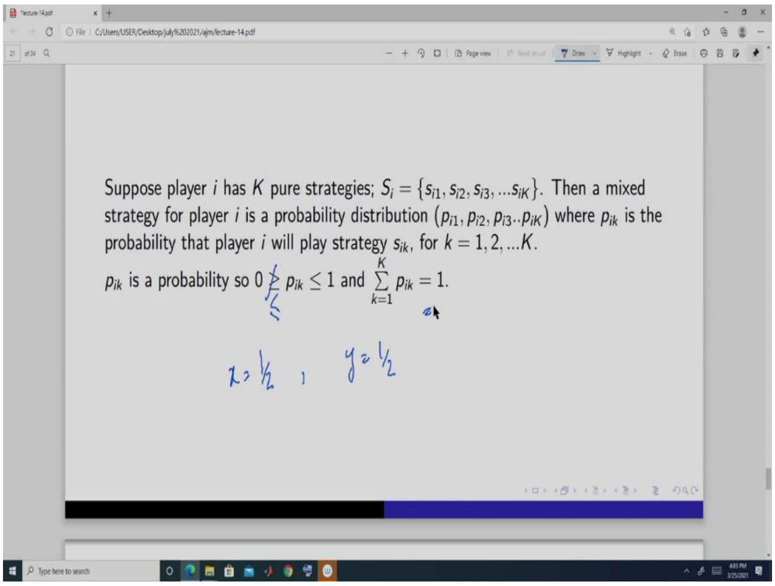

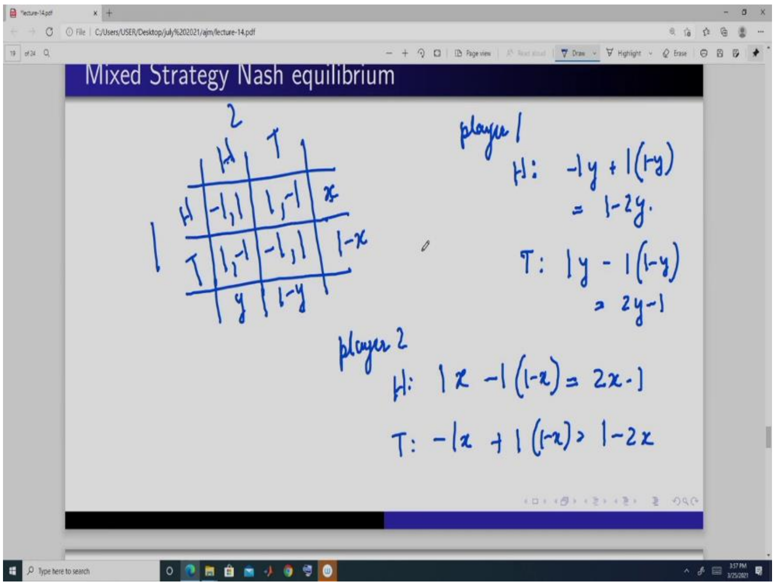

So, we see that whenever x, in this situation, we see that x is equal to half and y is equal to half is a Nash equilibrium and that is a mixed strategy Nash equilibrium, okay. So, how do we arrived at the mixed strategy Nash equilibrium? So, it is that what we do? We find that value of y where we are indifferent between head and tail. Player 1 is indifferent between head and tail. And we find that value of x where player 2 is indifferent between head and tail. And that value gives us the mixed strategy; because if we are not indifferent then we will attach the full probability to this A.

So, if H is greater than T in payoff so that is 1 minus 2y is greater than 2y minus 1 then, I will always, player 1 will always play head with full certainty. And if 2 minus, 2y minus 1 is greater than 1 minus 2y then player 1 is going to play tail with full certainty. So, when it is indifferent? When y is equal to 1 by 2. And when same for player 2 when x is equal to 1 by 2 player 2 is indifferent between head and tail. So, it is attaching some probability to this and this. Here also it is attaching some probability to this and this. So, at this point when x is equal to half and y is equal to half we get a mixed strategy Nash equilibrium.

So, formally suppose player i has K strategies so strategies set of player I is this- $S_i = \{s_{i1,}s_{i2} \dots s_{ik}$ then a mixed strategy for player i is a probability distribution this where pi is the probability that player i will play strategy this for k is equal to 1 to this, okay. And here this should always lie between this….oh; oh this inequality should be … this is wrong, okay. And this…. So, we are attaching some probability to each action, and when it is a best response compared to the other players strategies then it is…

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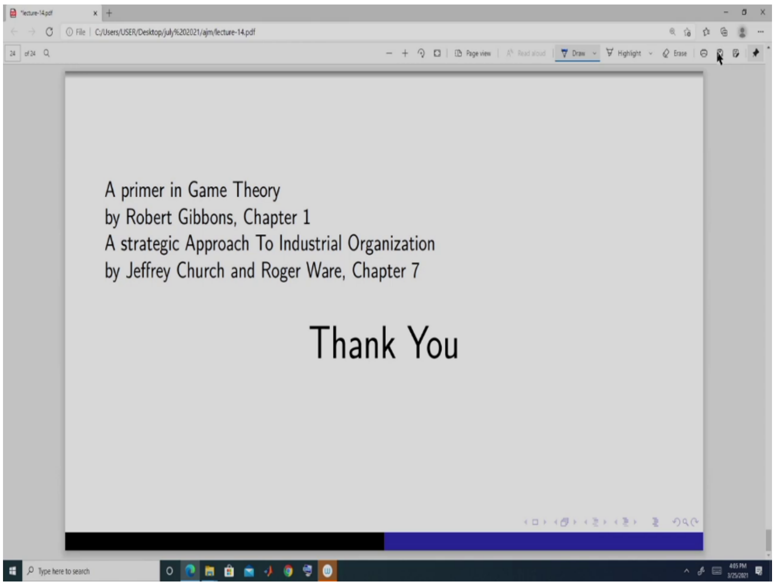

So, we will do one example in the next class, okay and for this you can read chapter 1 from Gibbons and chapter 7 from Church and Ware, okay thank you.