Introduction to Market Structure
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Lecture 17
Introduction to Game Theory, Iterated Elimination of Dominated Strategies

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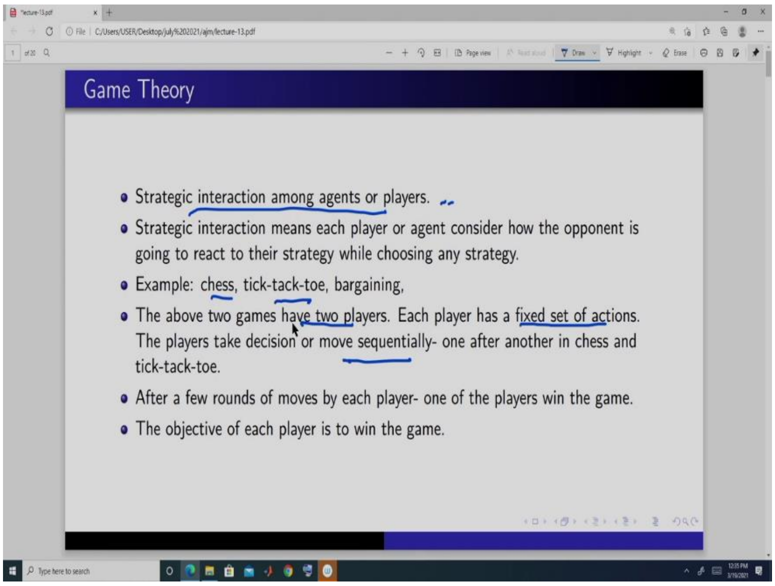

Today we are going to do game theory and this is module 5. So, in Game Theory it is mainly strategic interaction among players. So, what do you mean by strategic interaction it means that if we have suppose a number of players then while each of them taking any decision, they will always think how their opponent is going to react to the action they are going to choose.

So, for example, suppose in chess, what do we do? While making any move, I always try to think how if I moved in this way, how the other, my opponent is going to move in reaction or like in a tick-tack-toe. In tick-tack-toe what happens the players move sequentially and in each move they make a decision, right? and then after a set of moves you, we can find whether there is a winner or there is not a winner.

So, what happens in all these games we have a set of players. So, in these two examples, we have two players and each of them has set of actions that they can choose from and further we have to define whether they are taking the decision simultaneously or move sequentially or move one at a time, but in case what happened they move one after another in tick-tack-toe what happens they move one after another and after a few rounds, we know who is the winner and the objective here is to win the game.

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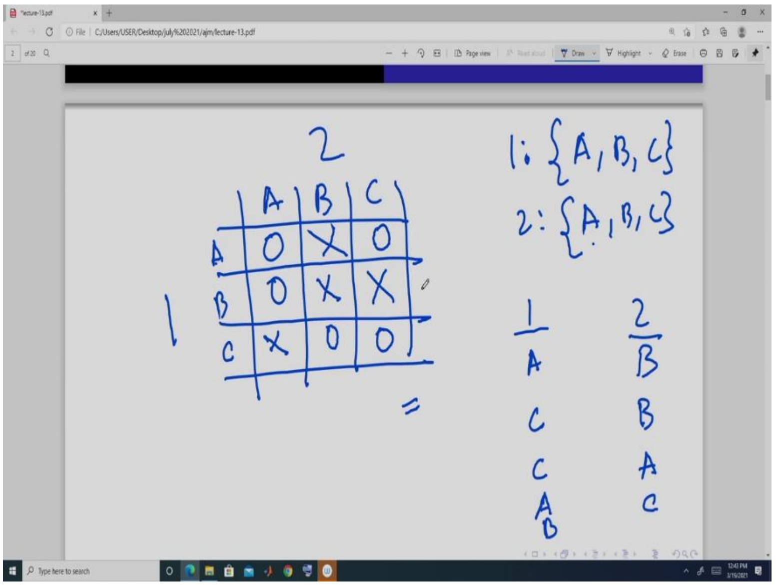

So, let us play one Tick-tack-toe game. So, here this is supposed be player 2 and this is player 1. So, this, okay? it can choose any place from A and player 2 can choose from these places A, B, C these columns and player 1 will choose from these row A, B, C. Suppose player 1 moves first as chosen and player 2 has moved second and it has chosen this place and next again player.

So, we can say that the action that the player can choose from these three places and action of player B is from these 3 places. And the moves are like this, suppose player 1 moves first A, player 2 has moved B then suppose player 1 is again moving and it has suppose chosen this place. So, it has chosen this. Now, player 2 will always choose this position. If he chooses something else, then player 1 will place here and it will win the game.

So, this is their strategic interaction. So, while choosing this position player 2 is deciding is thinking that if he if it does not do place its mark here, then player 1 will do and player 1 will win. So, this is what we mean by strategic interaction. Now, see, player 1 will always choose this position. This because if player 1 leaves this then player 2 will place here and it will make.

Next player 2 is going to move. So, player 2 if it leaves this then what is going to happen player 1 will choose this and it will win the game. So, player 2 will always choose this so it is A, next if player 1 leaves it, then what is going to happen. Player 2 will choose this and it will win it so win the game. So, player 1 is going to choose here.

So, it is again going to be A then if player 2 leaves this part, this block that is C for player 2, then player 1 will place here that is B for player 1 and it will win the game. So, so, there is only one position left and that is this it will be here. So, this is C and then again it will place at B. So, in this game we cannot find any winner, okay. But I hope you have understood what do we mean by strategic interaction that while making my move or making decision regarding my placement where I am going to place my mark, I have to think about what my opponent is going to place, where my opponent is going to place if I place my any specific place, okay.

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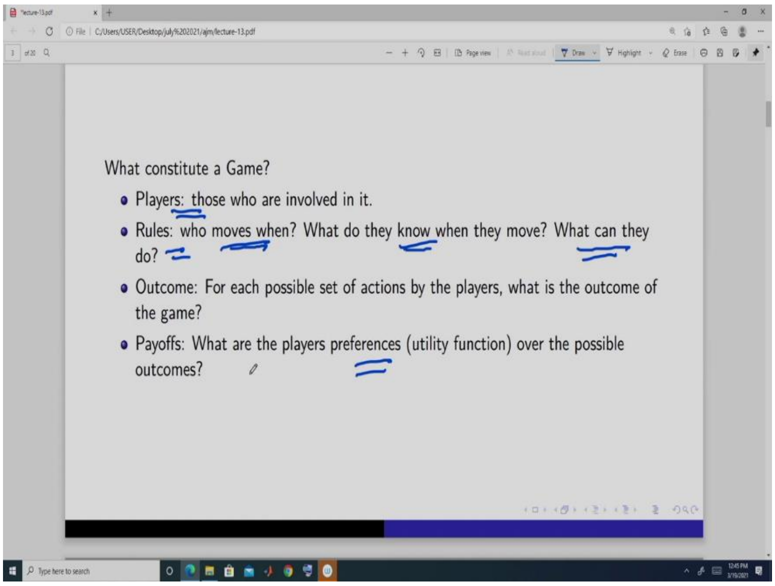

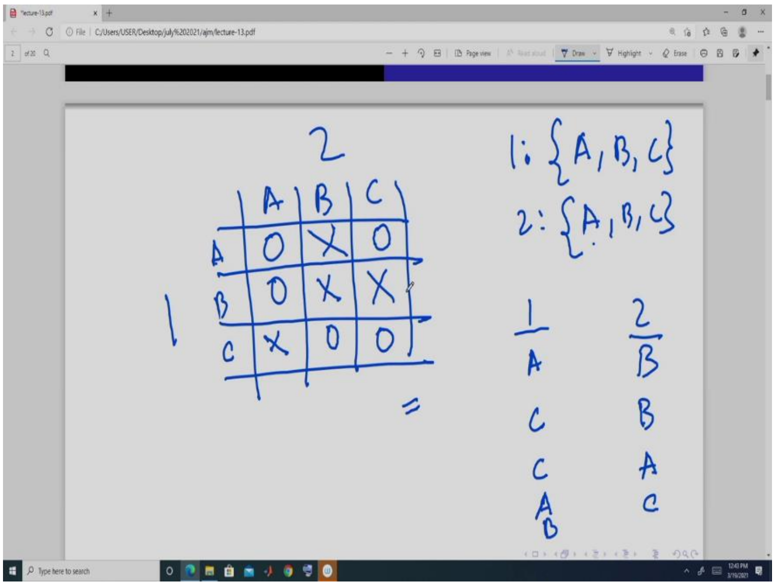

So, what mainly constitutes a game? It will have players who are going to play the game, we will have to define the rules like in tick-tack-toe, what are the rules? Rules are one player will move first and then the second player will move and then again the first player will move and then again the second player will move. So, this is one rule that is the sequence. So, in how the game is going to be played.

So, who moves when? So that has to be specified, then what do they know when they move. So, here when you play the tick-tack-toe, when player 1 was making this a player 2 while moving second, while player 2 was choosing this place, player 2 has observed where player 1 has placed his mark otherwise player 2 would have a means a different information if it does not observe this, but if it observes then its observation or the information set is going to be different.

So, this the role of the information set while making any decision plays a very important role and we will do this specifically when we do the extensivity, okay. And next we have to specify what they can do like here they have to choose from this player 1 has to choose from this A, B, C all these positions, okay. In A it has 3 positions in B it has 3 positions, in C it has 3 positions or places, okay.

So, these are the actions that they can take and we have to define the outcome. So, outcome means that when they are playing here, when they are playing So, if I suppose player 2 while player 1 has moved this player 2 has been moved this then player 1 has moved this this position, then if player 2 suppose placed somewhere else not here, then player 1 will win by placing it here and then it will win the game.

So, what is going to be the outcome here. So, if all these three matches in any line like this way or this way, this way or this way, then we say a player wins and if it does not match, then we say no one wins. So, in that way, we have to define the outcomes, okay. So, those are the outcomes and then we have to define the payoff, that if I win, what do I get? I get if I win, I get lots of satisfaction. So, we have to define the utility function, okay.

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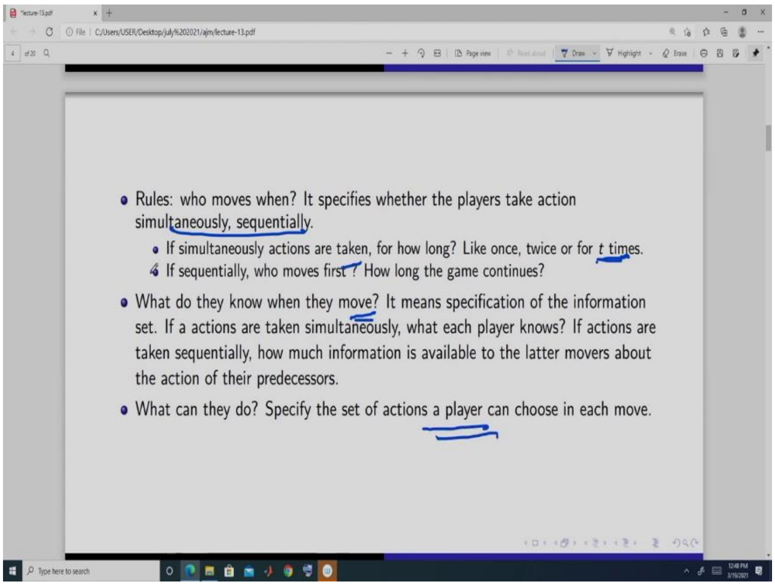

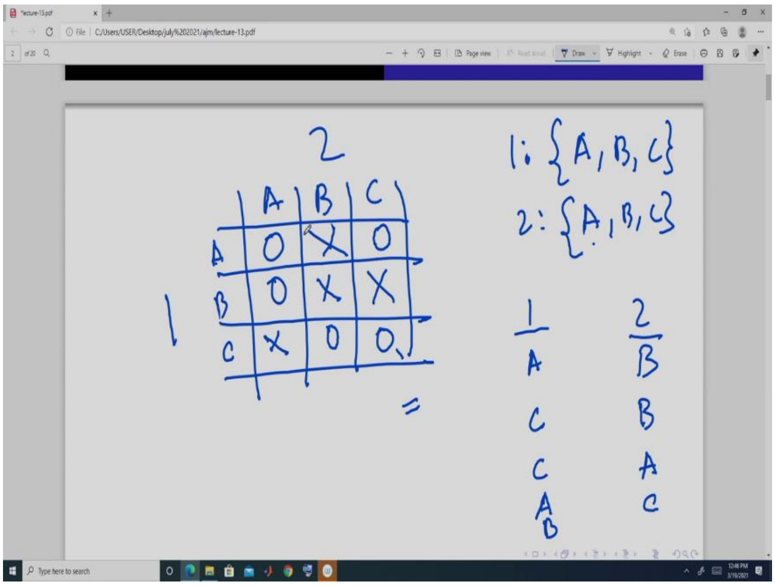

So, mainly a game constitutes this. So, when we say rule, when we say that who moves when it specifies whether the player takes action simultaneously or sequentially a little bit later we will see what do we mean by decisions are taken simultaneously. It means that both the player are choosing their action or their strategy at the same time. So, if they are simultaneously choosing then we have to specify whether they are going to play this game for one time or they are going to choose it for 2 times or they are going to play it for suppose some t times, okay.

So, it is like this that in this tick-tack-toe we know we cannot say beforehand how much but we say that the play will continue as long as all the places are not if any of the places is vacant or there is a winner if you get a winner then the player the game will end and if we have not got any winner, then the game will continue. So here we have specified that the game is going to go on each player is going to choose as long as there is no winner or there is any vacant place to be marked.

So, this is like a in and in sequential game we have to define who moves first and who moves second and how long the game is going to continue or how long the players are going to choose their actions. Now, what do they know when they move it means the specification of the information say like when I am choosing my action, I should know what are the possible actions that the player 2 can choose?

Or if I am playing after player 2 has played then I should know what are the actions that has been already taken by player 2 or I have to specify means if I do not know some of these actions or some of the payoffs then I will have to define some probability distribution over them. And that part we are not going to do in this course that is incomplete information game. Next, when we say what can they do we specify the set of actions for at each move, okay.

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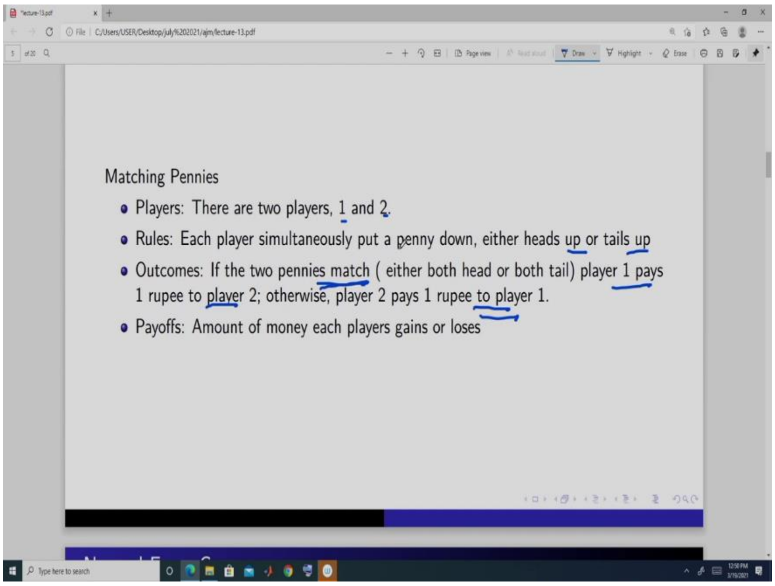

Now, let us look at the game that we are generally going to study in this course like one is these matching pennies. In the matching pennies what happens there are 2 players, player 1 and player 2 and they simultaneously what they do they toss a coin, okay or they will place a coin in a table, okay they will put a penny down.

So, what happens if both of them do that thing. So, either they will have both head or they will have tail, okay. So, what will happen if both the pennies matches that if either both are head and or both are tail then player 1 pays 1 rupee to player 2. If the pennies are not matching like 1 is head and another is a tail, then player 2 pays 1 rupee to player 1. So, this is the outcome and the payoffs are the amount of money each player gains or each player loses and what are the rules?

Rules here is the simultaneously place a coin in a table, okay. So, while making this action, both of them are taking their decisions at the same time or they are taking the decision together, but they know that what is going to be the outcome. So, they know all the possible outcomes. So, based on that they will take a decision, okay. So, this is one form of matching pennies, one form of a game which we call as matching pennies. It is a simultaneous move game and it is also single shot. So, generally we defined this kind of simultaneous move which is played only once as a normal form game.

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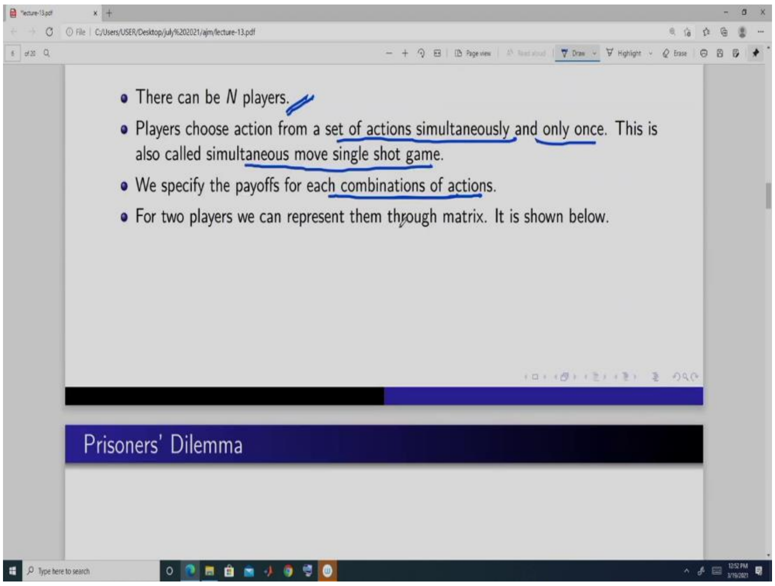

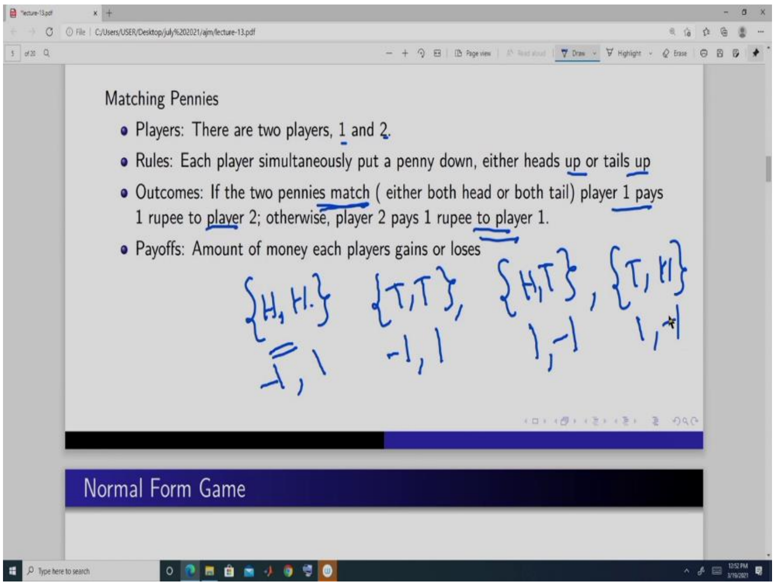

And it can be played between among N players. So, the number of players is not a restriction and player can choose a action from a set of actions simultaneously and they only choose only once. So, the game is played only once and it is played simultaneously. So, this is also called a simultaneous move single shot game, okay. And here we specify the payoff for each combinations of actions, okay like in this.

So, what are the possible outcomes? So, one possible outcome is head and head. So, we define the payoff here another is tail and tail, another is head and tail and the another is tail and head. So, if it is head and head, player 1 pays 1 rupee to player 2, so, it is minus 1 to player 1 and plus 1 to player 2. If it is tail and tail, so, it is minus 1 for player 1 and 1 for player 2 and if it is head and tail, then it is 1 for player 1 and minus 1 for player 2. And if it is tail and head again you will. So, these are the 4 possible outcomes and these are the payoffs they get in these outcomes, okay. So, this is a normal form game. So, normal form game can be represented through matrix and it is very convenient way.

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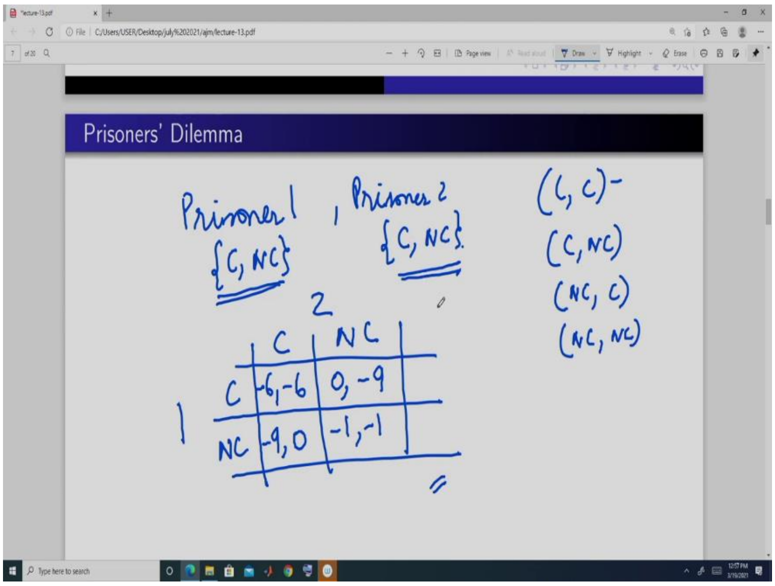

So, we will do a now I will specify a very famous game that is called a prisoner's dilemma. So, there are 2 prisoner, prisoner 1 and prisoner 2, okay and they are interrogated simultaneously in two separate, okay and they have committed some crime each one can either confess C or D or it may not confess So, it has 2 possible actions. Similarly, prisoner 2 has 2 possible actions either can confess or it may not confess.

So, how many combinations we have? Combinations of outcomes, so, this is C, C both of them confess, okay or it is let us define the combinations not in through this, **so** combination is C, C this is C not C then we have not C, C and not C not C, okay. So, there are 4 possible combinations from these 2 actions. So, these are called the action space, i.e (C,C), (C, NC), (NC, C) and (NC, NC) or strategy space, and these are the combinations of each element from this strategy or action space, okay.

And now we define some payoff from here. So, if this is the outcome what is going to be the payoff or what is going to be the utility for each of these individuals, okay what they are going to get if this is. So, this whole thing can be represented through a matrix in this form. Suppose here this column represents player 2, so, it has 2 actions, one is C and other is not C. Player 1 again it has 2 actions that is C and not C, and here this block gives me the outcome when both of them are choosing C and C.

This is when player 1 is choosing C and player 2 is choosing not C. This when player 1 is choosing not C and player 2 is choosing C. This outcome is when player 1 is choosing not C and C and the player 2 is also choosing and C that is not confess, non-confess, or this is confess, and if both of them confesses, then they get a little suppose 6 years of jail and so jail is a you get a disutility from jail.

So, that is why I have written it as minus 6. But if player 1 confesses and player 2 does not confess, then player 2 is not given any punishment, but player 2 is given a stricter punishment and that is 9 years of jail, okay. So, it is so if player 2 1 does not confess, but player 2 confesses, then player 1 gets 9 years of jail and player 2 is not given any punishment. If both of them does not confess.

But since the police know that they have committed a crime, but they are not very sure whether they are sure they will get some amount of punishment and that is 1 year of jail. So, if we look at this here, then each of them should prefer this outcome, right? because it is 1 year, 1 year. So, this first element is for player 1 that is for the player whose actions are specified in the row and the second element is for the player 2 whose actions are represented in the columns, okay. So, this is a representation of the prisoner's dilemma game. And this is called matrix representation of normal from.

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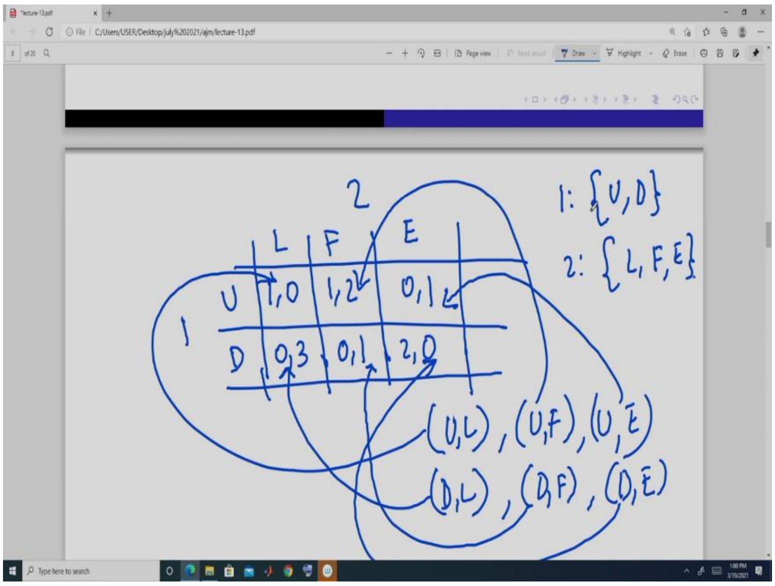

Another type we can think suppose again we have 2 player, player 2 and so, this is suppose U and D for player 1 for player 2, it is L F and E, okay. So, now this is a type of asymmetric game where action or strategy space of player 1 is U and D and for player 2 it is L, F and E, okay. And now suppose we define the payoffs. Suppose the payoffs are like this, okay. So, here so when player 1 place of the different total combinations that is possible combinations of outcome is this- (U, L), (U, F), (U, E), (D, L), (D, F), (D, E)

So, this outcome is represented here and these are the payoffs. This is given here and these are the payoffs. This is for player 1 who plays who have played **a** chosen action U and this is for player 2 who has chosen F. This is for this for this player 1 has chosen U and player 2 has chosen E, this outcome is this. 0 for player 1 and 3 for player 2, player 1 has chosen D. So, it has got 0 when player 2 has chosen L it has got 3.

This combination gives you this payoff this outcome. So, it is 0, 1; 0 for player 1 when it chooses D given that player 2 has chosen F and player 2 gets 1 when it chooses F and player 1 has chosen D. This is for player this combination is D, E when player 1 has played or choosing D and player 2 has played E, okay. So, this 2 is the payoff for player 1 and 0 is player 2. So, this is also another game where the action state or the strategy space are asymmetric, okay.

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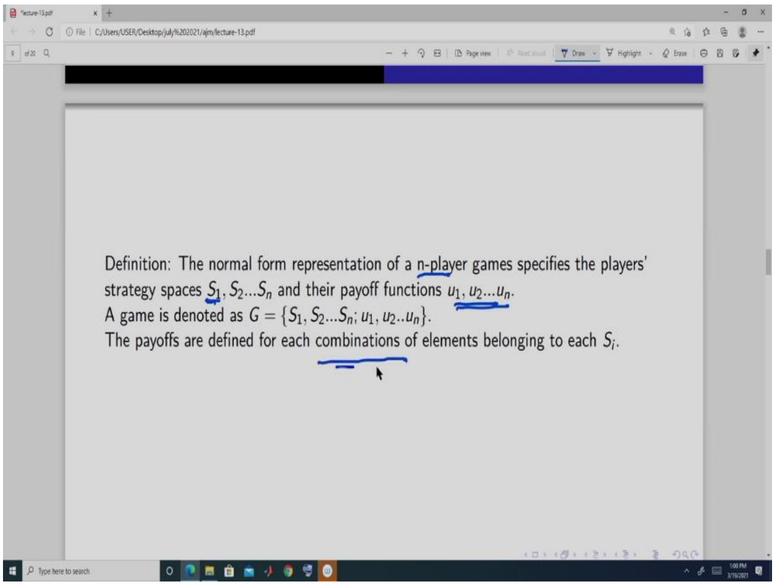

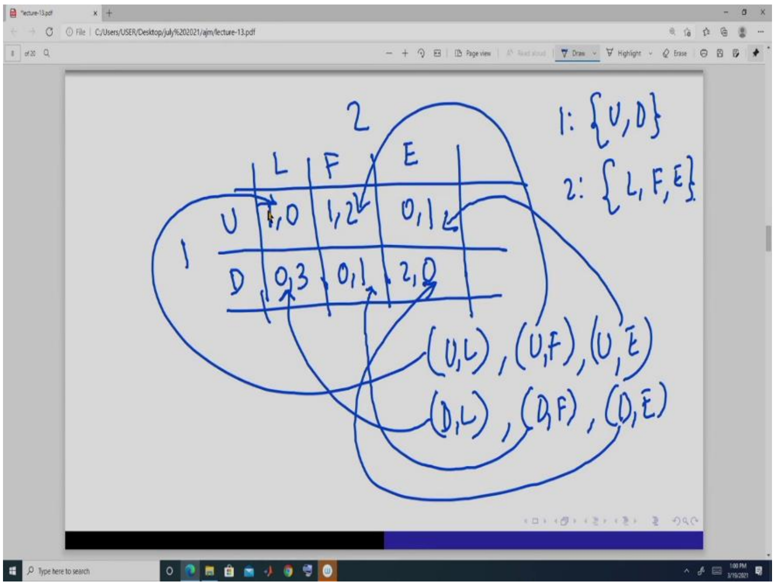

So, we define in a compact form the normal form game in this way that suppose there are n players and each player has a strategy space like this, this is capital S1 for player 1 capital S2 for player 2 and like this for gap n. For player n it is Sn and they have a payoff function which is which gives you this the amount you get when you play any when there is a specific combination of outcome like this.

So, there is a utility function defined for each player and payoffs are defined for each combination of elements belonging to. So, each combination means, so, here all these combinations all possible combinations. So, for this we have to define the utility how much the sorry, we have to define the payoff how much they get, okay. Now, we have to play this game. So, we have to define a solution concept.

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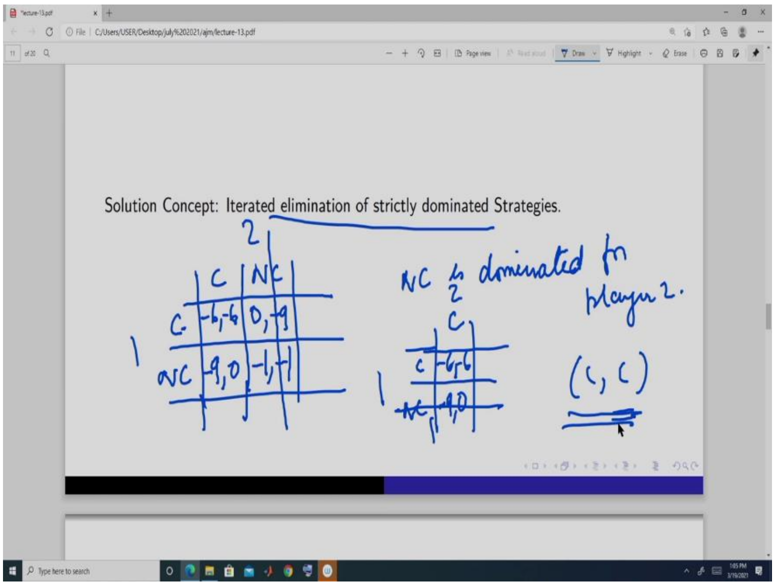

So, first solution concept that we are going to do is iterated elimination of dominated strategies. So, that is, so first let us consider the prisoner's dilemma game. So, player 1 it can confess or not confessed player to confess or not confess. If both of them confess confess payoffs and minus 6, minus 6, if player 1 confesses player 2 does not confess 0 comma minus 9, if player 1 does not confess player 2 confess, then it is minus 9, 0.

Both the players do not confess this. Now, what iterated elimination of dominated strategy says that you eliminate the dominated strategy. What do we mean by dominated strategy? So, for player 2, suppose it plays so in this case, minus 9 here minus 6, so this is less than this, right? Here, for player 2, this is 0 and this is minus 1. So again, this is less. So, this strategy is actually dominated.

So, NC is you can say dominated for player 2, so, if we start with player 2, then this can be removed. So, we have eliminated the dominant. So, that means when you specify a action and or a strategy of player 2, and you look at all the possible actions that the player 1 can do. It can choose C or it can choose this if it choose C then it will get this if it chooses this it will get you have to compare this.

So, in all these comparisons, what will happen minus 9 is greater, is smaller than minus 6, minus 1 is smaller than 0. So, this if it plays this if player 2 plays this strategy or this action, then it is going to get less than whatever player 1 is going to choose. If it chooses C it will get minus minus 9 it will get minus 9 which is again less than minus 6 if it chooses player 1 to NC it will get minus 1 which is again less than 0.

So, that is why this is dominated. So, this will remove then the game is so, player 2 has no option now, right? but player 1 has options. So, it will compare minus 6 and minus 9 and it know minus 6 is less than a greater than minus 9. So, it is so, what is the outcome? Outcome is going to be C, C always, player 1 will choose C after this elimination.

Because suppose player 2 starts this process of elimination then we remove this then we arrive at this reduced form and from here player 2 does not have any actions or strategies to remove it has only 1 A, player 2 has player 1 has to from it, it removes the NC and we get end up having only this outcome. So, this is the outcome of this game, okay. If there is iterated elimination of dominated strategies.

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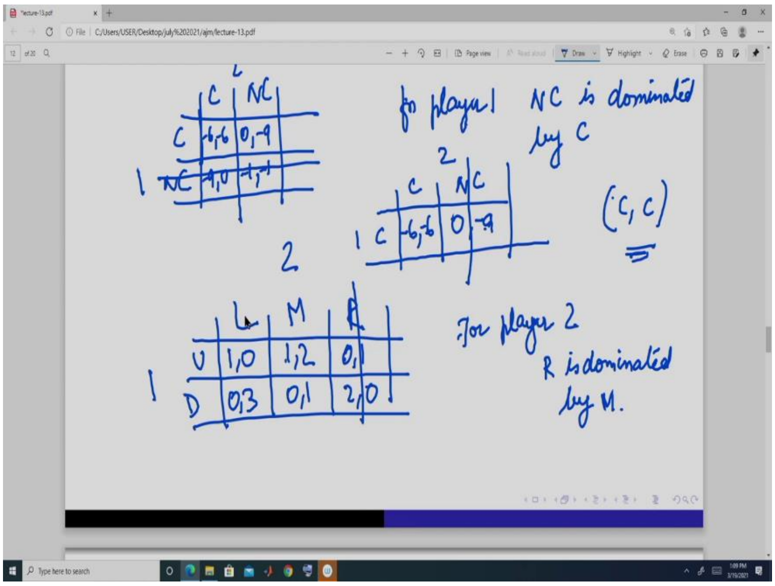

Now, here the same game if it is suppose the elimination is started by player 1, okay. So, player 1 it will player 1 suppose, chooses this NC. So, if it chooses this you will see that compare it with C minus 6, minus 9, 0, minus 1. So, if it plays this for each outcome, it will get less compared to this. So, that is why for player 1 and C is dominated by C. So, we eliminate this. So, next the reduced form is, is this.

Now, player 1 has only one action of strategy so it cannot eliminate further. Player 2 has 2, it has C and NC. So, if it plays C it gets minus 6 if it plays NC it gets minus 1. So, this can be eliminated. So, again the outcome is C and C. So, in this case we see that whether we start from player 2 we will start the process of elimination from player 2 or from player 1 we get the same outcome that is C and C.

Now, let us take another example, okay. So, this was this is for player 1 and, okay this is for player 2, okay. So here so, if we take player 1’s this U and compare it with D, so (1, 0) (1, 0,)(0, 2). So, for this 2, this dominates this or this is dominated by this but for this U gives more than D. So, we cannot eliminate any action for them, but if we compare this M and R, we will see that 2 is greater than 1 or 1 is less than 2, 0 is less than 1. So, for player 2, R is dominated by M. So, we remove this, so, this is our game left.

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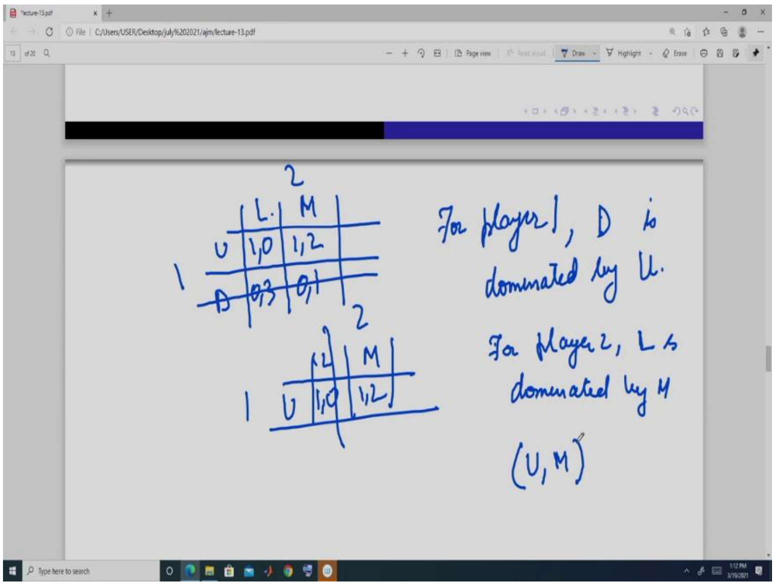

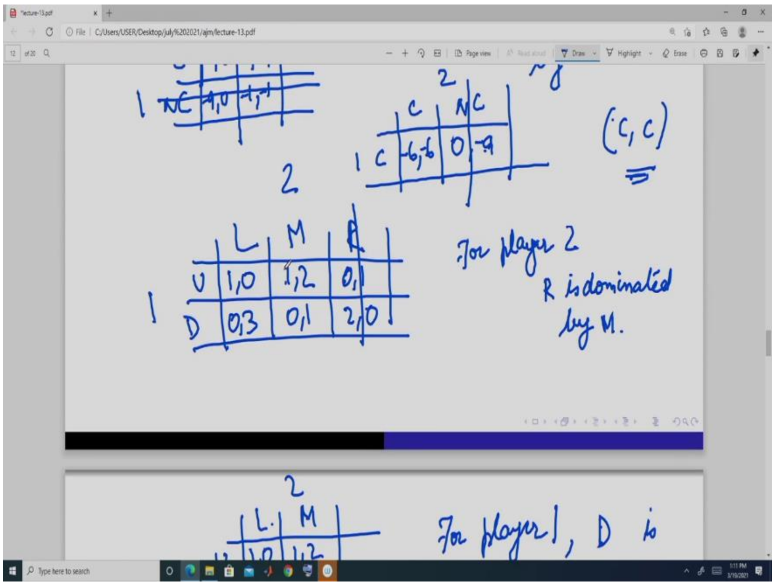

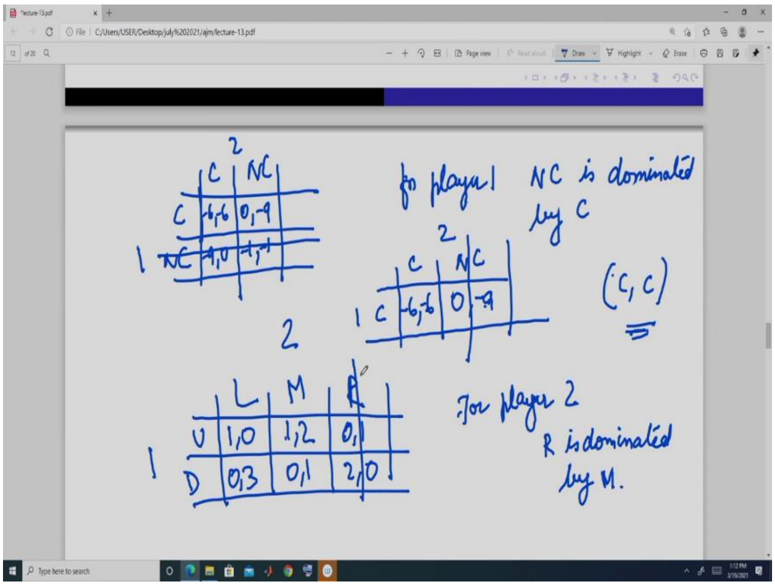

So, now, if you compare for player 2 again L 0, M 2, so, this is greater than, but here 3, 1 So, this is greater. So, 2 cannot decide, 2 cannot say whether this is dominating this or this is dominating this, and for player 1 it knows U 1, 1, D 0, 0. So, this is dominated for player 1, D is dominated by U. So, next our next is this. So, for player 1 there is only 1 action and that is U for player 2 there is L and M and from this we can remove L for now, for player 2 L is dominated by M. So, the outcome in this game is (U, M).

In this game it is going to be this. So, when players are playing once and they are playing simultaneously, all these eliminations will be done by on their own they will not interact, but they will know. So, here we are assuming that the player 1 knows that player 2 is also going to behave in this way, and player 2 knows that player 1 is going to behave in this way.

So, there is player 1 knows that player 2 first is going to remove this and then player 2 is also going to remove this and then player 2 again going to remove this and it is going to come end up having this outcome. Player 2 also knows that player 1 is will after looking at this game will know that player 2 since player 2 is the rational is going to is never going to pay this strategy.

So, this is important. So, this so, then it again knows that since player 1 is itself a rational player, so it will remove this so it is a left with this game. So, then it again knows that player 2 is also a rational player. So, it is not going to play this action and that is so they end up having this. So, since both the player behaves rationally and also both the player thinks that the other player is also rational and other player also thinks that he itself.

So, player 1 thinks or player 1 knows that player 2 is rational and player 1 knows that player 2 knows that player 1 is also rational and so using that argument, we can eliminate all the dominated strategies and we will end up having one outcome one this.

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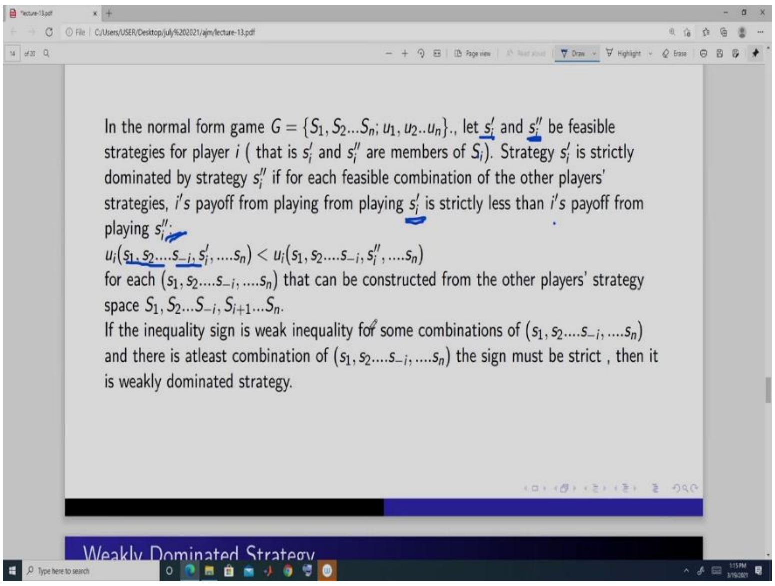

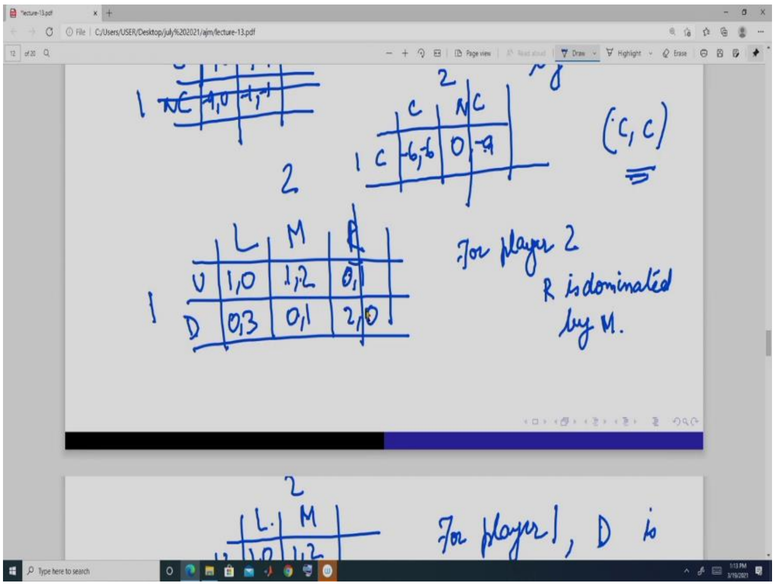

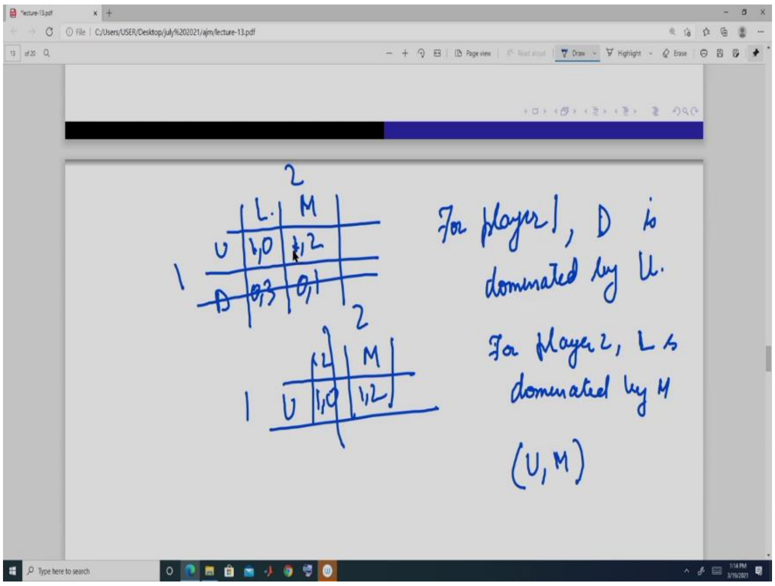

So, more formally, we can define a dominated strategy in this way suppose take any player i and let this Si dash and Si double state double dash be 2 strategy for player i, okay so they are 2 member of this strategy space i okay. Then strategy Si dash, single dash is strictly dominated by a strategy of player i double dash if for each feasible combination of other player strategies. So, it is like this.

So, here if this is dominated by this then all the possible combinations of player 1 that is this and this so, you have to compare this with this and this with this, right? So, that is what it says that for each feasible combination of other player strategies Si's payoff from playing this Si dash is strictly less than a Si's payoff from playing double dash. So, this utility from playing single dash strategy is less than the playing double dash.

So, single dash a player i is dominated by the strategy or action as double dash, okay. And so, here when we look at this, so, this is one combination, so for each of these combinations, okay for each Si that can be constructed from the other player this here. So, when we are eliminating this then what we are doing player 1 is eliminating. So, it is looking at all the possible combinations, okay. So, it for this this has to be less than this and this has to be less than this, okay. And when we say it is weakly dominated, then when some of them are equal and some of them are at least 1 must be unequal.

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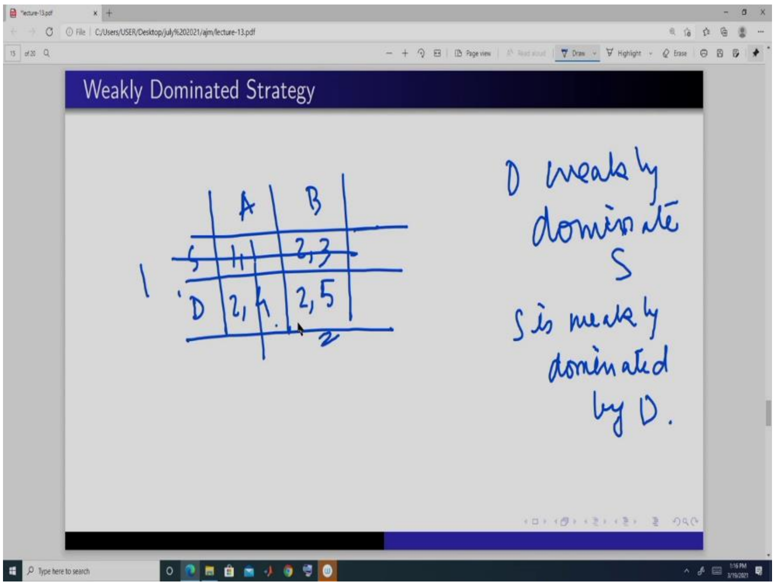

So, this can be 1 this is see now here so, if you compare the payoff for player 1 then you will see that this S and D so, this is greater but here it is same. So, D weakly dominates or weakly dominates S or S is weakly dominated by D in this case, okay. And then here again you can remove this so, you can remove here and so, this this will be removed. So, outcome is this and if you look at so, we have to do only this way, okay.

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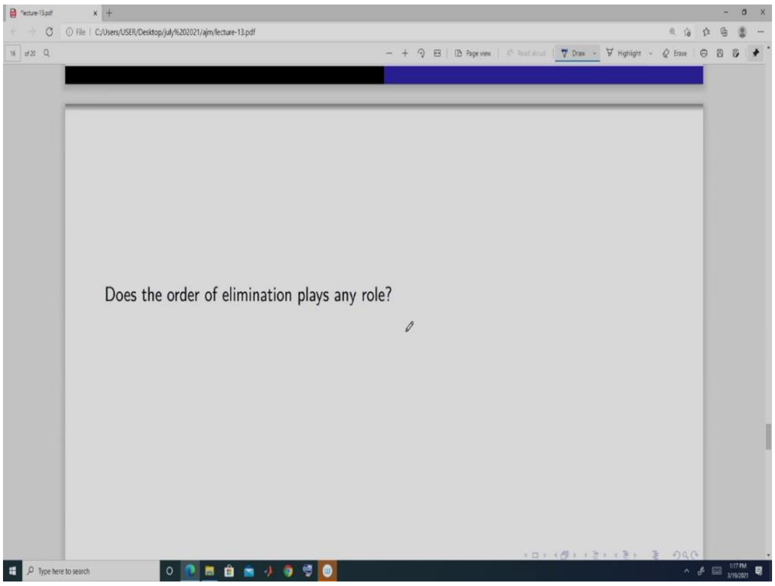

The next thing that we are going to do, and it is going to be in the next class, and that is whether the role or the order in which we eliminate the strategies or actions, whether that has any important role to play that if we change the order of elimination, then do we get a different outcome? Okay.

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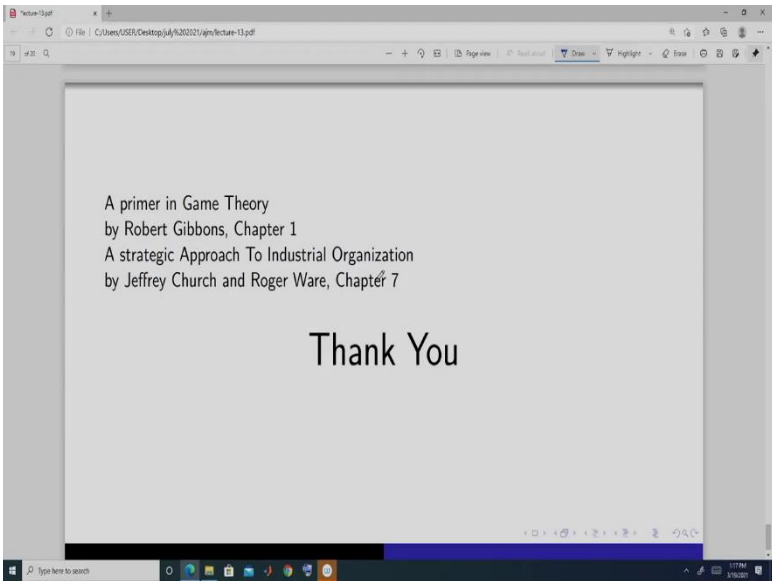

For this person, you can read chapter 1 of the book by Robert Gibbons, A Primer in Game Theory, or you can read chapter 7, from this book, A Strategic Approach to Industrial Organization by Jeffrey Church and Roger ware. Thank you.