Fundamentals of Semiconductor Devices
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Lecture - 05
Density of states

Welcome back. So, today is the third lecture of this course. In the last class, we had done quite a few of things. We had introduced the concepts of Fermi function which is the probability of finding an electron at an energy. We introduced the concept of Fermi level. We said that Fermi level is a statistical construct, where there is a 50 percent probability of finding an electron at any temperature ok.

We also elaborated on the E-K diagram in the presence of the real crystals you know the how electrons move into real crystal, and how the E-K diagram can capture the physics of electron transport by summarizing everything in an effective mass term ok. We can designate the effective mass of electron as m star, which will basically take into account the effect of the band structure the quantum mechanics of the transport. So, we do not have to worry, so much we can apply classical equation to that.

We also introduce some very important concept required for this entire course that is hole. We say that when electron goes from a valence band to conduction band for example, it leaves behind a vacancy that vacancy is called hole, it is like a quasi-particle. And it moves in the direction of the applied field, so it has a positive charge and holes are actually absence of electron, but it is convenient to model them as particles.

And so it hole has its own effective mass, hole has its own E-K diagram, and from the EK diagram if the valence band in the conduction band, they are the that the peak and peak positions at the same K, then we call it direct band gap, we call it indirect band gap, if they are not in the same position. So, all these things we discussed you know how indirect band gap, semiconductor like silicon cannot be used for making LED.

So, all the white light LED’s that you see in the town, in the market everywhere those are not made of silicon, because silicon cannot emit light. You need a direct band gap material, I told you that a direct band gap material if it has an energy of band gap Eg, then the wavelength of the light that is emitted also will be will give you HC by lambda will be easy the band gap material ok.

So, now I told you that the most important thing after that to learn today is density of states. So, we shall introduce density of states, and we shall introduce the concepts of doping. So, then we are basically ready to understand how electrons and holes fill up the bands, and how carrier transport takes place ok. So, will come through the whiteboard, and start off density of states. [FL]

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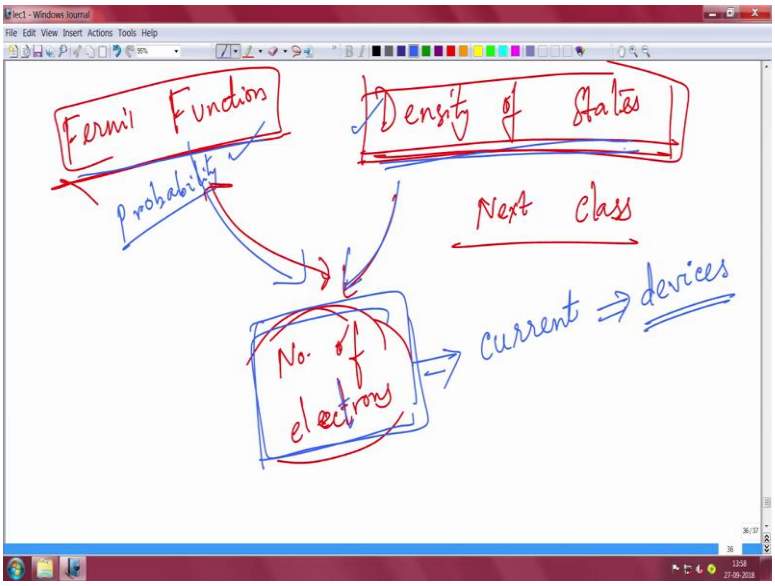

So, you might see in the last class you know, I told you that there is Fermi function, which gives you probability right which gives you the probability of finding electron. And then density of states is another very important concept. And together Fermi function and density of states, you can find out the number of electrons or number of holes ok.

And you need to know the number of electrons and holes, because only then you can find out the current density. And only when you can find out the current density, you can actually talk about devices right. Any device carries current right, whether it is a solar cell or BJT does not matter. So, we need to understand, how many electrons holes are there for that we need density of states. And we need the Fermi function, we derive the Fermi function, the Fermi probability in a very simplistic manner, and density of states is something will talk about right now ok.

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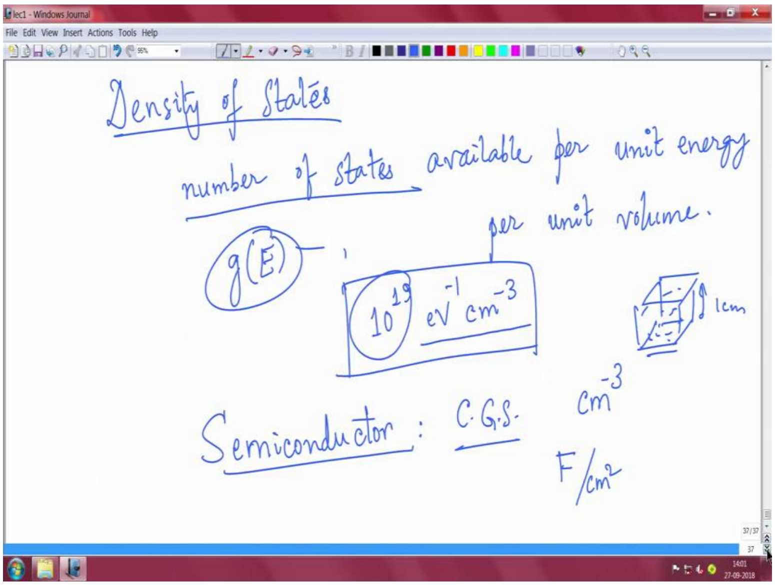

So, what actually is density of states, it is a very interesting concept right. Density of states: density of states actually means, if you know think about it, it means actually the number of states, the number of states available number of states energy states available per unit energy per unit volume ok.

So, in each unit volume per unit energy, how many energy states are there ok, how many states are there, which electrons can occupy? Now, this is a function of energy, I will call it g of E ok, this is a function of energy ok. And there are very very interesting ways to derive the expression as to how many states can be there, and for that you have to take help of k space the reciprocal space you take like a sphere, and then you find out how many discrete states are there and so on.

So, the derivation of the density of states is something, we will not do here I can I will put it up in my notes, which I will upload. So, you can study the notes. But, in general the density of states you know it is a number of available states. So, if the number of available states is for example: 10 to the power 19 states are there we in per unit energy, per unit volume, we will call it per electron volt per centimeter cube. So, the unit of density of states is per electron volt, per centimeter cube.

And if there is a 10 to the power 19 so, we know that a 10 to power 19 states per energy, unit energy per centimeter cube. One thing to note here is that in semiconductor devices or in semiconductor physics from the device point of view the units that are used are C G

S units not the M K S units, we use C G S units. So, it is always centimeter and not meter that we use.

So, for example, if we talk about the number of electrons or number of holes, then we normalize it with respect to volume. So, it will be number of electrons per centimeter cube or number of holes per centimeter cube, which means you know in 1 by 1 centimeter in 1 by 1 centimeter cube, 1 centimeter by 1 centimeter, how many electrons would be there. For example that is how we give the nomenclature of the units in semiconductor devices we do not generally use meter ok. So, similarly capacitance will be farad per centimeter farad per centimeter square.

For example, so this is something we should keep in mind ok, because it will be useful in doing calculations and numericals later on. So, I told you the density of states is the number of energy states available per unit energy per unit volume which is you know this and it basically has an expression I said that I will not derive it.

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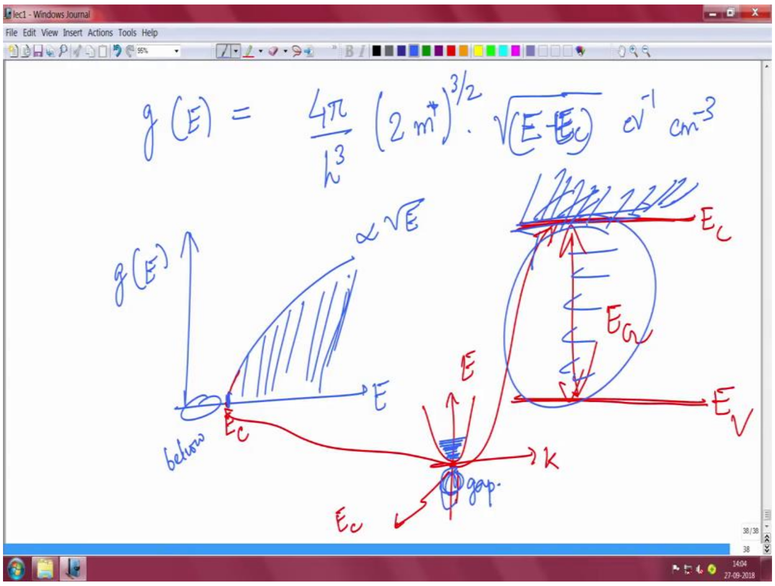

The density of states has an expression something like

$$g(E) = \frac{4\pi}{h^3}(2m^*)^{3/2}\sqrt{E - E_C}\ eV^{-1}\,cm^{-3}$$

Where g(E) is the density of states, m* is the effective mass of electron or hole, E is the energy, Ec is the conduction band energy.

So, if you plot density of states versus E, then you go as square root of it will go as square root of E. So, as energy increases the number of available states also increases as the square root. The number of available states per unit energy, this is per unit energy per unit volume, everything is normalized with respect to volume.

Remember electron or hole density is also normalized with respect to volume, because that is how you should do it I mean if you take a semiconductor crystal, it always makes sense to talk about unit volume you normalize to volume [FL]. And I started this not from 0, you see this, you see at this point I started did not as did not start from this point from 0, I started it from some point here. Do you know why, because this point is the conduction when E c the bottom of the conduction band.

What I mean is if I have by the way in semiconductor devices we do not you know from the device point of view we do not draw these boxes of convection band in valence band that we do not do. What we do in is a convention it is just an you know the way it is designated in semiconductor device physics. We just put sorry we just put a straight line as conduction band and a straight line as valence band. Here the gap between them is your energy band gap.

What this straight line means is actually the bottom energy band, this is the top of the valence band, this is the bottom of the conduction band. If you recall your E-K diagram right, this is E, this is k right. So, this bottom is actually this that is representative with respect to position. And this is this bottom most position is where your conduction band starts, below that there is no conduction band right. So, this position actually corresponds to this. What it means is that only from only from the bottom of the conduction band you can start filling up, you cannot fill up below it because below there is gap there is no nothing right.

So, below this portion below you cannot have any density of states. Why, because there are no states within the band gap you only have states here in the conduction band right. So, it starts from here ok. So, essentially it is E minus E c [FL] E c is the bottom the conduction band. So, only it starts filling from the bottom the conduction band, it cannot fill below the conduction become because this is gap, nothing is here right.

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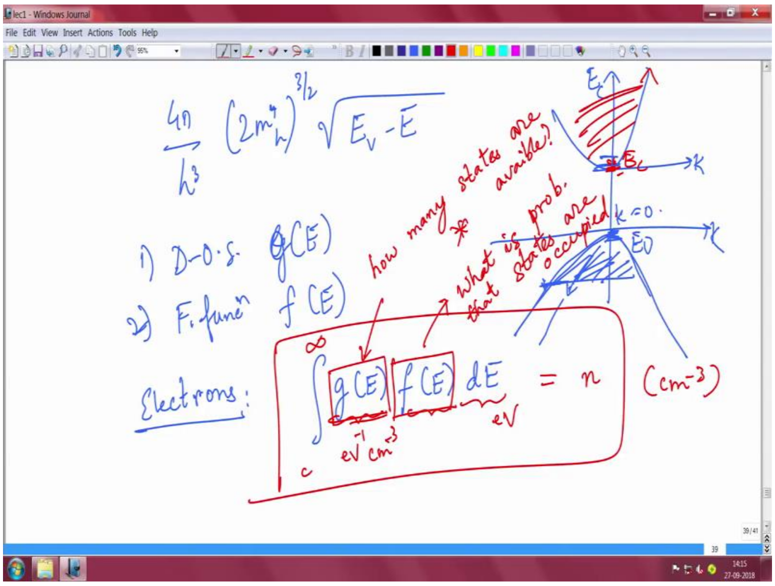

So, similarly, similarly for holes I told you right you have this k and then you have the hole here, I am not very good at drawing, but the thing it is a parabola here. So, this is the bottom of the conduction band at k equal to 0. So, you know for holes also it will be same thing

$$g(E) = \frac{4\pi}{h^3}(2m_h^*)^{3/2}\sqrt{E_V - E}\ eV^{-1}\,cm^{-3}$$

Where m^*_h hole effective mass, Ev the top of the valence band, E is the energy.

So, more negative it is like the higher energy it is so E v minus any point here for example,. So, it goes as square root of the E. So, this gives you an idea of the density of states how many states are available, how many states are available for electrons or for holes right electrons or for holes to for the electrons and holes to occupy the conduction band and valence band right.

So, now, we have two things right. So, now, you know what is density of states, we know density of states I call it D O S, density of states which is essentially g of E ok. And then the number two is the Fermi function Fermi function which is given by f of E. So, essentially if I talk about electrons, for example, I talk about electrons right, electrons, if I talk about, and how many electrons are there total number of electrons that are there in a conduction band will actually depend on the product of density of states and the Fermi function sorry and the Fermi function integrated over the entire energy from the bottom of the conduction band here from here to infinity E c, this bottom here to infinity.

This gives you the total number of electrons which I will call n ok. This gives you the total number of electrons n; and the unit is per centimeter cube. This is the energy electron volt; this has no unit. This is per unit energy per unit centimeter cube, so the unit is per centimeter cube this gives you the number of. What is actually happening, it basically tells you how many states are there, how many states are available right, how many states are available that is the first question multiplied by what is the probability that you will occupy those states what is the probability that states are occupied those energy states are occupied right.

So, essentially the product of how many empty states or how many energy states are there times what is the probability that those states will be occupied. The product of these two function if you take the integration over energy from this bottom to conduction band to enter infinity, then you get the number of electrons that is what is happening here ok. And it is a very easy concept interesting concept will not lose track of sight. We will always recap what we are doing, so that we do not lose track of sight.

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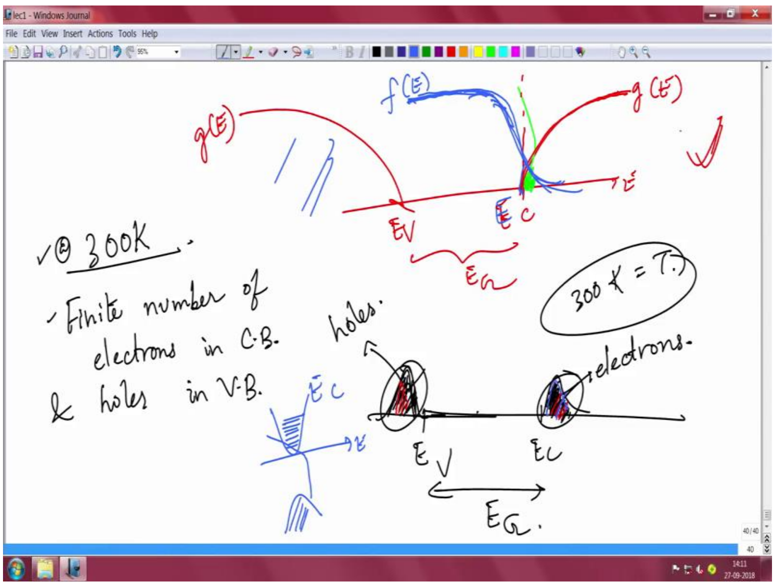

So, for example, you know essentially under energy exists if this is the energy this is E c for example, your density of states will start only from here g of E. And similarly this is holes for example, the band gap is this one right, this is the band gap ok, this is g of E for holes. Holes also have their own density of states, electrons also have their own density of states, because it depends on effective mass m star of electron and hole and that is different. So, let electron and hole density of states also be slightly different because the effective ones are different, this is your density of states. You have to multiply by the Fermi function, you have to multiply by the Fermi function.

So, of course, the Fermi function depends you know on the Fermi level also. So, how does the Fermi level go you know above the conduction band, you should not have generally electrons, but you know the Fermi function for electrons for example, will go something like this you know below the conduction band everything you know the idea is that your electrons will be if you take the Fermi level can be maximum here then it will be something like this that is a Fermi level it is a Fermi function the probability its very high probability. It is almost 100 percent probability electrons will be in valence band actually, the electrons will be in valence band right.

So, there is a almost 100 percent probability electrons will be in valence band. At absolute 0 of course, nothing will be in conduction band, but at finite room temperature there will be small tail I told you right in the last class. So, in small temperature in a finite temperature there will be slight this right. [FL] above E c also there will be some electrons that is the probability will be there that is what at finite temperature it means. At 0 Kelvin, it will be just a rectangle ok.

So, if you take the product of this red and blue, you get this green thing here this one right. How it looks like you know it looks like this is E c, you know. So, it will look like this, the product of these two one of them is decreasing one of them is increasing. Similarly, this is E v, you will have something like this. So, this is actually your electrons that are there in the conduction band. This is actually your holes that are there in your valence band. And this is your energy band gap Eg ok, there is some electrons here, there is some holes here ok.

I am talking about at say room temperature which is say 300 Kelvin. At 0 Kelvin of course, you cannot have anything you know at finite temperature will always have something ok. So, you have electrons in the conduction a little bit, you have holes at the valence band little bit, this is at room temperature. So, see there is a very critical thing at any temperature other than 0 Kelvin. At lower temperature, this will become smaller; at higher temperature this will become larger that is different thing, but at room temperature it will have it will be some amount right.

So, you see at room temperature at 300 Kelvin, it seems like there is always a finite number, there will always be a finite number of electrons in conduction band right, and there is a finite number of holes in valence band. This is a very important you know profound finding actually if you think it deeply. So, at room temperature I have not added any intentional impurity, I have not added any I am not shining any light not applied any magnetic field nothing, just a semiconductor crystal a semiconductor wafer in equilibrium, at room temperature I have not done anything to it. But still there is some electron that will be there, some hole that will be there in the valence band.

Do you know why, I told you this comes from the density of states and the Fermi function. So, density of states increases with energy above conduction band you know so this you see what I am trying to say essentially is that you have the E-K diagram here right this is E c of course, you know this is your k diagram. So, density of states increases as you go up, but a Fermi distribution dies down as you go up because all the electrons will be in valance band only. So, there is very few probability that electrons will be there. So, the Fermi probability comes down like this ok. So, the product of these two gives you this actually hump ok. So, now, that will answer our very fundamental question that we had raised some time back.

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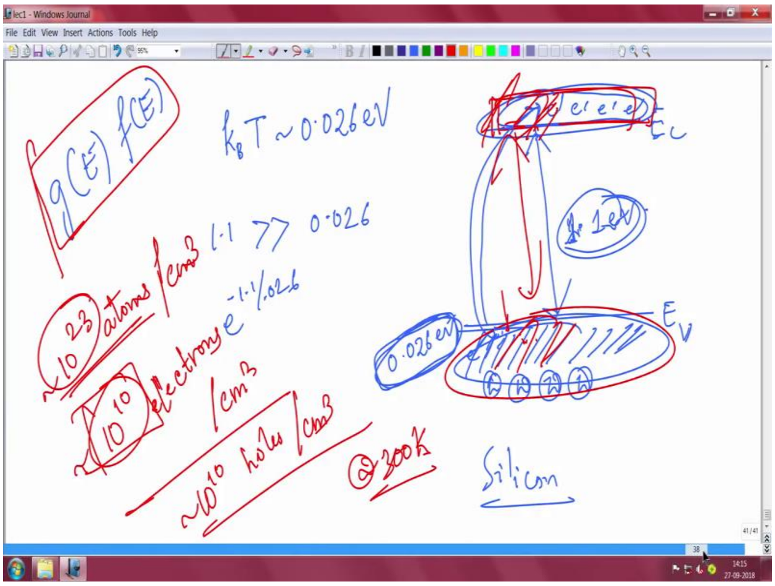

And that is if this is your conduction band and this is your valence band for say I will take silicon I will take silicon for example, whose band gap this is 1.1 electron volt it is a huge band gap I mean it is some decent band gap. Room temperature energy k, Boltzmann times temperature at room temperature is only 0.026 E v. So, the electrons that are there in the valence band, this is valence band right, they only have energy of 0.026 electron volt.

With this small energy thermal energy this is the thermal energy provided by the room temperature right, thermal vibration. So, 0.026 electron volt with that kind of small energy, how can electrons jump this huge barrier of 1.1 E v, because 1.1 is very large, 1.1 is very large very very large compared to 0.026. And in semiconductor in all these energy processes everything actually goes exponentially ok. So, this is a huge change exponentially it will go. So, it will go e to the power 1.1 by 0.026 that is the actually that is the probability that electrons will actually surmount the barrier ok.

So, this is very very small probability that electrons can actually which 0.026 e v small energy, it is very small probability their electrons will be able to surmount from the conduction band sorry valence band to the conduction band. Despite that there is always a finite number of electrons here and there is always a finite number of holes here that is obvious from the figure that I had drawn in the last slide you see this figure. There is always some electron, this electron corresponds to the edge of the conduction band, this is at the edge of the valence band holes. So, they always those electrons and holes actually exist here, electrons exist here holes exist here.

Despite a very very low probability that electrons can hop you know jump across this barrier and that is because of the fact that density of states is multiplied by the Fermi function, it is not only probability. It is not only probability that electrons will go from valence band to conduction band, it is also how many energy states are there that the probability and that energy states has to be multiplied. And it seems like when you multiply and take an integration, there is actually a finite number you know it is much less than the Avogadro number of course is much much less than what is the atomic density.

So, for example, silicon might have 10 to the power 23 atoms right per centimeter cube per centimeter cube of volume you have so many atoms. But the probability is very low that electrons you know will jump from one to the other thing. So, actually the number of electrons that will be able to jump because of this probability times the density of states function is only 10 to the power 10. So, you will have 10 to the power 10 electrons close to 10 to the power how does it come I will tell you ok, 10 to the power 10 electrons will be there approximately per centimeter cube in the conduction band.

And similarly 10 to the power 10 holes also will be there per centimeter cube in the valence band at room temperature. So, this is a much smaller number 10 to the 10 is a much smaller number compared to the atomic density ok. If one atom was to give one electron, then there should be 10 to power 23 but that is not possible because this gap is huge right.

So, the probability times the density of states gives you a realistic number of how many electrons are there here, how many holes are there here, it turns out that is around 10 to power 10 electrons and 10 to the power 10 holes that are actually there right. So, this number will derive how we derived that will come to there ok. So, now, before that I was actually in this if you recall I was telling you about the expression here right. So this expression, if you look into this expression, let us recall again holes also will have a similar expression, but we are talking about electrons here. So, first we will talk about electrons and holes will become the same thing.

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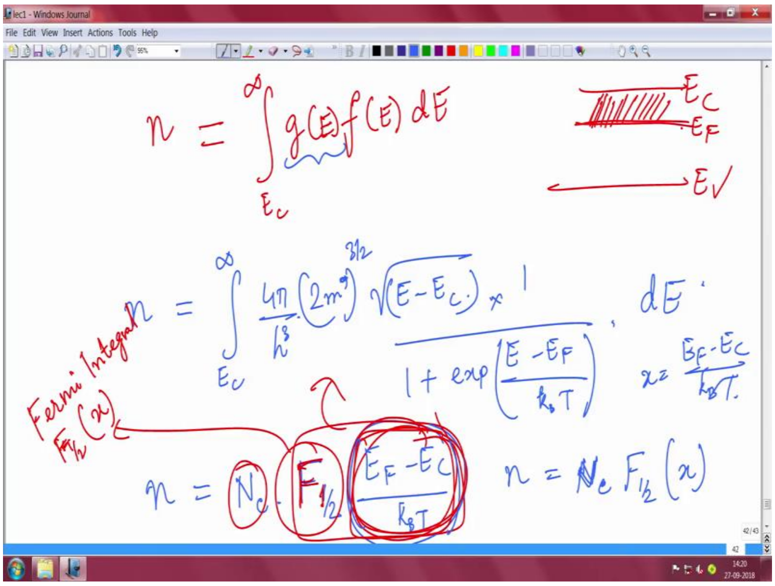

So, I telling you that electron at room temperature for example, at room temperature electrons in silicon we talking about silicon I need a semiconductor it is given by

$$n = \int_{E_C}^{\infty} g(E) f(E)$$

Where Ec the bottom of the conduction band, g(E) the density of states right and f(E) the Fermi function.

So, now if you do that calculation of course, we do not want to completely solve the integral here that is their mathematica and other functions to do that. It so happens that the density of states function is given by

$$n = \int_{E_C}^{\infty} \frac{4\pi}{h^3} (2m_h^*)^{3/2} \sqrt{E - E_C} \times \frac{1}{1 + \exp(\frac{E - E_F}{k_B T})} \, dE$$

Where E_C is the minimum of conduction band, m^*_h hole effective mass, E_F is the Fermi function, k_B is the Boltzmann constant, T is the temperature.

Now, this is an integration that we have to do actually. Ah And then this is a complicated integration that can be simplified by substituting some values, we will not do that. But eventually if you do that, what will come you know n will be equal to some number N c, I will come to that quickly into a Fermi integral of the half order of E F minus E C by k B T. Let us pause a second and go it into it.

This is actually an argument of the function ok. This F half you know this F half this is a Fermi integral this is actually Fermi integral of half order. So, it is like F of half, it is a Fermi integral of half order you can put any x there. So, this x is actually this E F minus E C by k T ok, so Fermi level. So, if this is your conduction band, this is your valence band, Fermi level could be anywhere. The Fermi level could be anywhere say it could be here, I will come to the explanation later on ok, Fermi level could be anywhere.

So, E F minus E c, so this gap negative of this gap E f minus E c divided by k T that is your x the argument here right. So, there are tables in an internet also you can find out, if you know this value if you know this gap divided by k T, k T of course, you know 0.026. So, if you know this gap divided by k T, a negative of that of course, then you can find out that is x basically. So, F of the half of integral of that you can find out from table so internet also and then you multiply with something called N c, I will come to that, then gives you n this is a very simplified expression. And this basically takes into account that is entire integral that is there ok.

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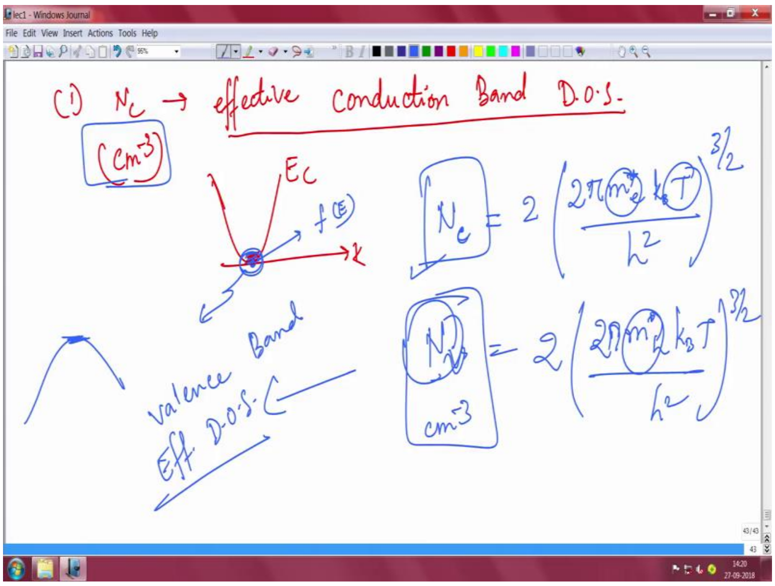

So, there are two things, number one thing is: what is actually N c? N c is called the effective I am talking about electrons right. So, it will be effective conduction band effective conduction band density of states. And the unit of N c is per centimeter cube, it is not per centimeter cube per E v, it is just per centimeter cube. It is called effective conduction band density of states.

What it means is that if this is the number of energy states at the bottom of the conduction band. So, if I have a E c and this is k, so the number of states at the bottom of the conduction band here which when multiplied by F of E here at this point will give you the number of electrons that are there at the bottom of the conduction band ok. So, it basically means that sort of you can say this is the effective density of states corresponding to the bottom of the conduction band, and its unit is per centimeter cube that is what it means.

And actually Nc has a very nice expression and that is given

$$N_C = 2(\frac{2\pi m_e^* k_B T}{h^2})^{3/2}$$

Where m^*_e is electron effective mass, m^*_h is hole effective mass, k_B is the Boltzmann constant, T is the temperature.

So, you see this is actually density of effective density of states conduction band density of states this depends on the effective mass of electron. So, if the effective mass of electron is more, your density of states at the conduction band edge also will be more it is understood, it also depends on temperature. If you increase the temperature their number conduction band density of states also will increase.

Similarly, you will have an N v and this is conduction band density of states, and this is effective valence band effective density of states. You can say valence band effective density of states. So, it is also like the density of states at the edge of the valence band here ok, and its unit also n v in is also is per centimeter cube ok. And this is given by

$$N_V = 2(\frac{2\pi m_h^* k_B T}{h^2})^{3/2}$$

Where m^*_h is hole effective mass, k_B is the Boltzmann constant, T is the temperature.

So, this density of states conduction band density of state valence band density of states are numbers that can be precisely estimated or determine for different semiconductor crystal. Because if you know the effective mass you know the temperature then, you are done you can find out. And physically this represents the number electrons you can say that are actually at the edge of the conduction band. If you multiply the density of states times the if you multiply this by the probability of finding the electron there it will give you that it is a very large number in general, but this is your effective conduction band density of states.

So, what as I talking about? I was talking about this thing you know you have you have your total electron concentration as your effective density of states conduction band density of states ok, this is per centimeter cube remember. Fermi Dirac integral has no unit. And this is Fermi Dirac integral of x, where x is equal to E F minus E c by k B T ok. Now, this looks little daunting or mathematically little scary, but do not be scared actually because this is a very simple expression that a lot of things can be inferred for the from this ok.

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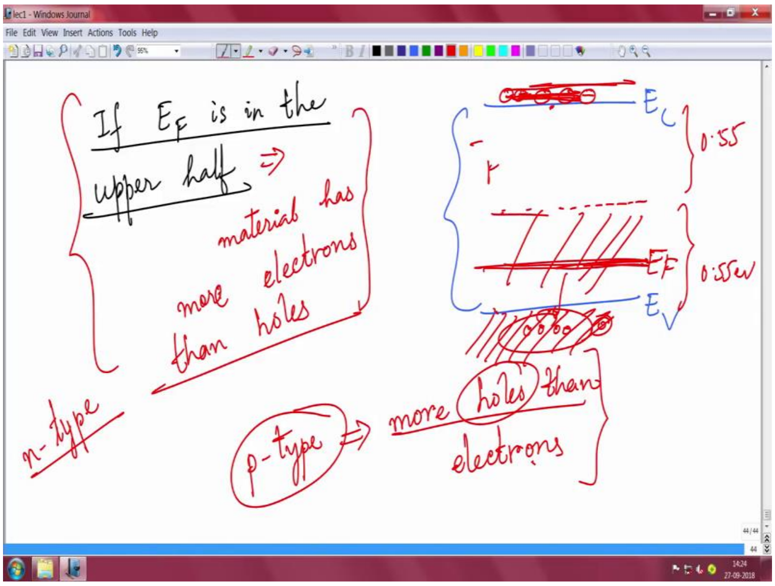

So, now in a material in a material, so this is a in a semiconductor material. This is conduction band; this is valence band ok. Exactly at the middle this is suppose the band gap of the material which is 1.1 e v, which is the band gap. You think of exactly the middle almost the middle ok, this is the middle of the band gap. And this is Eg by 2 like this is 1.1 by 2. So, this will be 0.55 sorry this will be say 0.55 e v this will be also 0.55 ok.

So, if the Fermi level, if the Fermi level is somewhere here in the upper half, if the Fermi level is in the upper half is in the upper half, what does it mean? You know if the Fermi level is in upper half it means suppose the Fermi level is here ok, I will talk about Fermi level. Suppose this is a Fermi level ok, suppose this is a Fermi level E F. It means the probability of finding the electron is 50 percent at this level ok, it means the probability of finding the electron is 50 percent in this level that probability the fifty percent probability of finding the electron is closer to conduction band than to valence band.

What it means physically is that, there is a higher likelihood that there will be more electrons here there will be more electrons here than there are holes here. So, if this is the case, then the material has the material has or the semiconductor has for whatever reason more electrons more electrons ok, it has more electrons than holes and vice versa. Which means if the Fermi level if the Fermi level is in the bottom half say if the Fermi level is suppose here it means 50 percent probability of finding electron is actually here which means the 50 percent probability of finding electron is closer to valence band than to conduction band that means, there will be fewer electrons here, but more holes here in the valence band. In that case if the Fermi level is in the lower half, it means there will be more holes than electrons there be more holes than electrons that we should keep that in mind.

So, whereas I, yeah I was here right. So, there will be more electrons and holes. So, these are very unique things actually when a material or a semiconductor has more electrons than holes, we call it n-type semiconductor ok, there are more electrons. And if your material has more holes than electrons we call it p-type p-charge right, when there are more holes than electrons more holes than electrons and the Fermi level is in the lower half.

So, that means, there are more holes with the probability of finding the electron is closer to valence band and valence band is always filled up with electrons right. So, it means there will be much fewer electrons in the conduction band, more holes in the valence band; so, more holes than electrons means that the material is p-type and more electrons than holes means that the material is n-type.

Now, you cannot we cannot arbitrarily have higher electrons than holes or higher holes and electrons. We have to add essential external impurities to do that. If you do not add the external impurities, then what will have you know suppose you do not add any impurity, no light is shining, no magnetic field is applied nothing, no electric field is applied nothing, then ideally the electron and hole concentration will be identical ok. So, what I am trying to say is that let us come to a new page here.

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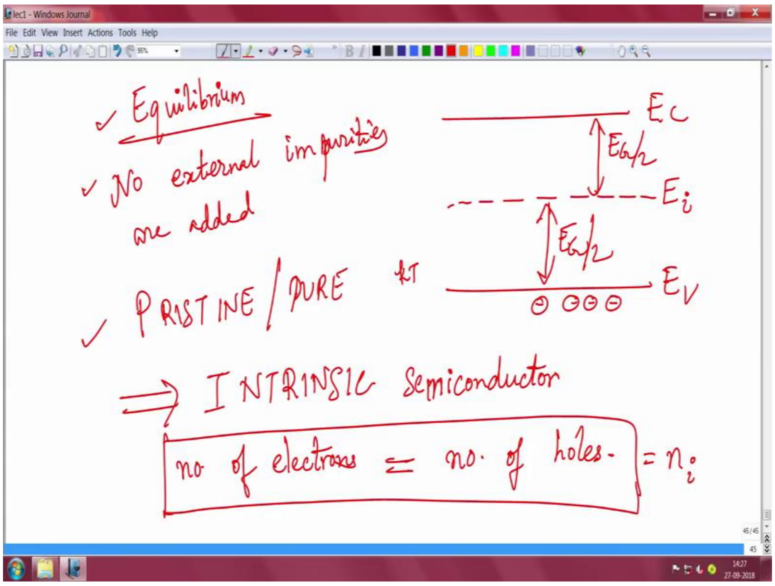

I told you this is a conduction band; this is your valence band. You take exactly the middle here that level is called E i is exactly middle here right. This is Eg by 2; this is Eg by 2 ok. If your semiconductor does not have any external impurity, no light is shining, no electric magnetic field, and it is in equilibrium, it is equilibrium and no external impurities have been added ok, no external impurities have been added nothing has been done. It is a pristine a pure crystal no impurities are added. So, it is a pristine item it is like the pure form of semiconductor right that is called an intrinsic semiconductor, intrinsic semiconductor sorry semi conductor ok.

An intrinsic semiconductor, in an intrinsic semiconductor, the number of electrons is exactly equal to the number of holes. I told you the electrons are here in the valence band. Because, of thermal energy k T there is a probability that they might go up very small probability, but because of density of states remember, I keep telling about density of states every, every often right.

Because of density of states there will be a finite number of electrons this hump you see there will be finite number of holes here right the area under this electron and hole curve will be the same actually you see. There will always be some electron that will try to go there and leaving behind hole. So, the number of electrons that are formed in the conduction band has to be equal to the number of vacancies left behind in this right, because if 10 electrons go up here then 10 holes are remaining. So, number of electrons and number of holes has to be exactly same at room temperature for an intrinsic semiconductor. And this number comes because of the probability times the density of states that are there density of states that are there.

So, you see here the number of electrons and number of holes has to be exactly equal we call that n i, it means intrinsic carrier concentration it is called intrinsic carrier concentration. So, now this is a very interesting we are entering into very very interesting and exciting semiconductor device concepts. So, let me wrap up the class today here lecture will wrap up the class here. And the next class will start from here ok. The electrons and holes are equal in an intrinsic semiconductor. So, in what case will electrons be more than holes, or in what case will holes be more in electrons that is important right p-type and n-type semiconductor, we will start from there ok.

So thank you for your time, here again we will wrap up the class today with the concept of intrinsic semiconductor ok.

Thank you.