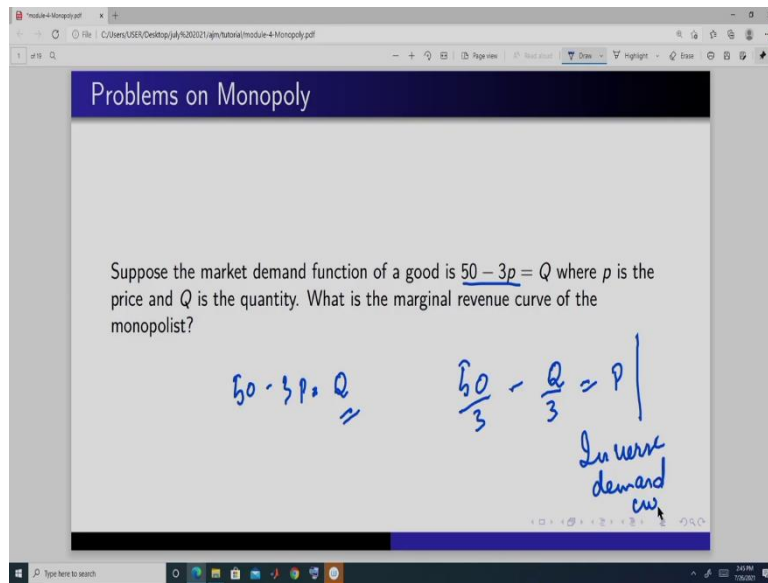


Introduction to Market Structures
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Lecture 16
Tutorial on Monopoly

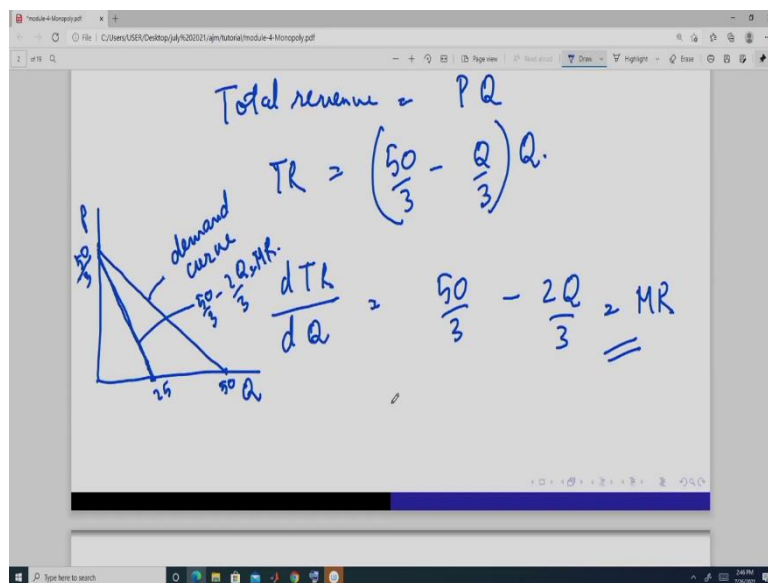
We will discuss few problems on monopoly. So, let us look at this first problem.

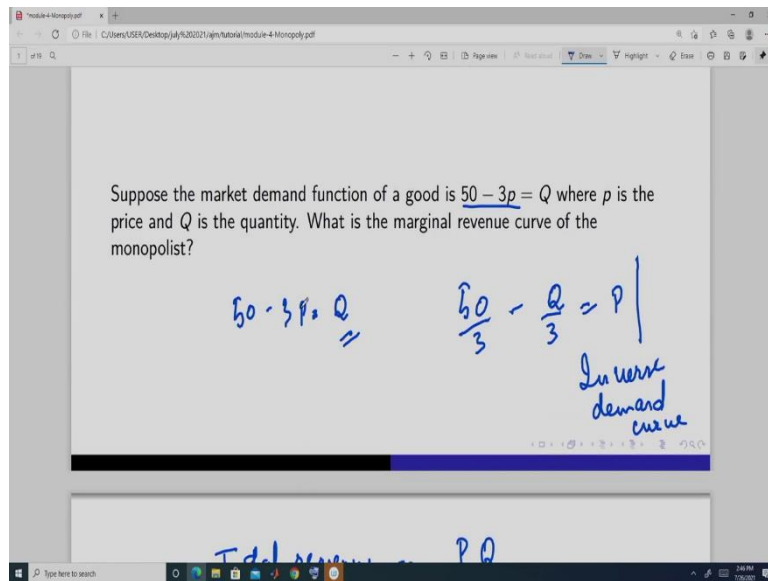
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Suppose the market demand function of a good is this- $50 - 3p = Q$ where p is the price and this Q is the quantity demanded, okay. What is the marginal revenue curve of the monopolist? Now, from this, if this is the demand curve, then this is the inverse demand curve- $\frac{50}{3} - \frac{Q}{3} = P$, okay.

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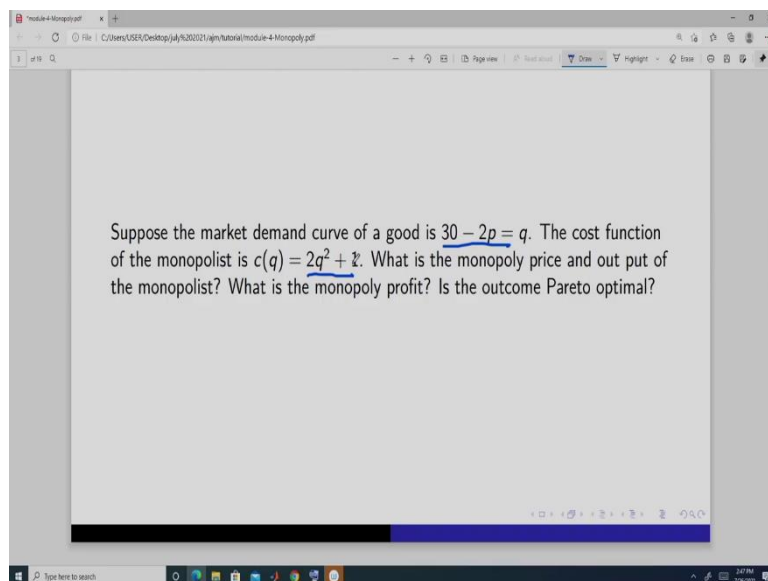




Now, total revenue is equal to price into quantity. So, here you apply the, this is the price- $\frac{50}{3} - \frac{Q}{3}$ and quantity is this- Q because total revenue is always a function of the quantity. We get this, this is the total revenue- $TR = (\frac{50}{3} - \frac{Q}{3})Q$ and since it is differentiable take the derivative of this with respect output that is q we get, this, this is $MR = \frac{50}{3} - \frac{2Q}{3}$ that is the marginal revenue and if we want to plot this we will get something like this if it is price and this is quantity.

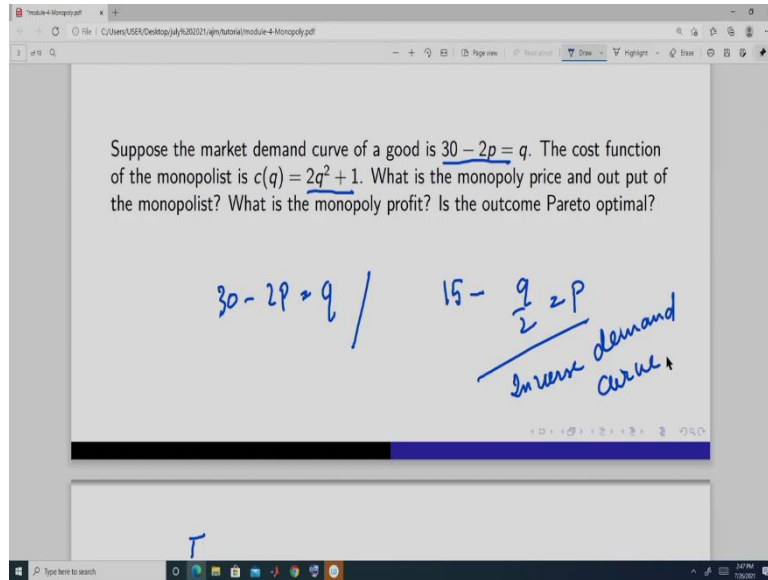
This point is 50, it is this, it is this divided by 3 this point is 50. This is the demand curve and this is the marginal revenue curve this curve and this point is going to be 25 from here, okay, this is how we get the marginal revenue curve of a firm our monopolist.

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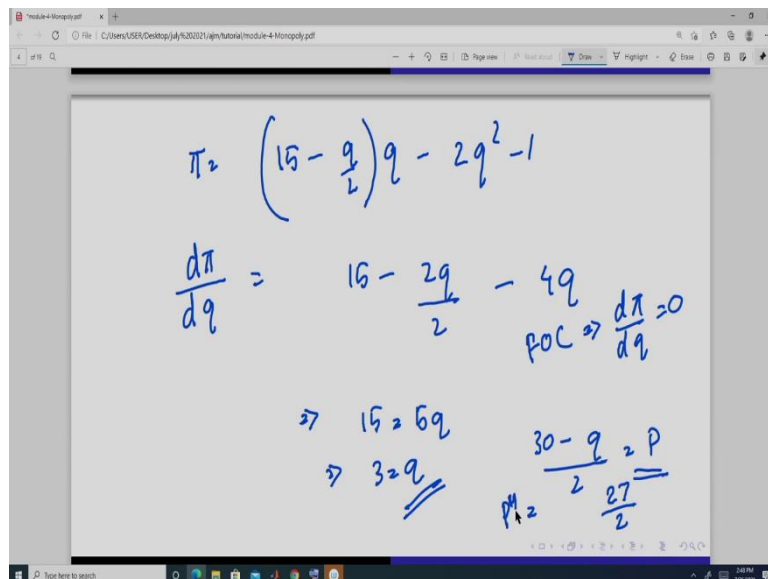
Next suppose next problem suppose the market demand curve of a good is this- $30 - 2p = q$, cost function of the monopolist is this- $c(q) = 2q^2 + 1$. So, marginal cost is strictly increasing here, this, what is the monopoly price and output of the monopolist. So, this and we have to find the monopoly profit also.

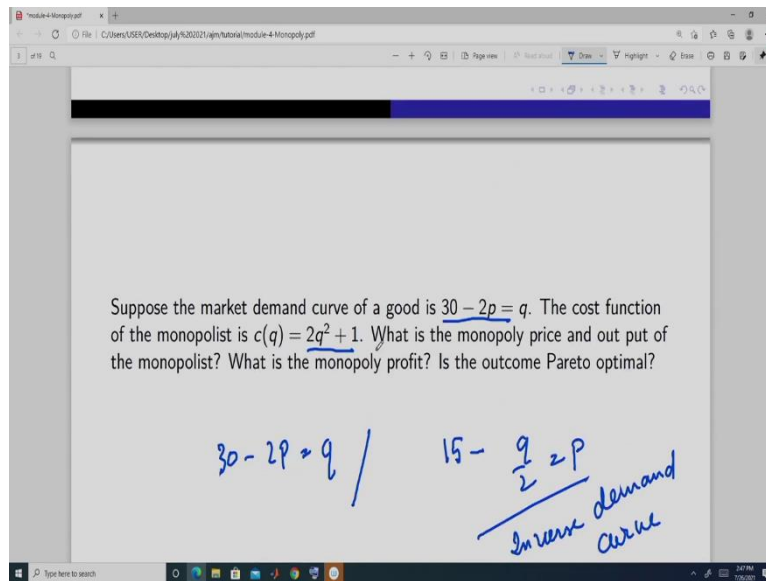
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So, the profit function so, from here let us first derive the. So, if this is the demand curve- $30 - 2p = q$, then the inverse demand curve is, is this- $15 - \frac{q}{2} = P$, this is the inverse demand curve, okay.

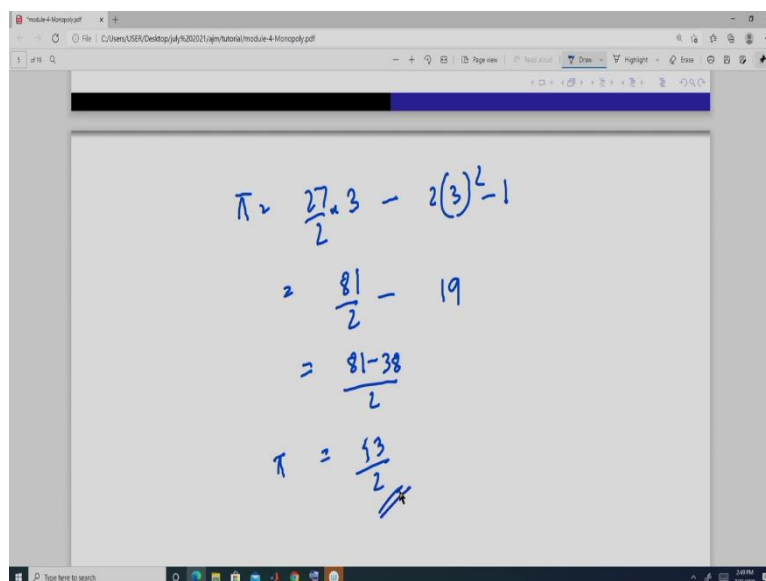
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Now, so, the profit function is $15q - 2q^2 - 1$, cost function is $2q^2 + 1$ and this is the fixed costs. So, this is the profit function- $\pi = \left(15 - \frac{q}{2}\right)q - 2q^2 - 1$ it is a differentiable function. So, when the monopolist will optimize this or maximize this with respect to output and we will get this first order condition will give that this equal to 0, i. e. $\frac{d\pi}{dq} = 15 - \frac{2q}{2} - 4q = 0$. So, we get this is the monopoly output $= 3 = q$ and what is going to be the monopoly price, monopoly price is going to be this- $p = \frac{30 - q}{2}$. So, plug in here it is 27 by 2 . So, this is the monopoly price- $p = \frac{27}{2}$ and this is the monopoly quantity- $3 = q$.

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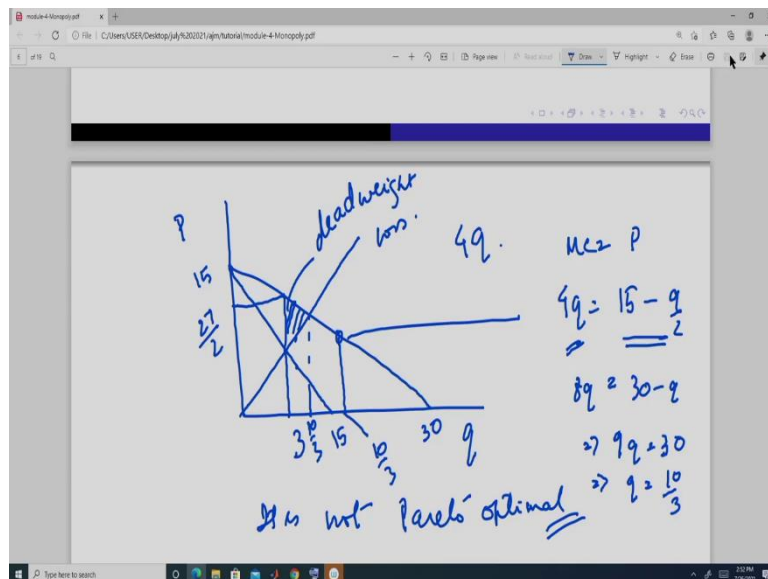
Suppose the market demand curve of a good is $30 - 2p = q$. The cost function of the monopolist is $c(q) = 2q^2 + 1$. What is the monopoly price and output of the monopolist? What is the monopoly profit? Is the outcome Pareto optimal?

$30 - 2p = q$ / $15 - \frac{q}{2} = p$
Inverse demand curve

$\pi = (15 - \frac{q}{2})q - 2q^2 - 1$

Now, what is going to be the monopoly profit? Monopoly profit is price is this, output is this and fixed cost is 1, so it is going to be this- $\pi = \frac{27}{2} * 3 - 2(3)^2 - 1$. So, this is 19. So, it is, so, it is going to be what? 43 by 2, this is going to be the monopolist profit- $\pi = \frac{43}{2}$.

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$\Rightarrow 15 = 6q$
 $\Rightarrow 3 = 2q$
 $q = \frac{3}{2}$

$P = \frac{30 - q}{2} = \frac{27}{2}$

$\pi = \frac{27}{2} \cdot 3 - 2(3)^2 - 1$

Now, whether it is a Pareto optimal point or not how do we decide that? See, this point is if you look at this is 15, this is 30, this is 15, marginal cost this is $4q$, it is this this is 3, this point is 27 by 2, this point is what this is $4q$ equal to 15, right? this where marginal cost intersects the demand curve. So, it is MC is equal to price at this. So, this if you solve this you will get this a and this is what sorry, it is this- $4q = 15 - \frac{q}{2} \Rightarrow 8q = 30 - q \Rightarrow 9q = 30 \Rightarrow q = \frac{10}{3}$.

So, this point is 10 by 3. So, this diagram will be slightly like this, suppose this is the point 3 and this point is 27 by 2 and this is 10 by 3. So, this reason is the deadweight loss. So, that is why it is, it is not Pareto optimal, we get this, okay. Now, let us solve another problem.

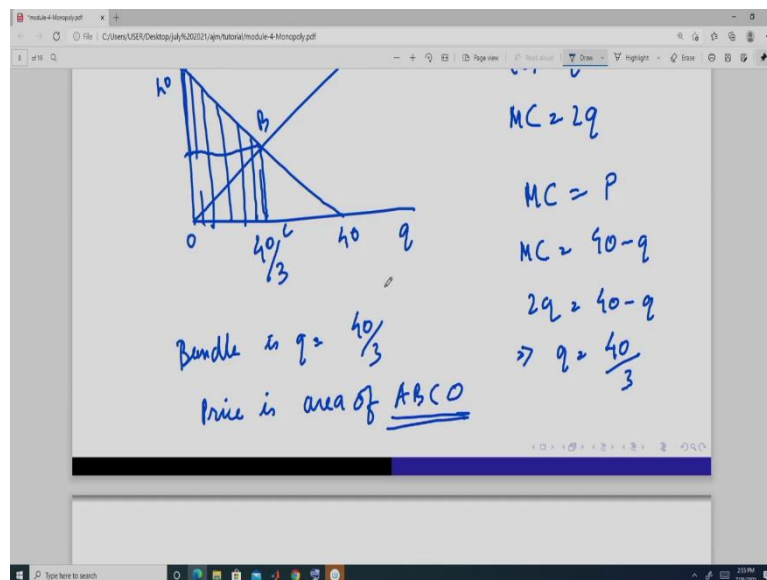
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Suppose the demand function of a consumer is $40 - p = q$ where p is the price and q is the quantity demanded. Suppose the cost function of the monopolist is $c(q) = q^2$. Suppose there are m number of such type of consumers. Suppose the monopolist can do first degree price discrimination. What is the bundle monopolist is going to sell? Is it Pareto optimal?

Suppose the demand function of a consumer is this- $40-p=q$, it is a downward sloping, where p is the price and q is the quantity demanded and suppose the cost function of the monopolist is this- $c(q) = q^2$, okay. And suppose there are m number of such type of consumers and suppose the monopolists can-do first-degree price discrimination. First degree price discrimination means that it can charge different prices for each unit of good and it can charge different prices to each person, okay.

Now, here all the persons are similar. So, it can charge different unit price to different amount of unit that it is selling. So, in this case, we have seen that the monopolist is actually doing a kind of bundling. What is the bundle monopolist is going to sell and is it a Pareto optimal? okay?

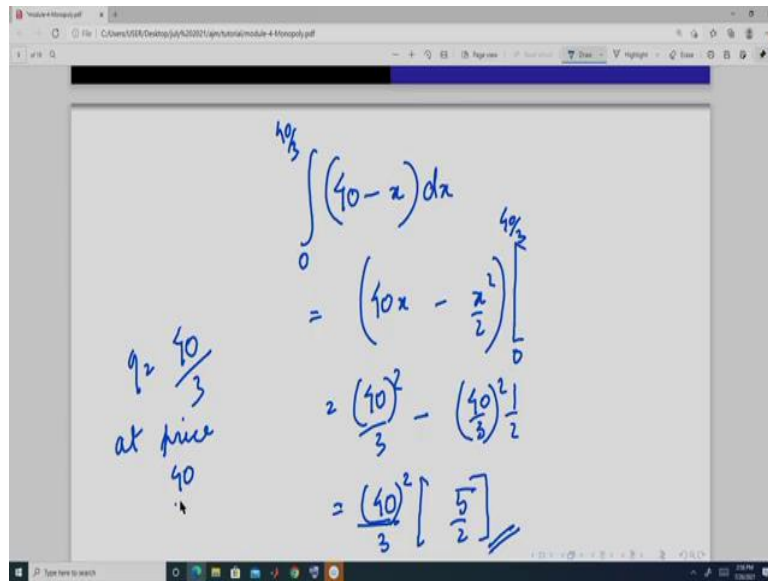
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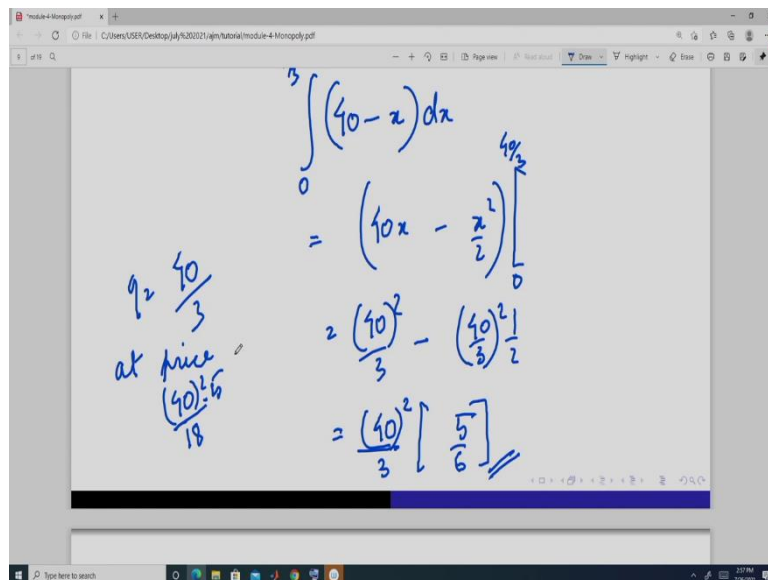
So, this is we will see, this is suppose quantity, price this point is 40. This point is 40. This is the demand curve and the marginal cost is because cost function is q square So, marginal cost is $2q$. So, this is MC, it is this, this point is where MC is equal to price. So, it is equal to 40 minus q , MC is $2q$, so this is equal to. So, this point is 40 divided by 3 , okay. Now, monopolist we know first degree will charge this whole amount.

So, monopolist will do it is the bundle is like this you get so, the bundle is q is equal to 40 by 3 and you get a price which is given by this whole area this is suppose A, B, C and O . So, price is area of A, B, C, O this, right?

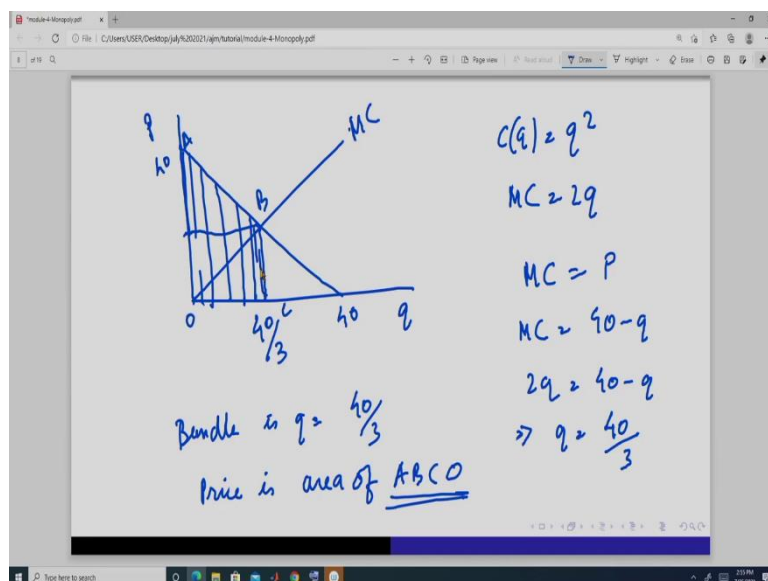
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Handwritten integral calculation for profit at $q = \frac{40}{3}$. The integral is $\int_0^{\frac{40}{3}} (40 - x) dx$. The result is $\left(40x - \frac{x^2}{2}\right) \Big|_0^{\frac{40}{3}} = 2 \left(\frac{40}{3}\right)^2 - \left(\frac{40}{3}\right)^2 \frac{1}{2} = \frac{(40)^2}{3} \left[\frac{5}{2}\right]$. To the left, it says "q = 40/3 at price 40".

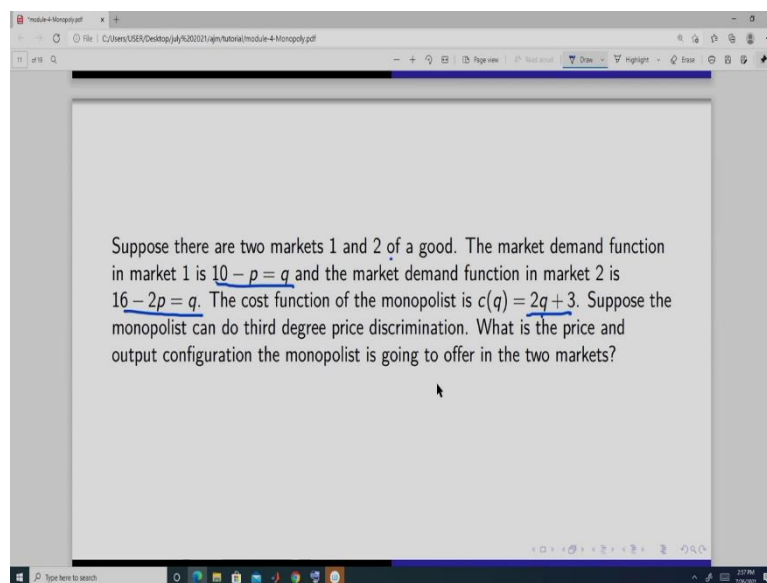


Handwritten integral calculation for profit at $q = \frac{40}{3}$. The integral is $\int_0^{\frac{40}{3}} (40 - x) dx$. The result is $\left(40x - \frac{x^2}{2}\right) \Big|_0^{\frac{40}{3}} = 2 \left(\frac{40}{3}\right)^2 - \left(\frac{40}{3}\right)^2 \frac{1}{2} = \frac{(40)^2}{3} \left[\frac{5}{6}\right]$. To the left, it says "q = 40/3 at price (40)/6".



So, this area you can find out it simply 0, 40 by 3 area under the demand curve and demand curve function you can write integrating over x here x is the quantity- $\frac{40}{3} \int_0^{40} (40 - x) dx$. So, it will give me this whole region and this is what? this is going to be so, this is you will get this and this amount is going to be this much amount. So, the price is bundle is that quantity 40 divided by 3 at price 40 square divided by sorry it is going to be 6, you can say it is divided by 18 into 5 this this is the price of this good, i.e $P = \frac{(40)^2}{18} \cdot 5$.

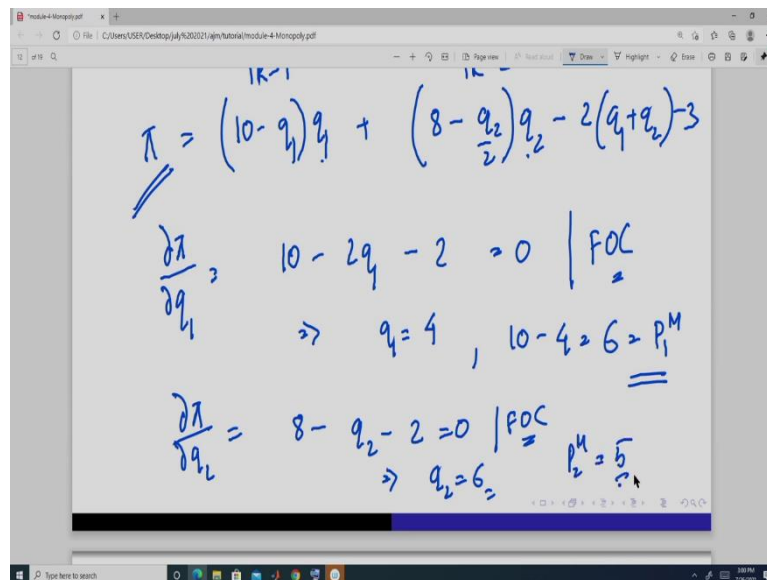
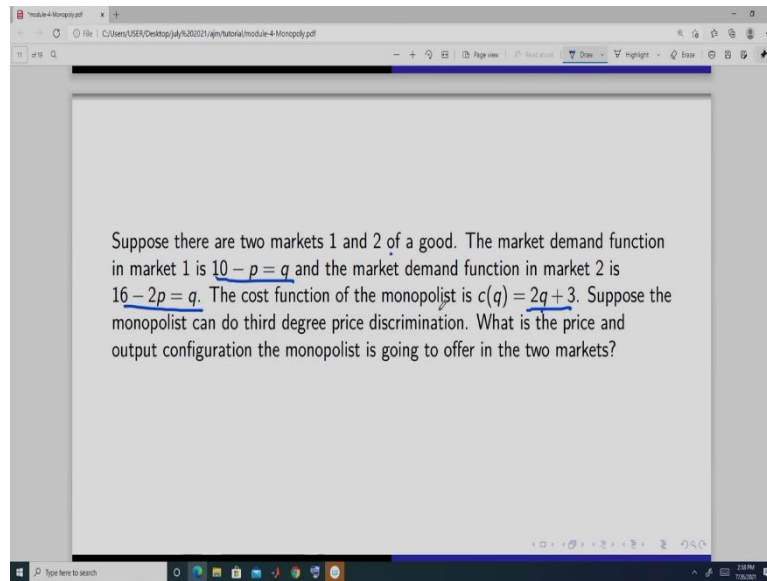
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Now, let us do another problem related to price discrimination and suppose this is this problem is related to third degree price discrimination. So, suppose there are two markets, market 1 and market 2 and demand function of market 1 is this- $10 - p = q$ and the demand function of market 2 is this- $16 - 2p = q$, cost function of the monopolist is this- $c(q) = 2q + 3$, okay. So, it is a constant marginal cost and suppose the monopolists can-do third-degree price discrimination it means what?

That the buyer of market 1 cannot go to buyer of market 2 and buyer of market 2 cannot go to buyer of, means buyer market 1 consumers which are there in market 1 cannot go to market 2 and the consumers who are there in market 2 cannot move to market 1, okay. This is like because of specific characteristics of the buyers like old person and the young person or the students or non-students or like geographically different location and they are very far apart, okay.

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Now, we will get this here. Market 1's demand function is this- $10 - p = q$ market 2's demand function is this- $16 - 2p = q$. So, market 1's output is q_1 , this is the total revenue TR from market 1- $(10 - q_1)q_1$ plus this is, this is total revenue from market 2- $\left(8 - \frac{q_2}{2}\right)q_2$, minus $2q_1$. So, this is the total mono profit earned by the monopolist- $\pi = (10 - q_1)q_1 + \left(8 - \frac{q_2}{2}\right)q_2 - 2(q_1 + q_2) - 3$. So, monopolist will choose q_1 and q_2 such that this profit is maximum. So, it will be this.

So, we will get this is first order condition- $\frac{d\pi}{dq_1} = 10 - 2q_1 - 2 = 0$ and then this will give me q_1 is equal to 4. So, the monopoly price is going to be this in market 1- $10 - 4 = 6 = P_1^M$.

Then it is again this is the first order condition will give me this- $\frac{d\pi}{dq_2} = 8 - q_2 = 0$. So, q_2 is equal to 6. So, plug in q_2 here. So, p_2^M is actually 5, i. e. $P_2^M = 5$.

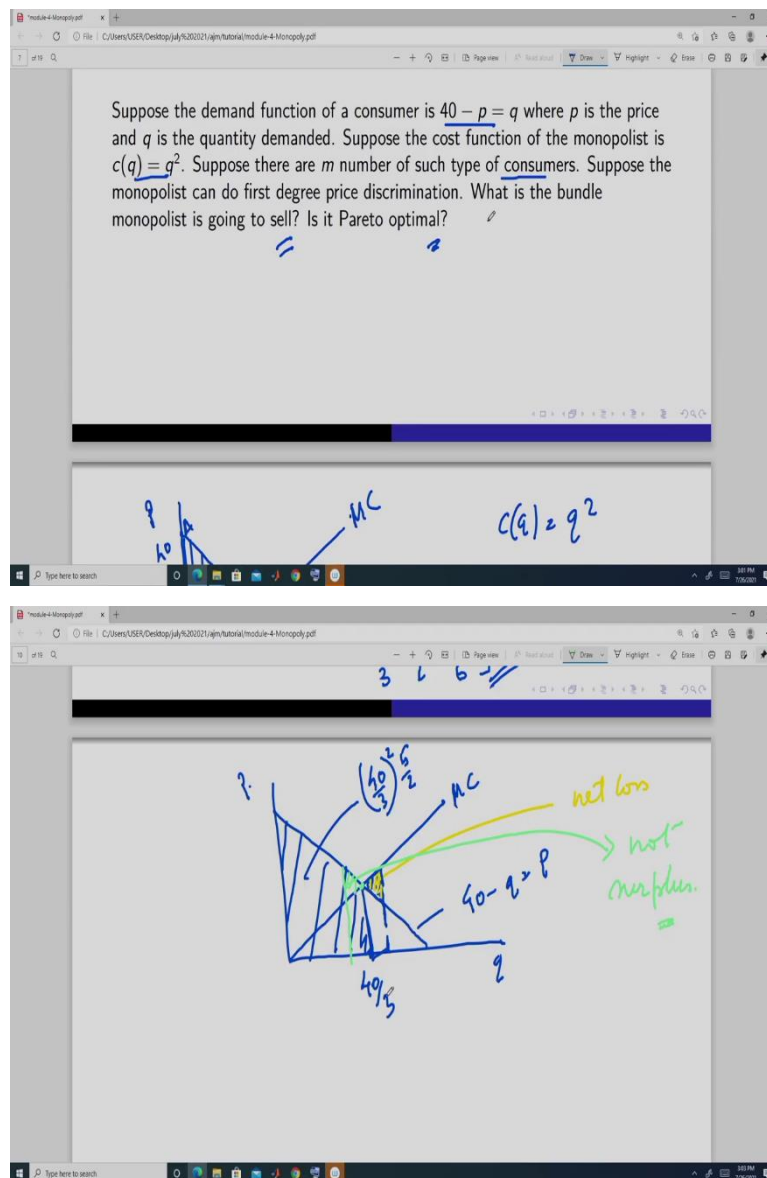
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The first screenshot shows the first-order conditions (FOC) for profit maximization with respect to quantities q_1 and q_2 . The FOC for q_1 is $\frac{\partial \pi}{\partial q_1} = 10 - 2q_1 - 2 = 0$, which yields $q_1 = 4$ and a corresponding price $P_1^M = 6$. The FOC for q_2 is $\frac{\partial \pi}{\partial q_2} = 8 - q_2 - 2 = 0$, which yields $q_2 = 6$ and a corresponding price $P_2^M = 5$. The profit π is noted as 6.4.

The second screenshot shows the calculation of the total profit π by substituting the optimal quantities and prices into the profit function: $\pi = 6.4 + 5.6 - 2(4+6) - 3$. This simplifies to $\pi = 24 + 30 - 23 = 31$.

So, what is the monopoly profit here now? Monopoly profit is price is 6 here and output is 4, right? 6, 4 plus price is 5 output is 6 minus 2, 4 plus 6. So, this is 24, 30, 24 plus 30 minus this is 23 So, this is 31. This is the profit if it does third degree price discrimination- $\pi = 6.4 + 5.6 - 2(4 + 6) - 3 \Rightarrow 31$, okay.

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I have left one thing in the previous question and that is this question that is, is it Pareto optimal? That we have already discussed. So, what is the outcome we have got? This is the marginal cost, this is demand and this is the price, okay. This whole shaded area is the price and that price is 40 by 3 square 5 by 2, this- $\left(\frac{40}{3}\right)^2 \cdot \frac{5}{2}$. Now, if it produces here then net loss it makes is this amount, this yellow amount is the net loss if it produces here is the net loss.

Because revenue it is getting his additional revenue is this, but additional cost is this much so, this is the net loss. If it is producing here, then it is foregoing this much revenue and the cost is this much so this much is the net surplus it is foregoing. So, that is why this point here it is not foregoing any loss for it is not foregoing any surplus neither it is making any losses. So, that is

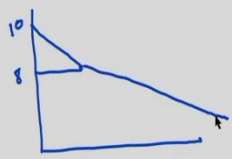
why this point is the Pareto optimal in the sense that it is making the monopolist getting all the surplus and the consumers are not getting any surplus.

But if you want to move anywhere, if, then there are some amount of surplus that they can make, but no one is making or they make some losses and the monopolist is making losses here. So that is why this is the Pareto optimal position, okay.

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Consider the previous example and suppose the monopolist cannot do third degree price discrimination. What is the monopoly price and output? Compare the profits and find out whether it is profitable to do third degree price discrimination.

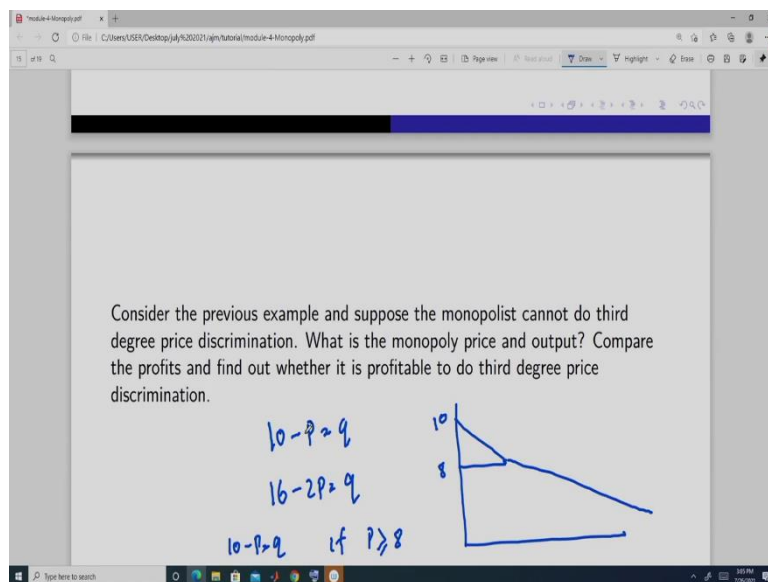
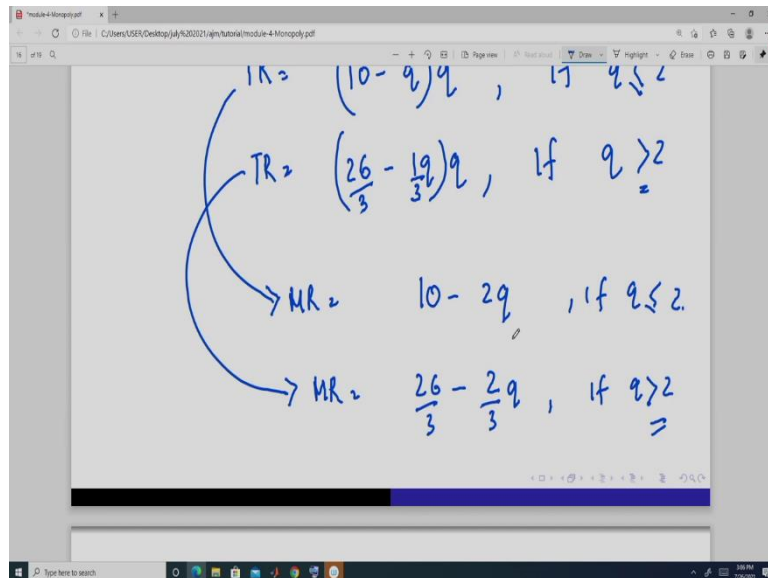
$10 - p = q$
 $16 - 2p = q$
 $10 - p = q$ if $p \geq 8$
 $26 - 3p = q$ if $p < 8$



Suppose there are two markets 1 and 2 of a good. The market demand function in market 1 is $10 - p = q$ and the market demand function in market 2 is $16 - 2p = q$. The cost function of the monopolist is $c(q) = 2q + 3$. Suppose the monopolist can do third degree price discrimination. What is the price and output configuration the monopolist is going to offer in the two markets?

Next, let us solve another problem. That is this consider the previous module where this this problem these demand curves, there are two demand curve this- $10 - p = q$ and this- $16 - 2p = q$ and suppose here the firm cannot do third-degree monopoly, third-degree price discrimination then what is going to be the market? So, the if you look at this, it is this, okay. So, market demand curve is 8 and it is like this, this is the market demand curve- $26 - 3P = q$, if $P < 8$.

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So, from here what do we get? We get the total revenue curve is- $TR = (10 - q)q$, if $q \leq 2$ if you plug in this equal to 8 here quantity is, if q is less than equal to 2 and the total revenue is this- $TR = (26 - 3q)q$, if $q > 2$. Now, let us look at this marginal revenue curve, this marginal

revenue curve is this- $MR = 10 - 2q$, if $q \leq 2$ if q is this marginal revenue curve is it is this, right? So, it is this now, if we look at this, if you plug in 2 here two this is equal to 5.

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$TR = (10 - q)q$, if $q \leq 2$
 $TR = \left(\frac{26}{3} - \frac{1}{3}q\right)q$, if $q > 2$
 $\rightarrow MR = 10 - 2q$, if $q \leq 2$
 $\rightarrow MR = \frac{26}{3} - \frac{2}{3}q$, if $q > 2$

$16 - 2p = q$
 $10 - p = q$ if $p \geq 8$
 $26 - 3p = q$, if $p < 8$

$TR = (10 - q)q$, if $q \leq 2$
 $TR = \left(\frac{26}{3} - \frac{1}{3}q\right)q$, if $q > 2$
 $\rightarrow MR = 10 - 2q$, if $q \leq 2$

$MR = MC$

$p \geq \frac{26 - 10}{3}$
 $= \frac{16}{3}$

$\frac{26}{3} - \frac{2}{3}q = 2$
 $\Rightarrow 20 = 2q$
 $\Rightarrow q = 10$

And if you plug in 2 here this is equal to 26 minus, so, this is 22 by 3- $MR = \frac{26}{3} - \frac{2}{3}q$, if $q > 2 \Rightarrow \frac{26-4}{3} = \frac{22}{3}$, right? and if you plug in 2 here it is 5. So, if you look at price here, total revenue, marginal revenue here this point is suppose, this is 2, then this is somewhere here, this is suppose 5 and this is 7 point something, so, it is this. So, this is the marginal revenue curve. This for here and this till this much and marginal cost is this is 2.

So, the monopoly profit is, monopoly output is this because marginal revenue is equal to marginal cost it is here. So, it is this would be equal to 2. So, this is q is equal to 10 this point is 10 and this price is plugin price in this portion 10, so, it is price is 26 minus 10 divided by 3. So, it is 16 by 3. So, this point is actually 16 by 3, okay.

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Handwritten calculation on a digital whiteboard:

$$q = 10, P = \frac{16}{3}$$

$$\pi = \frac{16}{3} \times 10 - 20 - 3$$

$$= \frac{160}{3} - 23$$

$$= \frac{160 - 69}{3}$$

$$= \frac{91}{3}$$

Handwritten calculation on a digital whiteboard:

$$\pi = 6.77 \times 0.6$$

$$= 24 + 30 - 23$$

$$\pi = 31$$

So, here monopoly output is 10 and monopoly price is 16 by 3. What is the profit here? Profit here is this- $\pi = \frac{16}{3} * 10 - 20 - 3 \Rightarrow \frac{160}{3} - 23 \Rightarrow \frac{160-69}{3} = \frac{91}{3}$ price 13 this revenue cost is 20 fixed cost is 3. So, this is, so this is, 71 by 3. If we do this because to be 10 and it will be this, what is so, this is 30 point something 30.33 and the profit here is 31. So, if we compare the a what do we get?

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Handwritten notes on a whiteboard:

- 3rd degree price discrimination = 31
- $\pi^M = \frac{91}{3}$
- $\pi^{3M} > \pi^M$

We get that the profit in third degree price discrimination is equal to 31 and simply monopoly price is 91 by 3. So, third degree M is greater than by simply M. So, there is always going to be a tendency among the monopolist to do third degree price discrimination if it is possible for them, okay. Thank you.