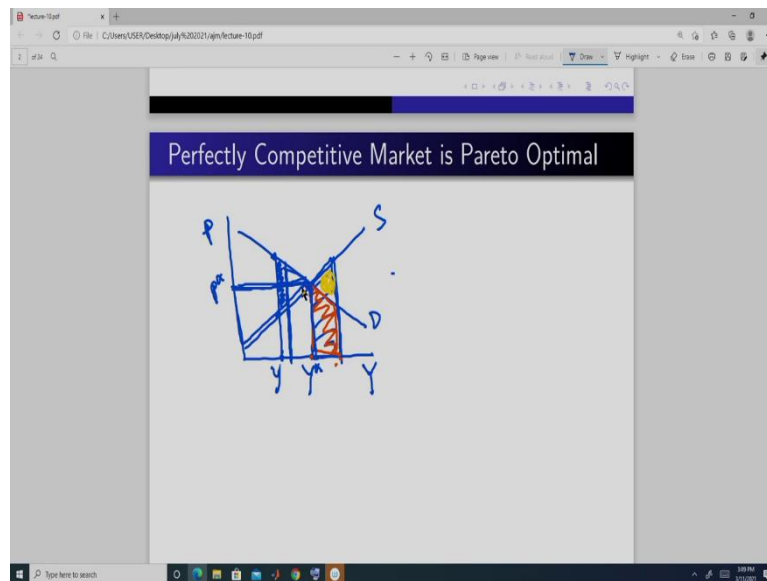


Introduction to Market Structures
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Module 04
Monopoly Price

Welcome to my course Introduction to Market Structures. Today we will do, first do the last portion of perfectly competitive market, and then we will do monopoly.

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In a competitive market we know how the price, market price is determined. If this is the market quantity, this is the price, this is the market demand and this is the market supply, then the equilibrium price in the market is this and the aggregate output produced and sold in the market is this y^* .

Now, we know that this point is in the demand curve and also in the supply curve. So since it is in the supply curve it means it is a point in the marginal cost of each firm. And at this price marginal cost is equal to the price. So it is maximizing the profit of firms. And also at the same time, since it is in the demand curve so it is also maximizing the utility of the individuals.

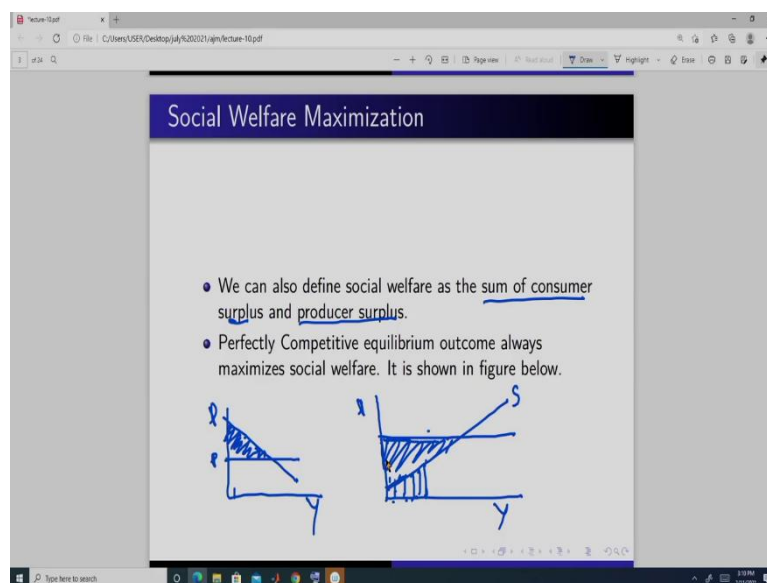
But suppose we take an output like this, this output. This output, this, at this output people willing to pay is this much and the cost is this much, okay. Now if they can produce a slightly more, here, so what is happening? This much addition is coming. Like this is the, we know this amount is the producer surplus and this is the consumer surplus. So this much is the

addition to the surplus. So that is why this output is not Pareto-optimal because we can make both the individual better off.

Similarly, instead of this if we are producing here then the amount of additional cost is this, and the amount people are willing to pay is only this portion. So this yellow portion is the additional cost which cannot be borne by the seller, because the producers, buyers are only willing to pay this much amount and seller wants this whole amount. So this is the loss that they are going to make.

So, that is why to produce more than y^* , and suppose it is this it is not Pareto-optimal. You cannot make anyone better off without hurting anyone, okay. So that is why since we cannot move to any other point, then from this, so that is why this point is a Pareto-optimal point, okay. This we had done.

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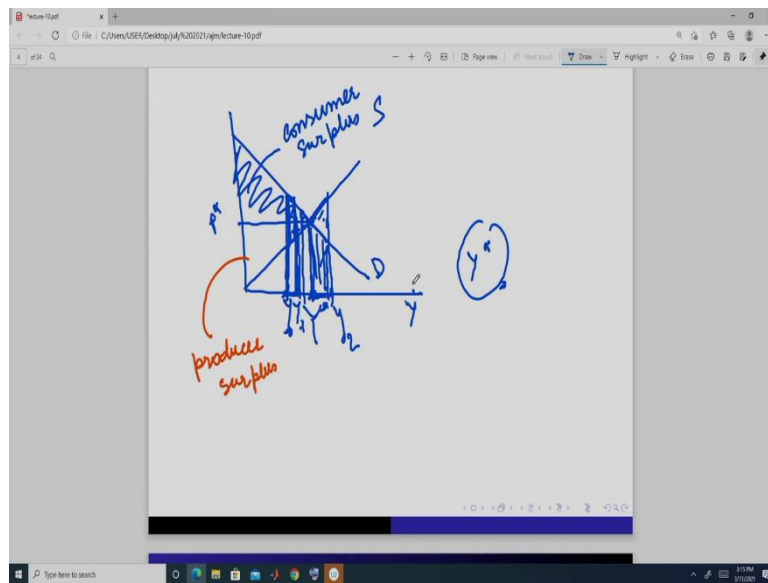
Now, we will do what do we mean by social welfare. This we also have, we have just defined it in the last class. So social welfare is the sum of consumer surplus and the producer surplus. And suppose this is the demand curve, market demand curve. And this is the price. And suppose this is the market price.

Then this triangle is the consumer surplus, because for each unit consumers are willing to pay this point which are in the demand curve. But they are actually paying this. So this much is the surplus that it they are getting. And if we take this price and suppose this is the supply curve, market supply curve and the market price is this, suppose, then this region is the producer surplus.

Why? Because their cost is only this much for each of these units, marginal cost, right? Sum of the marginal cost, horizontal sum of the marginal cost gives me the supply curve. So these heights are the marginal costs for producing this much amount of output.

Now, but they are getting this much prices for each of these units. So this whole region is the total producer surplus. So competitive outcome, the outcome in the competitive market, what it does? Actually maximizes this sum of consumer surplus and the producer surplus.

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How that happens? Suppose this is the demand curve and for simple exposition we are taking them to be the straight line. So this is going to be our market price. And this is going to be the market output, optimal market output or the equilibrium output. Here this region is consumer surplus. And this region, this orange color is the producer surplus, right?

Here instead of producing this, suppose we produce this much. Then if we increase the output by little bit amount, this much, the market output then what we are doing? We are adding this portion, this whole portion. Now here cost is only this much which is given by, suppose this. So initially this is the equilibrium output, market output. And suppose instead of this we produce this, y^* .

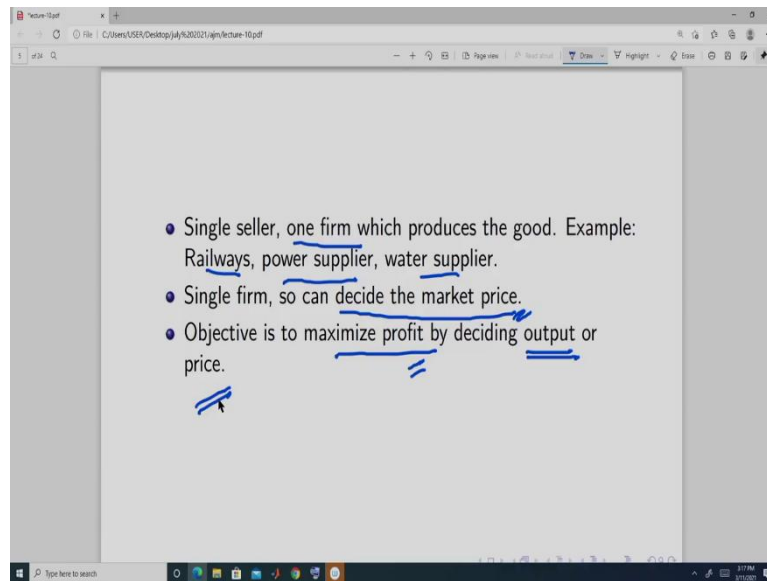
At y^* what we are doing? We are producing this much. Now suppose instead of this we produce y_1 . Then we are adding this portion. Here this much is the cost, and this much is the total revenue, because consumers are willing to pay this much amount. And so this into this, so this whole the producer can take because they are willing to pay this whole amount and this is the cost. So this much is the surplus.

So, if we stick to this then we are not adding this much. So if we switch from this y naught to y_1 we are adding some additional surplus which is sum of producer; and the sum of producer and the consumer surplus. So this is not social welfare maximizing. So we will move to this. So like this we can go on arguing till we reach this point, right? Because if we are here again we can add this triangle as a surplus which is sum of consumer and producer surplus. So we will continue till this point.

Now, suppose we produce this much amount of output which is y naught. Now here instead of producing y star that is the equilibrium output if we produce this y_2 then additional cost borne by all the firms is this much. An additional amount the consumers are willing to pay is only this. So this triangle is the loss, right? That is the additional cost, part of that extra cost which cannot be borne by the amount consumers are willing to pay. So that is why this is going to be something like a loss.

So, if we move right to this y star then there is a possibility that we may make some loss. So that is why it is not optimal to move in the right of this y star. So that is y star, this amount that is being produced in a perfectly competitive market is the social welfare output, okay. So this is the perfectly competitive market.

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Now, we will do another topic and that is monopoly market. And monopoly market is where, as a special form of a, in a monopoly there is only one firm okay, one firm or there is a single seller. Or you can say that there is only one producer.

Example like Railways it is a single firm or it is a only one, we have only one seller that is the Indian Railways. It may be owned by a government but it is only one provider, okay. Power supply in our house it is mainly by, we do not have the option to choose from multiple sources or from multiple firms. We have only one power supply. Water supplier, all these are monopolies. Water supplier in most of the places, it is done by the municipal corporations. So all these are form of a monopoly.

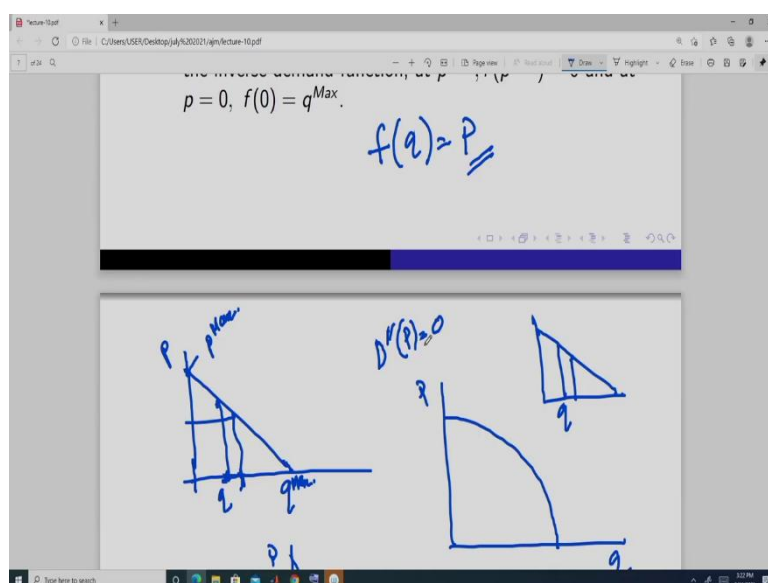
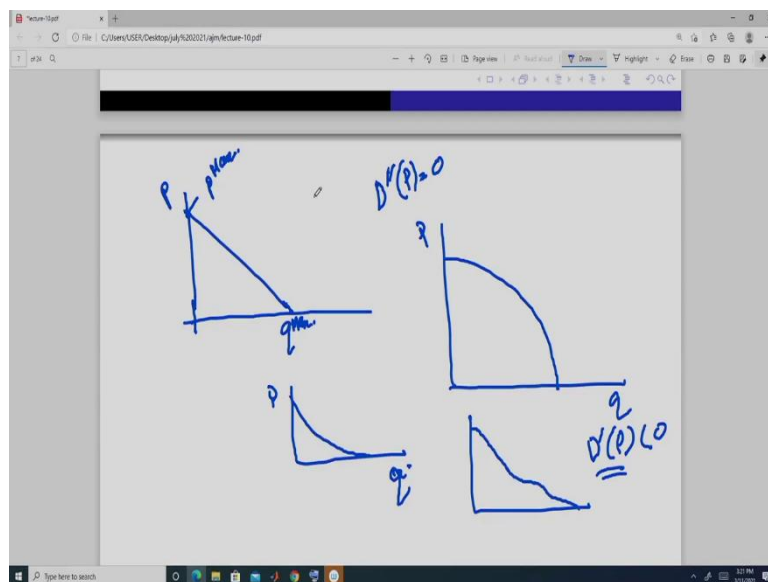
So, in a monopoly there is only one firm. And since there is only one firm they can decide the market price, okay. Now what the objective of this firm is, which is a monopoly firm, is to maximize profit. And it can maximize its profit either by deciding the amount of output it is going to sell in the market or the price that it is going to set in the market. So either it can decide the profit maximizing price or it can decide the profit maximizing output. These both will give the same outcome, okay.

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The general market demand function is $D(p) = q$.

- $D'(p) < 0$ for all p .
- There exists p^{Max} such that at $p = p^{Max}$, $D(p^{Max}) = 0$.
- At $p = 0$, $D(0) = q^{Max}$.
- We have inverse demand function, since demand function is strictly decreasing in price. It is given as $f(q) = p$. In the inverse demand function, at p^{Max} , $f(p^{Max}) = 0$ and at $p = 0$, $f(0) = q^{Max}$.

$f(q) = p$



And next we will specify the market that is the nature of the firm in this market. So first we will specify the market demand. So this is our demand curve, demand function you can say which is given by this function $D - D(p) = q$. So you plug in the prices and you know the market output is this q , market demand.

So, at each price p , how much is demanded in the market of this good, of this particular good is q . So if this is suppose electricity you can say, you plug in the price of electricity, how much amount of electricity is been demanded with this.

We have got this from the individual demand curve which we have got from the consumer optimization. There are each consumer does, that is the utility maximizing, maximization subject to a budget constraint. And to this function see we will keep a general function here and then we will solve.

Later on we will solve one example and we will also give a diagrammatic explanation of this thing. So then things will be more clear, if you are getting a bit confused or if it is a bit vague for you, right now.

So, we specify this demand function in such a way that it is downward sloping. So that is why the slope is negative. And this demand curve has a price that is P^{Max} . If you plug in P^{Max} it is so high that there is no demand at this price. So this is demand is 0, i.e $D(p^{Max}) = 0$. And at a price, zero price, so the maximum demand that is generated is this q^{Max} , i.e $D(0) = q^{Max}$, okay.

Now, since this is a strictly decreasing because function because of this slope is negative so we can have an inverse of this, and the inverse function is this- $f(q) = P$. So what this inverse does? Inverse allows us to make this function. Now it is a function of q . So we plug in quantity. We know what price the consumers are willing to pay. If we plug in this much output in the market in this function then how much maximum the consumers are willing to pay is this price, okay.

Now, given this specification of demand curve we will get this if we take price here and quantity here, this price is the p^{max} . So at this price the quantity demanded is 0. And this price is q^{Max} . So, when the price is 0 this is. So this demand curve is downward sloping if it is like this. It is a straight line, if it, second derivative is 0. It can take a form like this also, right? Or it can take a form like this also, okay. We can, any one of this will do.

In fact, it can take a form, not like this so much curl is not possible but at least like this it is possible. As long as the derivative is negative, it is, it can take any form, right, so all these variations are possible. Out of all these variations we will again assume only few and that we will come to later. We will specify later. But all these demand functions are possible.

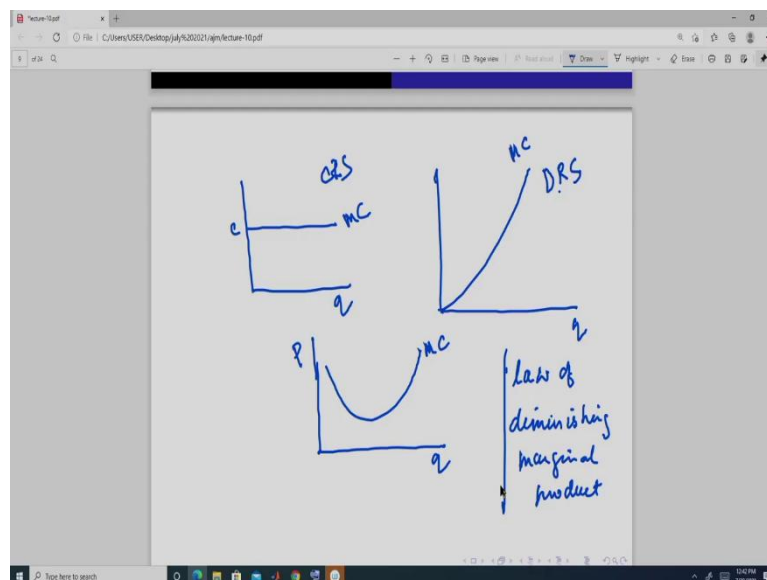
Next we specify, so the demand situation scenario is clear. So it is a downward sloping demand curve. So if you want to sell more you have to reduce the price. So if you are selling this much amount of output, now if you want to increase the market output then you have to reduce the price. At this price you cannot sell more than this, this amount. So this is, this idea is very important.

Like when we are saying downward-sloping it means that if you are selling this much amount of output at this price and if you want to sell more output you have to reduce the price, the market price, okay. Otherwise it is not possible.

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- The cost function of the firm is $c(q) + F$.
- $c(q)$ is the variable cost and F is the total cost. We also assume $c(0) = 0$.
- We further assume that $c'(q) > 0$ and $c''(q) \geq 0$.
- $c'(q)$ is the marginal cost.
- It means that cost function is constant returns to scale (CRS) or decreasing returns to scale (DRS).

$c(q) = cq$ $MC = c$
 $c(q) = cq^2$ $MC = 2cq$



Now, cost function is of this nature- $c(q)+F$. So this portion is the variable cost and this F is the total cost. And we assume that if output is 0 this c is equal to 0. So that means variable cost is only incurred whenever there is a positive amount of output, okay.

Now, we assume that it is, this is the marginal cost- $c'(q) > 0$. It is always positive, this is marginal, and the second derivative of this cost is always non-negative, this- $c''(q) \geq 0$. So what does this mean? It means that either this can take a 0 value or it can take some positive value. It means that either it the technology, cost function is CRS or it is like DRS, decreasing returns to scale.

So, this portion you can specify either like this- $c(q)=cq$, when this is positive because marginal cost here is c and the derivative of marginal cost is 0. So this is also satisfied. Or

you can take a form like this- $c(q) = cq^2$. Now here marginal cost is like this- $MC = 2cq$, okay. These are the examples of marginal cost. And this we get when we have CRS or DRS and also when one of the factor is fixed and the other is variable.

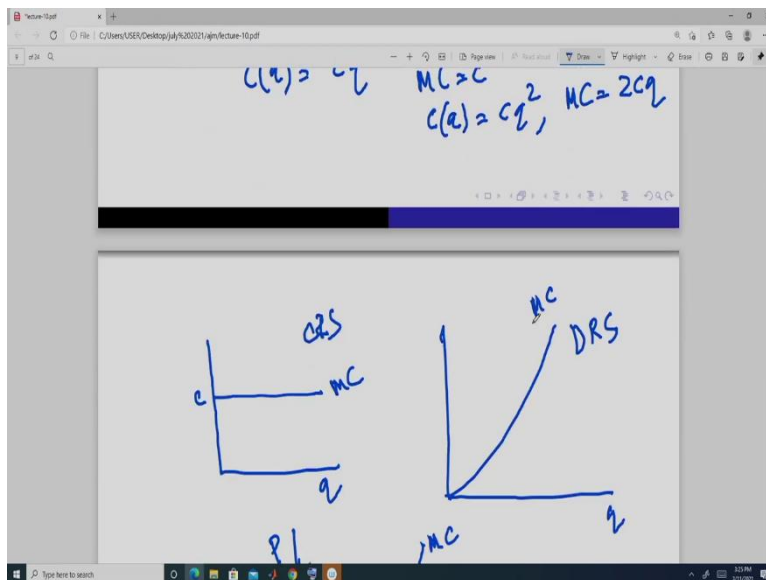
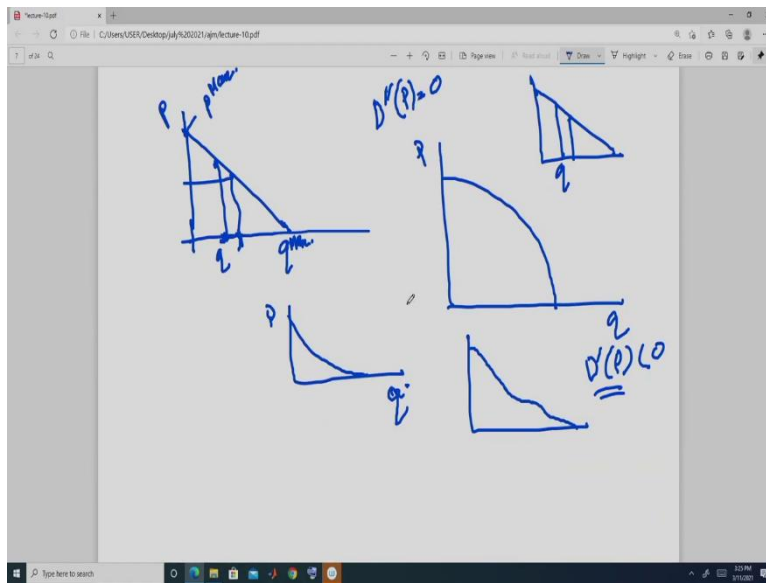
Like a capital is fixed and labor is variable and due to operation of law of diminishing marginal product we get a upward-sloping marginal cost function. And here when we take this F to be a fixed component it is coming from, suppose the rent that we pay for the building, that. Or when we have only one factor is fixed that is giving us this component. Together that with rent is giving us this component. And the second derivative is also positive, right? and it is $2c$.

So, this means that our, if we plot output here then marginal cost can be of this nature when it is CRS, or it can be of this nature when there is DRS. This is the marginal cost and this is the marginal cost. So we consider only these two forms.

But there is a possibility that we have a marginal cost like this. So we do not introduce that the optimization that we are going to do but we will consider this case separately. Later on you will see. So this is one example of DRS we have, marginal cost.

But this kind of marginal cost function may be due to law of, due to law of diminishing Marginal product, marginal product. When one input is fixed, another input is variable. Like capital is fixed and labor is variable. Then also we will get this. So we have, there is or we may have here this kind of situation, okay.

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So, what we have? We know the demand curve in the market is of any one of this form. And the cost curves are; marginal cost is either this or this, okay.

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$f(q)$

- The profit function of the monopolist is $\pi = f(q)q - c(q) - F$.
- The above profit function is function of monopolist output q . In this case the monopolist maximizes with respect to quantity q .
- We can also write the profit function as function of price. $\pi = D(p)p - c(D(p)) - F$.
- When profit is taken as function of price, the monopolist maximizes profit with respect to price.

$D(p)$ $D(p)p - c(D(p)) - F$

- Suppose the profit function is a function of quantity. So $\pi = f(q)q - c(q) - F$. Since $f(q)$ and $c(q)$ are differentiable so we use calculus to find the optimal q .

$$\frac{d\pi}{dq} = f(q) + f'(q)q - c'(q)$$

First order condition says gives that

$$\frac{d\pi}{dq} = f(q) + f'(q)q - c'(q) = 0$$

$$\Rightarrow f(q) + f'(q)q = c'(q)$$

We get the optimal output q^* by solving the equation

$$f(q) + f'(q)q = c'(q)$$

At q^* , $f(q^*) + f'(q^*)q^* - c'(q^*) = 0$

Now, we specify the objective of the firm. So objective of the firm, we know is to maximize profit. Now we can specify the objective in this form- $\pi = f(q)q - c(q) - F$. So this is, here this portion is, so this is price into q , amount of output produced by the monopolist. So this is the total revenue- $f(q)q$. Cost is c minus f , this is the total cost- $c(q) - F$, okay. So this is the profit of this firm, total revenue minus total cost.

Here you notice that this profit function is a function of output, so output that firms are producing. So if we want to maximize this produce, this, maximize the profit, since the profit function is a function of output so we will maximize it with respect to quantity that is q . So the firm will decide the amount of output it wants to produce. And from that we can derive

the market demand. And that is the monopoly price by plugging in the demand curve that is this, inverse demand curve.

Or we can do, write the profit in this form- $\pi = D(p)p - c(D(p)) - F$. Here this amount, this DP is the, you plug in the price, you get the quantity. So this is the quantity into price. So this is the total revenue- $D(p)p$. Here cost is always a function of quantity. So it is this- $c(D(p))$. So this is the variable cost minus, so this portion is the total cost and this is the total revenue. So this is the total profit- $\pi = D(p)p - c(D(p)) - F$.

But here this profit function is a function of price. So if monopoly considered this, its profit function in this form then the monopolist will decide the price, and given the market demand, if you know the price you know the amount that the consumers will buy from here. So in this objective function the price will be determined and then also, corresponding output will be, we will get the corresponding output from this demand curve.

In this here what will happen? The output will be determined monopoly output and corresponding monopoly price will be determined from this, okay. So this is the objective of the monopoly's, objective function of the monopolist, okay.

Now monopolist is going to maximize this. And it is a conventional in the whole, in the literature to take this function- $\pi = f(q)q - c(q) - F$, okay. We generally do not consider this function- $\pi = D(p)p - c(D(p)) - F$. So all the monopolist decides the or the monopolist decides the output. And the corresponding monopolist price is determined based on the demand curve, okay.

So, suppose this is the profit function- $\pi = f(q)q - c(q) - F$, okay. And we know this function, this inverse demand curve and this cost, variable cost function they are differentiable. We have assumed that. So we now use calculus to find the optimal q .

How do we do? We take the derivative of **the** profit function with respect to q . If we take the derivative we will get this- $\frac{d\pi}{dq} = f(q) + f'(q)q - c'(q)$. So derivative of this is this $f q$ plus f dash q into q . And this is the. Now here first order condition will give that this should be equal to 0. So this is the first order condition- $\frac{d\pi}{dq} = f(q) + f'(q)q - c'(q) = 0$. From here we can write this- $f(q) + f'(q)q = c'(q)$

Now here see, so if we are given this first order condition, now what we can simply do? We can solve this first order condition. So we will get an equation. If you plug in specific form of this here we will do that later on when we do an example.

But generally also you can do. You can solve this equation and you will get a q^* . And this q^* is, should be a profit maximizing output, right? So we will get this. So at q^* , it should be, first order condition should be satisfied- $f(q^*) + f'(q^*)q^* - c'(q^*)$., okay, by solving this equation, okay.

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Suppose the profit function is a function of quantity. So $\pi = f(q)q - c(q) + F$. Since $f(q)$ and $c(q)$ are differentiable so we use calculus to find the optimal q .

$$\frac{d\pi}{dq} = f(q) + f'(q)q - c'(q)$$

First order condition says that

$$\frac{d\pi}{dq} = f(q) + f'(q)q - c'(q) = 0$$

$$\Rightarrow f(q) + f'(q)q = c'(q)$$

We get the optimal output q^* by solving the equation $f(q) + f'(q)q = c'(q)$.

At q^* , $f(q^*) + f'(q^*)q^* - c'(q^*) = 0$

$$f(q) + f'(q)q - c'(q) = 0$$

- The first order condition implies marginal revenue equal to marginal cost at the optimal point.
 $f(q) + f'(q)q =$ marginal revenue. It is derivative of total revenue.
- Marginal revenue (MR) is the additional increase in revenue received by the firm when it sells one more additional unit.
- It lies below the demand curve since the demand curve is downward sloping. To increase sells, the firm has to reduce price. So additional revenue at the margin falls.

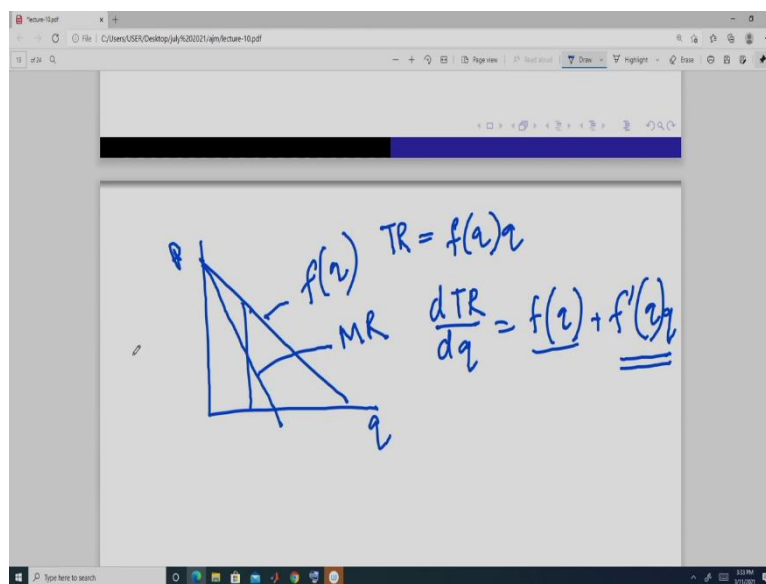
Now, here you will see what is this portion. If you look at this, this is actually something called the marginal revenue- $f(q) + f'(q)q$. So the first order condition says that marginal

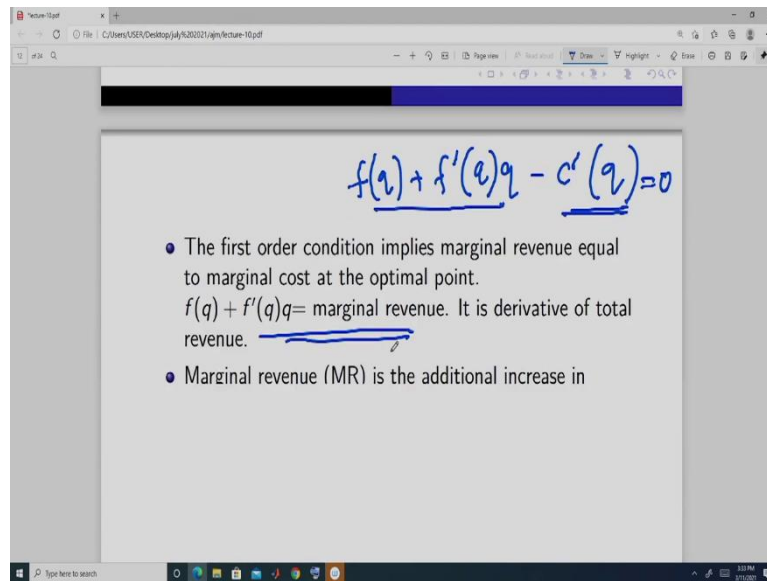
revenue, first order condition is what? Is this, is this- $f(q) + f'(q)q - c'(q)$. So this says that this portion is the marginal revenue should be equal to, because it is equal to 0, should be equal to, this is the marginal cost because it is the derivative of the cost function.

And what is the marginal revenue? Marginal revenue is the additional increase in revenue received by the firm when it sells one more additional unit. If you want to sell one more unit at the margin then how much additional revenue you are going to get.

Now, marginal revenue curve is always going to lie below the demand curve. Why? Because we know from the demand curve that if you want to sell more you have to reduce the price. So that means if you want to sell one more unit you have to reduce the price. So the price is falling. So price into quantity is going to give you the revenue. So the revenue is going to go down.

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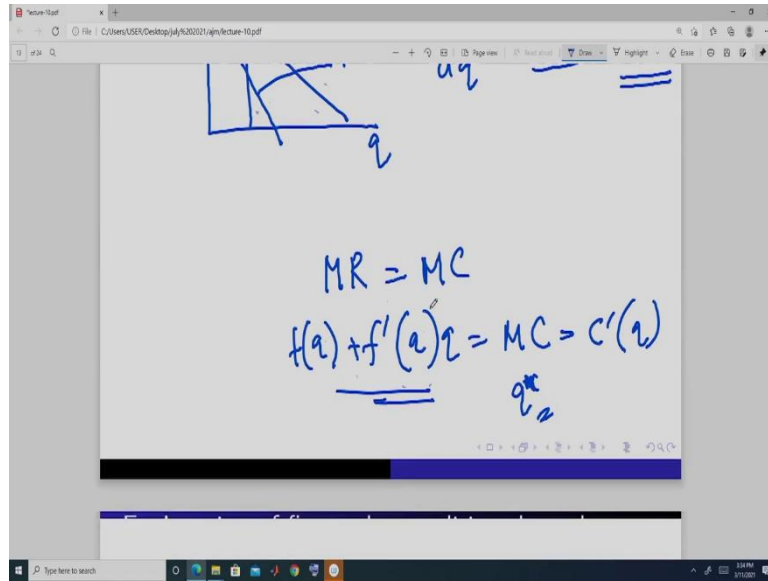


So which you can see from here, so if this is the price and this is a and this is the, supposed inverse demand curve like this, marginal revenue curve is. How do we get marginal revenue curve? So total revenue is this when it take it to be a function of output of a. Total revenue, this is the price into quantity, right? Total revenue.

Now if you take derivative of this with respect to output you will get this- $\frac{dTR}{dq} = f(q) + f'(q)q$. We know demand curve is downward sloping. So this portion- $f'(q)q$ is going to take a negative value. Now plug in outputs. This is the price plus some negative here. So that is why it is going to be something like this. And this is the marginal revenue curve, okay.

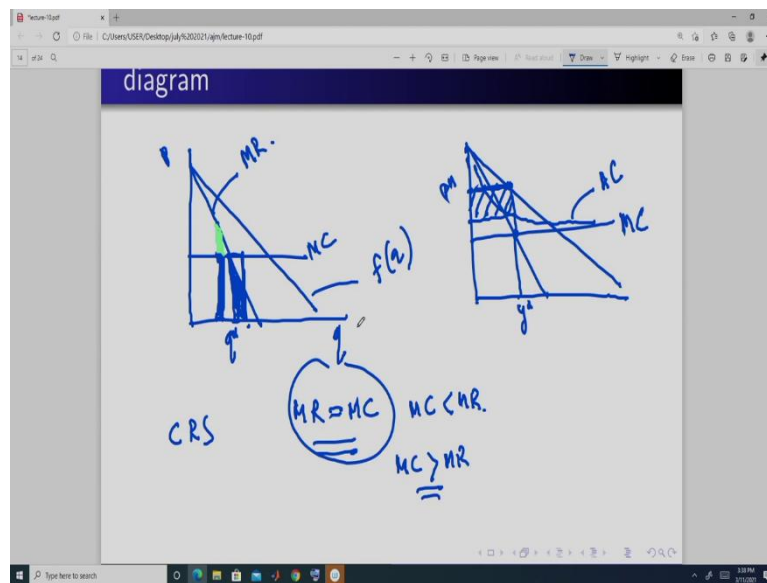
So, the idea is something like this. Since the demand curve is downward sloping, so from here if you want to increase some more amount of output so you have to reduce the price. So the additional revenue you are going to get, that is going to go down, okay. So that is the idea. So this is the marginal revenue and this is the, this portion is the marginal cost.

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So, first order condition says that the profit maximizing output will be such that the marginal revenue should always be equal to marginal cost, okay. And we can solve this function, sorry this equation which we get like this- $f(q) + f'(q)q = MC = c'(q)$. And solving this we will get the optimal output q^* , okay.

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Now, let us diagrammatically explain this how the optimal output is determined. Suppose output here, price here and this is the demand curve. So we will get a corresponding marginal revenue curve. So this is suppose the marginal revenue curve, okay.

Now, take the case of CRS. So the marginal cost is this. So the optimal output is always determined here, this. This is the optimal output q^* , so or the profit maximization output. And this output, why this is a profit maximum? Because suppose instead of this monopolist produced this much output. At this much output its marginal cost is this.

Marginal revenue is this height. If it slightly increases then output, then this much is the additional cost. It is here. Additional revenue is the total region it is going to get. So this green part is the additional revenue. So the monopolist will be at better off by increasing his output. So it will go on doing that. So it will be, so the marginal revenue should always be equal to, sorry marginal revenue should be always equal to marginal cost.

But if it produces the output more than, where, here, so here marginal cost is this and the marginal revenue is at this height. So if it produces this much amount of output extra then this is the additional cost, right, because this much amount of output and this is the cost. So this whole region is the total extra additional cost. But the revenue is this total area which is below the marginal revenue curve. So this triangle is the loss that they are making if it increases output more than q^* .

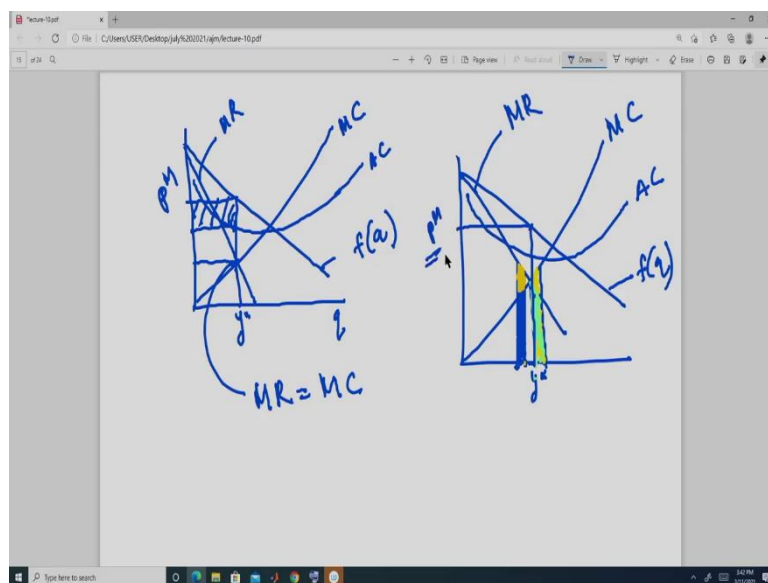
So that is why it is not optimal or profit maximizing to produce output which is more than q^* where, because at q^* this condition is satisfied- $MR=MC$. If you produce more output

more than q^* then what is happening? MC is greater than MR . So at this, profit is not maximizing.

If you produce less than this, then what is happening? MC is less than MR . Here also profit is not maximizing. In this case the monopoly, so profit is maximized when we have this condition. So the actual monopoly profit in this situation is something. So this is the marginal cost and suppose the average cost here it is going to be something like this. And this is the monopoly output y^* .

So, this is the monopoly price. This is AC . So this price is the monopoly price p_m . Because if it produces this much amount of output the market people are willing to pay this much from the demand curve we get. And so this is the monopoly price. And this is the average cost. So this rectangle is the monopoly profit, okay. So this is how the monopolist decides the output. Moment it decides the output it gets the monopoly price from the market demand curve.

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Now, let us take the case when we have decreasing returns to scale. So suppose again this is output. This is the market demand curve. This and suppose this is the marginal revenue curve. And suppose this is the marginal cost because it is decreasing cost, decreasing returns to scale and this is the average cost. This is again I have made mistake, okay. This is the AC .

Now, here you notice that at this point, now this output, at this output marginal revenue is equal to marginal cost. So firm should always produce y^* units of output and the monopoly price is going to be this much, this p_m and the monopoly profit is going to be, this

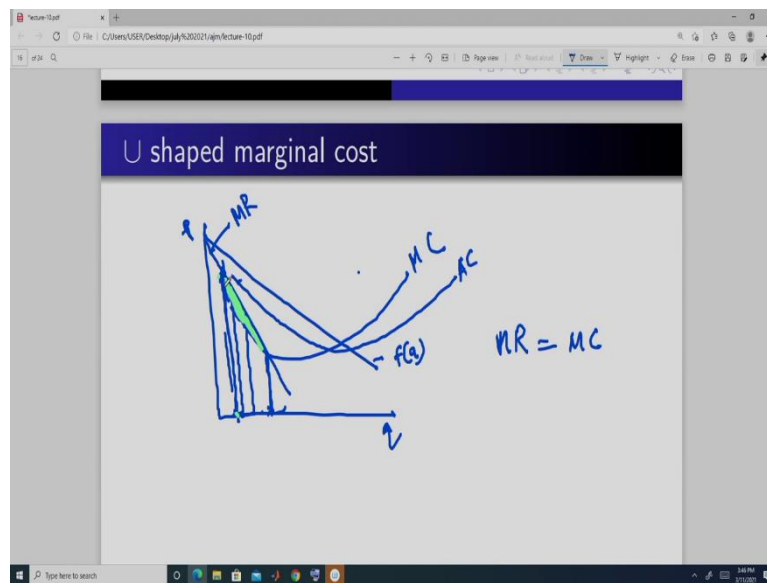
is the average cost. If it produces this much amount of output so it is going to be this rectangle, okay.

Now, why this is the profit maximizing output? Because if we take the same case here, this is market demand, and this curve is marginal revenue, and this is marginal cost, and this is average cost. So this is the profit maximizing output of the monopolist.

If it produces something less than this, here, this much amount of output then instead of producing here if it increases slightly till this then additional cost is this region below the marginal cost because it has already incurred the fixed cost, so additional cost is only this much. Additional revenue is this total region below the marginal revenue curve. So this whole, so this portion, this is the additional profit that the surplus, that the monopolist is going to get by producing this instead of this.

So, the monopoly should not produce this much. So it should continue like this until this point y^* . If it produces more than this y^* here then what is happening? Total additional cost is given by this portion, this whole yellow portion. But the total additional revenue is only given by this, this region. So this yellow extra portion is left as the additional loss that the firm is going to, the monopolist is going to make if it produces more than y^* . So that is why the optimal output it is going to be y^* . At y^* price is this. So this is the monopoly price. So this is how the monopoly determines its optimal or profit-maximizing output and the monopoly price, okay.

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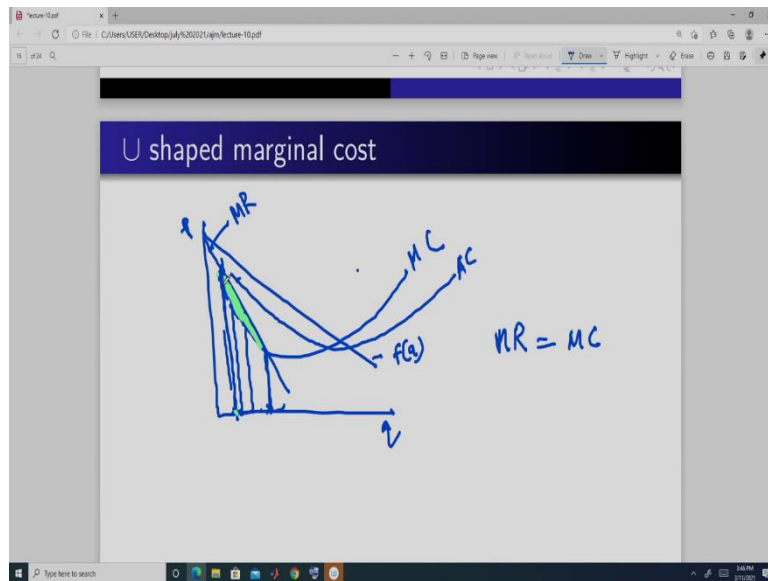
Next we will take the case of U shaped marginal cost, okay. So let us take the same demand curve like this and this is the marginal revenue. Let me make it slightly flatter so that it is easier to, okay suppose this is marginal cost. And this is the average cost. Now here notice we have two points at which marginal revenue is equal to marginal cost. One point is this. Another point is this. This is the demand curve and this curve is the marginal revenue curve. Here we are plotting output and the cost also.

But if we, the monopolist produce this output then what is going to happen? If it increases further this much, so this much is the additional surplus it gets. If it produces this much then additional cost is this region below the marginal cost and additional revenue is this region below the marginal revenue. So this portion is the additional surplus.

So, that is why the monopolist will not choose this output but instead it will choose this output. Because at this output we know if it increases more than this then it is going to make a additional loss. If it produces less than this then it is, it can by producing a little bit more, it can generate some additional surplus. So that is why this is, we know, it is a monopoly output.

Now, whether this is a monopoly output or not? But this is not. Because if it produces more than this then it can generate some more surplus given by this a. And it will go on this, this whole region. This whole region is the additional surplus it can generate. So that is why this output is not a.

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• Second order condition;
 $\frac{d^2\pi}{dq^2} = 2f'(q) + f''(q)q - c''(q) < 0$ at $q = q^*$, the optimal point.
 From the assumption, $c''(q) \geq 0$, so $-c''(q) \leq 0$.
 $f'(q) < 0$.

• Further we assume that if $f''(q) > 0$ it should not be big enough. It means that Demand curve is downward sloping and convex to the origin. If $f'(q) < 0$ and $f''(q) > 0$. We assume that it should not be highly convex for the second condition to be satisfied.

• If $f''(q) \leq 0$, we do not have any problem. The second order condition is always satisfied.

Now here, so this gives rise to our second order condition. And second order condition says that the, when the marginal cost intersects the marginal revenue curve it should be upper sloping, or it should not have a negative slope, okay. Here it is the slope is negative, right? So second order condition is given in this form- $\frac{d^2\pi}{dq^2} = 2f'(q) + f''(q)q - c''(q) < 0$, so which we have again taking the derivative of the first order condition.

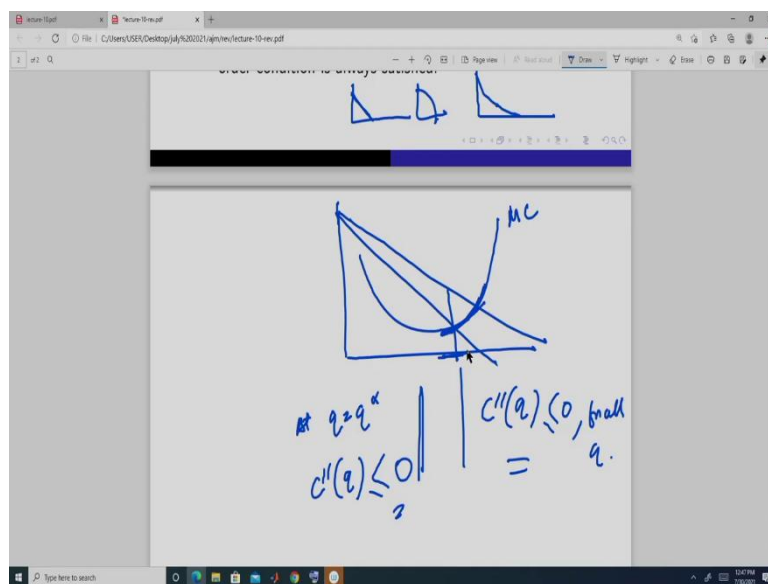
Now, here we know this portion, this portion- $2f'(q)$ is going to be negative. Why? Because downward sloping demand curve. This- $f''(q)q$ we are not very sure. We have not yet specified, okay. So let us be silent. This we have already specified that our marginal costs are of this nature or marginal cost are of this nature. So this 2 kinds satisfies this, okay.

So, from the assumption this, we get this portion is this. So this portion- $c''(q)$ is negative because they are either 0 or it is positive. This kind of marginal cost curve when we have CRS, this when we have either DRS or one factor is fixed and we have law of diminishing marginal product operating, okay.

Now, this portion- $f''(q)q$, so this- $2f'(q)$ is negative already and this- $c''(q)$ is negative. So this is negative. But what happens to this- $f''(q)q$? Now this we have also assumed that our demand function can be of this. So in this case it is this. It is convex to the origin.

So, what happens in this case? In this case what do we get that, for this overall to be a negative at the optimal point, i.e. $\frac{d^2\pi}{dq^2} = 2f'(q) + f''(q)q - c''(q) < 0$ at $q = q^*$ we need this curvature to be less convex. So even if it is convex should not be that convex. So then we will get this whole to be a negative number, okay. But in case of a concave demand function which is like this or like this, this is always going to be negative, so second order condition is always satisfied, okay.

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• Second order condition;
 $\frac{d^2\pi}{dq^2} = 2f'(q) + f''(q)q - c''(q) < 0$ at $q = q^*$, the optimal point.
 From the assumption, $c''(q) \geq 0$, so $-c''(q) \leq 0$.
 $f'(q) < 0$.

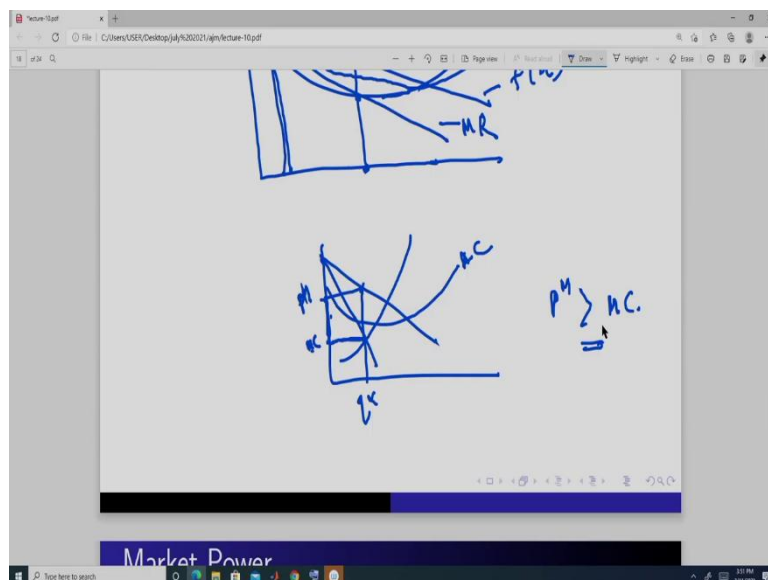
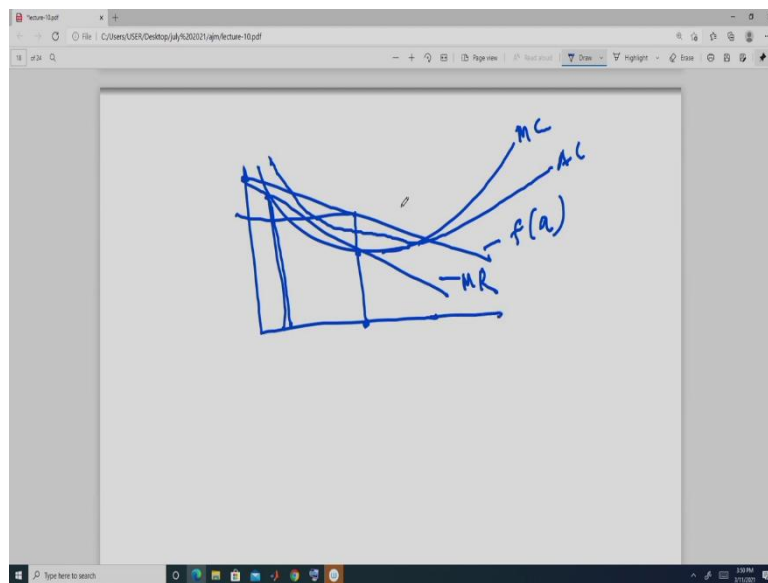
• Further we assume that if $f''(q) > 0$ it should not be big enough. It means that Demand curve is downward sloping and convex to the origin. If $f'(q) < 0$ and $f''(q) > 0$. We assume that it should not be highly convex for the second condition to be satisfied.

• If $f''(q) \leq 0$, we do not have any problem. The second order condition is always satisfied.

Now, we have taken another example and that is when our marginal cost curve is U shaped. It is like this. So in this situation what do we get in the monopoly outcome? Monopoly outcome is somewhere here and it is like this. So marginal cost is increasing at this. So in this situation, even if this condition- $c''(q) \leq 0$ for all q is not satisfied, at least for this it has to be satisfied, okay.

So, if we take this, when it is satisfied for this so then again when, for at q is equal to q^* should be this- $c''(q) \leq 0$. So this is a local assumption that is within in this range. So this ensures again that the second order condition is satisfied. So when we have a U shaped marginal cost then also the second order condition is satisfied, okay. So this is the importance of the second order condition.

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So, mainly the second order condition will allow us to, if we, our marginal cost curve is like this, demand curve is like this, marginal revenue, this is demand, this is marginal revenue, this is the, this is the average cost, then this is never going to be the optimum output. This is going to be the optimum output. From the second order condition we get that. And also we can argue logically why this is not going to be that.

So, we know how the market price is determined. If we are given a demand curve like this we will get a marginal cost like this and the cost function is, marginal cost is like this and the average cost is like this then the monopoly output is this y^* and the monopoly price is this p_M , right? We know this.

Now, the question is, suppose the cost function is similar. Then our cost functions are different. When a monopolist can charge a higher price and when it can, it has to charge a lower price? Because here if you see that this price is always greater than the marginal cost, because this quantity marginal cost is this and the monopoly price is this. So the price, monopoly price is greater than the marginal cost. How big it is going to be, what determines that.

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Market Power

The first order condition gives
 $f(q) + f'(q)q = c'(q)$ →
 $\Rightarrow p = \frac{c'(q)}{1 - \frac{1}{|\xi_d|}} = \frac{MC}{1 - \frac{1}{|\xi_d|}}$, where $|\xi_d|$ is the price elasticity of demand.

If $|\xi_d| = 1$ then $p(1 - \frac{1}{|\xi_d|}) = c'(q)$ becomes $0 = c'(q)$. Firm lost its power to set price. So the monopolist always produces when the price elasticity of demand is elastic.

Monopolist sets higher price for the goods with higher price elasticity of demand.

$P = \frac{MC}{(1 - \frac{1}{|\xi_d|})}$

So, that is based on the market power. And how this market power is given? We will do this. So first order condition gives us this condition- $f(q) + f'(q)q = c'(q)$, and if we do a little bit manipulation we can derive this- $p = \frac{c'(q)}{1 - \frac{1}{|\xi_d|}} = \frac{MC}{1 - \frac{1}{|\xi_d|}}$.

So, we can say that the monopoly price is always equal to marginal cost divided by 1 minus 1 by price elasticity at that a. So we know this output. So you plug in this output here. So you know the elasticity at that point. So this will give you the price, how much price. And this should always take a value which is less than 1. So the monopoly price will be always greater than the marginal cost.

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$f(q) \left[1 + \frac{f'(q)q}{f(q)} \right] = c'(q)$
 $f'(q) = \frac{dP}{dq}$
 $\Rightarrow \frac{dq}{dP} = \frac{1}{f'(q)}$
 $\Rightarrow \frac{P \cdot dq}{q \cdot dP} = \frac{P}{q \cdot f'(q)}$
 $\Rightarrow E_d = \frac{P \cdot dq}{q \cdot dP} = \frac{P}{q \cdot f'(q)}$

Market Power

$P = \frac{MC}{\left(1 - \frac{1}{|E_d|}\right)}$

The first order condition gives
 $f(q) + f'(q)q = c'(q)$
 $\Rightarrow p = \frac{c'(q)}{\left(1 - \frac{1}{|E_d|}\right)} = \frac{MC}{\left(1 - \frac{1}{|E_d|}\right)}$, where $|E_d|$ is the price elasticity of demand.

If $|E_d| = 1$ then $p\left(1 - \frac{1}{|E_d|}\right) = c'(q)$ becomes $0 = c'(q)$. Firm lost its power to set price. So the monopolist always produces when the price elasticity of demand is elastic. Monopolist sets higher price for the goods with higher price elasticity of demand.

How do we derive this? We derive, first order condition gives us this- $f(q) + f'(q)q = c'(q)$. This portion- $c'(q)$ is equal to marginal cost, okay. Now we know this is equal to price. So if we take that common we get this- $f(q)\left[1 + \frac{f'(q)q}{f(q)}\right] = c'(q)$. yes. Now here, so using this- $f(q)=P$ we can write, we can write it in this form- $P\left[1 + \frac{1}{\frac{f'(q)q}{P}}\right] = MC$. You can say marginal cost

Now, this is what? Suppose we can take the reciprocal of this- $f'(q) = \frac{dP}{dq}$. So we can write this- $\frac{1}{f'(q)} = \frac{dq}{dP}$ and to this if we do this we can write it in this form- $\frac{P \cdot dq}{q \cdot dP} = \frac{P}{q \cdot f'(q)}$. So this is

what? This is the point elasticity which we by big epsilon, capital epsilon. It is like this-

$$\xi_d = \frac{P \cdot dq}{q \cdot dP} = \frac{P}{q f'(q)}$$

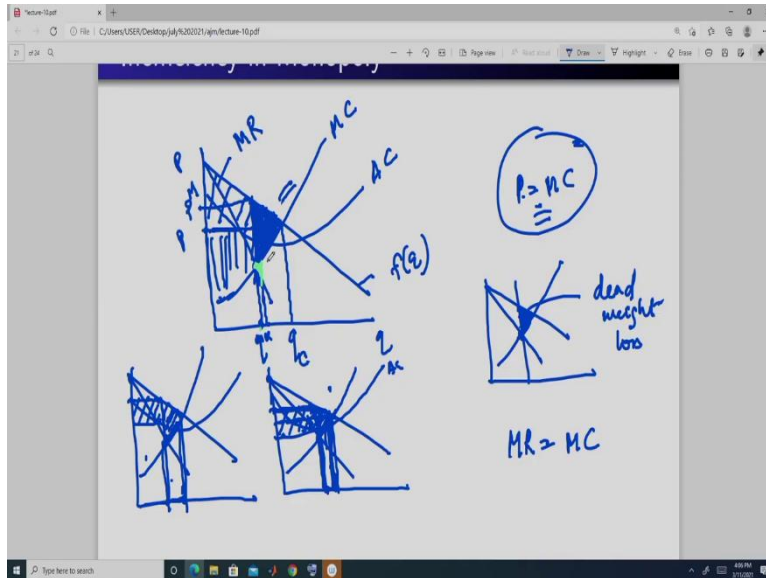
So, here we plug in this, this- $P \left[1 + \frac{1}{\xi_d} \right] = MC$ and elasticity of demand, since this is always negative so we can write it in this form- $P \left[1 + \frac{1}{|\xi_d|} \right] = MC$. We have done this, right? So price you can say now we have got this is equal to marginal cost that is this. So here this is the elasticity.

Now, if suppose at that quantity, price elasticity of that demand is unitary elastic. So if this is equal to this unitary elastic this takes 1, so this 0. So 0 is equal to this. i.e $0 = c'(q)$. So a monopolist will not have any power to set the price. So the monopolist will always produce when the price elasticity of demand is elastic that is this price elasticity is greater than 1.

The moment it is greater than 1, so this is a less than 1 so it is, so this price is always going to be greater than marginal cost. So this price is always, since it is going to be greater than marginal cost, so this price is going to be greater than the perfectly competitive market price because at the perfectly competitive market the price is always equal to marginal cost. Here this is due to this here.

So, what do we get? That the monopolist, means it only has only a high power, it has a higher market power that is high power to set a higher price when the elasticity of demand, or price elasticity of demand is more elastic. The moment the good has a more elastic demand at that price, so it can, at that quantity it can charge more prices. If the elasticity is less then it will charge or set a less price, okay. So this is actually, this condition is called the Marshall Lerner index, okay.

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Now, we will show the inefficiency that is generated in the monopoly, okay. So this is the output. This is price. This is the market demand. This is the marginal revenue. Suppose this is marginal revenue. This is marginal cost. And this is the average cost. Optimal output is this much q^* , monopoly optimal output or profit maximizing. Monopoly price is this much, right.

Now, suppose in this case there is one firm and that and that firm has to produce at that output where price is equal to marginal cost, suppose, okay, because had it been a competitive market the outcome would always be this, price is equal to the marginal cost, right?

So at that point output would have been this. So this is suppose q_c , right? At this, market price is this. Now here this is the monopoly price. And if we know that this is the optimal, or profit maximizing price. But suppose firm is producing this much, then see the revenue it is, extra revenue it is getting is this much, this region. And this, this is the cost. So this because the cost, there is a net cost or net loss. So that is why the profit maximizing is this.

But if you look at it slightly differently you will see that consumers are willing to pay this much here. So if this is the a , then total producer surplus is, consumer surplus is this much and the producer surplus is this whole region. And from here we minus the fixed cost, okay.

But if we leave apart the fixed cost this is the producer surplus. And this is the consumer surplus. So this additional surplus is, we are not taking the benefit of that. We are not generating this a . So this is the amount of surplus we are foregoing.

So, here if we take this case, so this triangle is something called the dead weight loss. And because the monopoly price is, marginal revenue equal to marginal cost instead of price equal to marginal cost there will always be a dead weight loss, okay. Because we get the optimization optimal output at this, and not at this point. So this additional amount, sum of consumer plus producer surplus is, monopoly is foregoing that much amount of surplus, okay. If it uses this pricing then this benefit, this additional surplus could be borne, okay.

If we take the sum of the surplus that is producer and consumer surplus, because here you will notice that the moment price. So this is the monopoly price, okay. And suppose this is the AC, so this is the monopoly profit. Now if the price is this suppose for some reason. Then the monopolist is going to get, this is the average cost, so this rectangle is going to be the monopolist profit. So this rectangle is less than actually this rectangle. So that is why this is not a profit maximizing condition. So here profit is maximized.

But if you look at in terms of social welfare you will see that here this extra surplus is not, no one is getting, neither the producer nor the consumer. But there is a possibility we can get this, right? So that is why and if we get this much amount here, if we take the sum of this a, so we can make the producer. So what we are doing?

So if we take this, so monopoly is charging this price. So consumers surplus is this triangle, right. Now what happens? You ask the monopolist to produce at this much level of output. Suppose. Then and consumers are, and you set the price to be at this level, okay.

Now, what you can do here? So what I have to show that this is an inefficient outcome. Inefficient outcome means this is not Pareto-optimal. So if it is not Pareto-optimal that means I can make at least someone better off if I move from this monopoly to this outcome, right.

Now that is, how that can be done? So here if the price is here then there may be some person whose demand, who cannot buy this much amount, who cannot buy at this price, so, but who are willing to pay this much prices. So what we do? We charge, we give this much amount to those people in aggregate and charge them this whole amount. So cost is here. So these people are going to pay this whole amount. And the monopolist cost is this much. And if we pay, ask them to pay this whole thing then this will be received by the monopolist. So this is received by the monopolist. So we are making the monopolist better off, right?

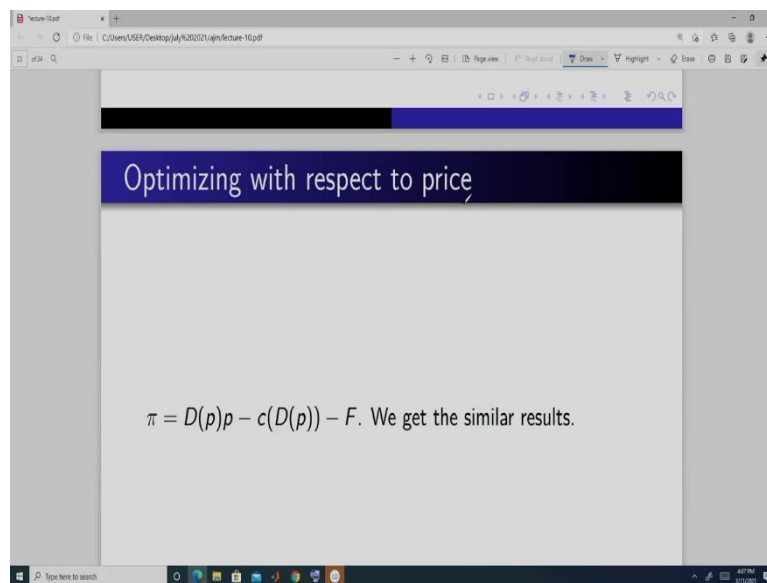
Or instead we can do like this also. If we take this and take this and take this here, so what we do? Monopoly market price is there. And we take some individual who cannot afford this

price. And what we do? We give them this much amount, and we charge them this much amount of price.

So monopolist is making this amount of profit. But it is producing this much amount of output and also this much amount. This much is going to someone else. And those people who are willing to pay this much, they are getting a benefit of this. So these people who were earlier not able to afford it, now they are getting this much amount and they are getting a surplus of this much. So we are making at least someone better off without hurting the monopolist, some consumers.

So, in this situation we are benefiting the consumer. In this situation we are benefiting the monopolist without hurting the consumer, here without hurting the monopolist. So in both the cases we can move to a Pareto-optimal situation. So that is why monopoly outcome is not a Pareto-optimal. So that is why it is an inefficient outcome. And there is this problem of dead weight loss, okay.

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So, next we will do natural monopoly and the optimization with respect to price. You will see it is more or less same and that we will do in next class. Thank you.