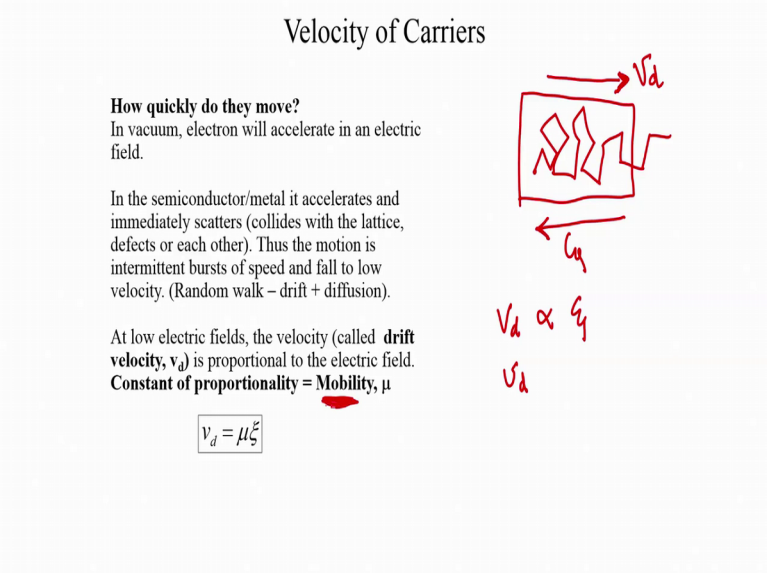
Semiconductor Devices and Circuits
Prof. Sanjiv Sambandan
Department of Instrumentation and Applied Physics
Indian Institute of Science, Bangalore

Lecture - 07
Solids: Electrons and Holes – Continued

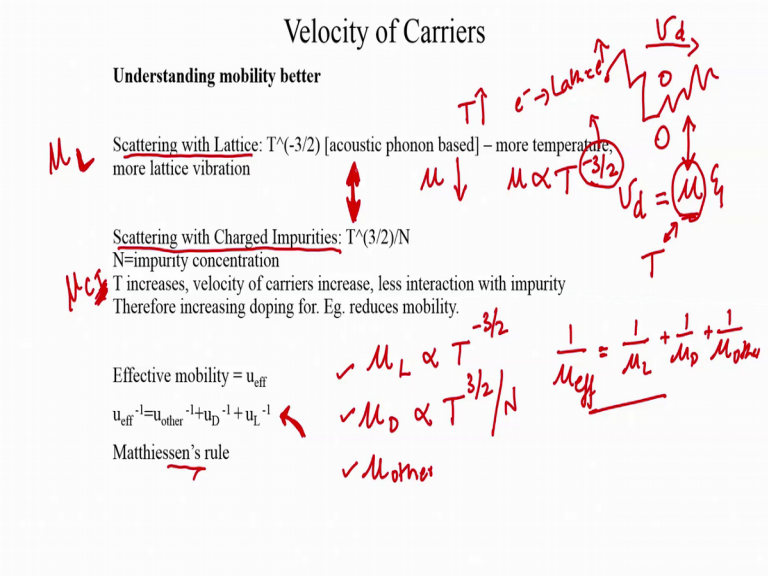
(Refer Slide Time: 00:14)



So, we now do realize that the movement of the electron in the crystal is quite complicated. The electron scatters through all the lattice atoms and it sort of makes its way through the crystal and it gives you a feeling that it is moving at a constant velocity which we said was the drift velocity. And this is in response to the applied electric field in the crystal.

Now how do we quantify the drift velocity? It so happens that for low electric fields, the drift velocity is proportional to the electric field for low electric fields and this constant of proportionality is a quite an important parameter and that is something called as the mobility.

(Refer Slide Time: 01:02)



So, let us understand this mobility a little better ok. So, we have now said that the movement of the electron through the crystal has got all this complex physics of it polarizing the atoms and its scattering with the lattice and its scattering with any impurities etcetera. And that movement of the electron in response to the electric field, allowed it to move at some constant velocity and that velocity was called as a drift velocity. And that drift velocity was related to the electric field by this parameter mobility which is mu. So, how does this parameter mobility relate or how does it connect with the physics of all the scattering ok?

So, we see that the parameter this mobility is a very strongly temperature dependent parameter ok. It is very temperature dependent and why is that?. So, we now we consider the different scattering mechanisms or the mechanisms by which the electron interacts with the lattice and in particular we look at two kinds of scattering mechanisms: one is the scattering with a lattice and the other is the scattering with the charged impurities in the lattice. So, you have these silicon atoms in the lattice and as I mentioned these silicon atoms are all vibrating around ok. So, although I am using the word silicon, it is true for any semiconductor lattice ok.

So, these atoms are all vibrating around and they are all connected to each other because of all the bonding and you have this massive spring mass structure that is vibrating

because of the thermal energy. And we have this electron that is interacting, that is scattering with all these vibrations.

Now, since the source of this vibration or the cause of this vibration is temperature as the temperature increases the vibration becomes more and more vigorous and the scattering probability also increases significantly. Therefore, it is expected that as temperature increases, the scattering of the electron with the lattice would also increase. And if one were to define a mobility or the drift velocity of the carriers in response to the electric field due to lattice scattering alone to the purely due to the lattice scattering. We will find that the mobility decreases with the temperature and that relation is approximately given as mu is to be proportional to T to the power minus 3 by 2. Of course, in reality you want to extract this coefficient in a more empirical manner.

Now the electrons also interact with charged defects in the lattice and a good example of charge defects is are ionized dopants and we will look at ionized dopants again in great detail further down the course. So, let us just assume that you have the lattice and at some place the lattice atom which is say silicon has been replaced by some other atom which has actually got a charge. And therefore, the electron is moving through the crystal and it tries and it interacts and it interacts with this charged ionized impurity. And that scattering is also temperature dependent ok, but it is got a different argument or a different kind of relation with temperature.

So, in this case it is quite important to see as to how much time the electron spends in interacting with the charge defect. If the electron spends a long time interacting with a defect, the scattering will be larger and the mobility will come down. But as temperature increases as temperature increases, the velocity the thermal velocity of the electron also increases. And therefore, since the electron has got a larger energy the electron gets past this impurity without spending too much of time interacting with it ok. So, you might if you would like to imagine it, you might want to imagine a very rapid interaction with the with the impurity as the electrons sort of flies past in its vicinity. And therefore, as the temperature increases the mobility due to it charged impurity scattering alone. So, if you were to define a mobility and a drift velocity due to charged impurity scattering alone, we will find that that mobility increases, because the electron is going to have a larger velocity and its going to spend lesser time interacting with that impurity.

Now clearly the number of such impurities also matter. So, as the number of such impurities increase or if the impurity concentration increases, it is more likely that the electron will spend larger amount of time interacting with these impurities. And therefore, the mobility will go down as the concentration increases. So, the mobility is said to be due to defect or charged impurity interaction is said to be proportional to T to the power 3 by 2 divided by, where N is the concentration or the number of impurities per unit volume of the semiconductor.

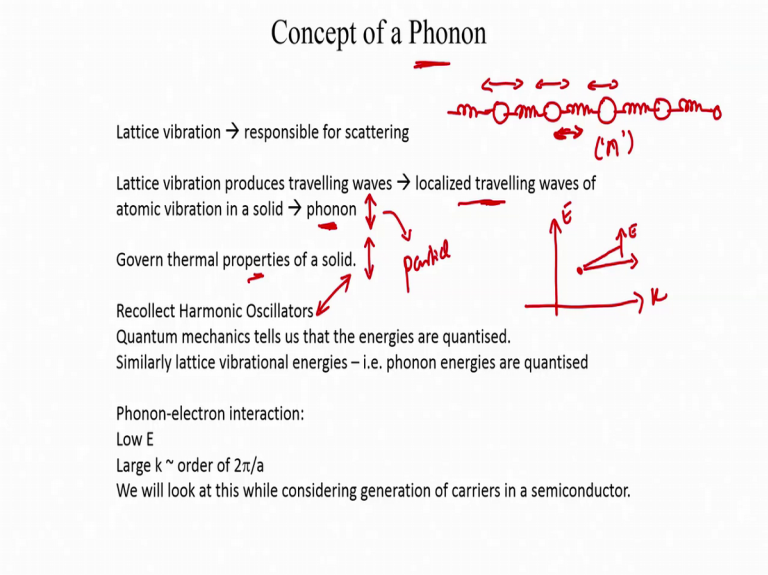
Now there could be many other scattering mechanisms. For example, they you could have something called as a piezoelectric scattering. You could also have electrons scattering etcetera and although we have not considered every scattering mechanism in detail. Let us just say that all the other scattering mechanisms affect mobility which is defined as u in this little expression as u other.

So, let us say there is a mobility due to all the other scattering mechanisms. So, these are the individual definitions that we gave that is the mobility due to lattice alone is proportional to T to the power minus 3 by 2. The mobility due to defects or charged impurities alone is proportional to T power 3 by 2 by N ok. And then we could have other mechanisms that contribute you know that define a different physics for the electron interaction with the lattice.

So, given all these different separate impacts on the electron mobility, what is the effective mobility of the electron in the crystal? So, that effective mobility the calculation is given by a very simple empirical formula which is got a small logical argument with regards to you know how the electron interacts with lattice. And it is given by something called as the Matthiesen's rule which says that if you have these different individual mobilities due to different mechanisms, then the effective mobility of the electron in the semiconductor is given by is related to these different mobilities as 1 by mu effective is equal to 1 by mu L plus 1 by mu D plus 1 by the mobility through any other mechanisms. And this is something called as Matthiesen's rule and it says in some sense it does make some good predictions, but it need not always make accurate predictions. But it is a useful method or an approach to know.

Now we said that the scattering with a lattice is dependent on acoustic vibrations or is dependent on thermal it is dependent on thermal energy and it is dependent on the vibrations of the atoms in the lattice.

(Refer Slide Time: 09:23)



So, since we are talking about the electron interaction with the lattice and in particular land the vibrating atoms or the lattice sites, it is useful to define a concept of an entity called as a Phonon.

Now, it is the vibrations of all the atoms in the lattice that results in the scattering of electrons and that results in one of the methods that determine the one of the phenomena, the determine or limit the mobility of the electrons in the semiconductor. Now these atoms could be thought of or could be modelled as a large spring mass a networked spring mass system and by looking at you know the vibration and the vibration modes of all these atoms, one could write a set of this equations and differential equations that included the displacement of all these springs ok. And you have a n atom periodic a linear periodic arrangement of n atoms and we could sit down and write out a differential equation that described this spring mass system.

Now as a consequence of such a differential equation it so, happens that the behaviour of atoms in a crystal a vibrating due to due to a source of external energy, results in localized travelling waves in the crystal and these localized travelling waves of atomic vibration in a crystal are is something called as the phonon ok. And therefore, since the

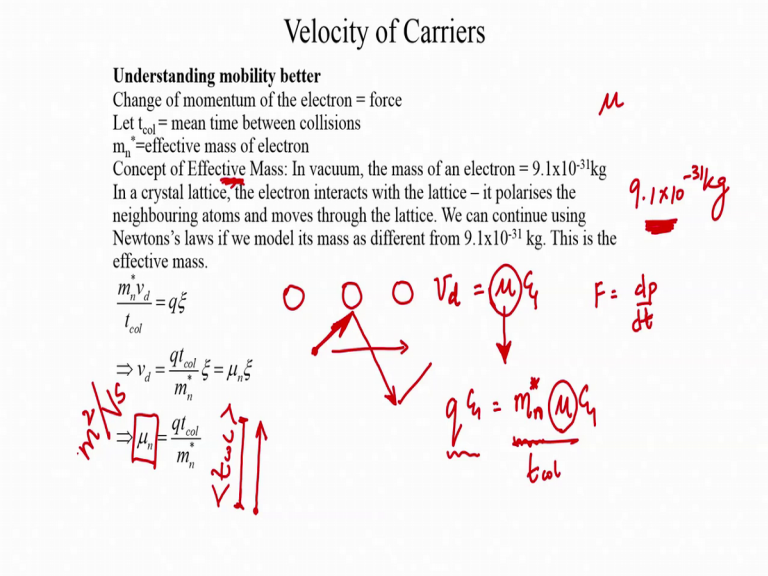
phonon is directly associated with the lattice vibration it is clearly going to be playing an important role with regards to the thermal properties of the solid. For example, let us say the thermal conductivity of the solid. So, therefore, the phonon can be thought of as the particle equivalent of lattice vibrations.

Now, since we are talking about vibrations we can make a direct connection or to the harmonic oscillator which was the example we considered by looking at Schrodinger's equation and we saw that the energies in the harmonic oscillator were all quantized. And therefore, it can be expected that the lattice vibrational energies in a crystal are also quantized or in other words the phonons can only take certain discrete energy levels or in the phonon energies are also quantized.

Now off interest to us in this course is the interaction of the phonon with an electron which we will look at when we talk about thermal recombination generation mechanisms. But a clue or you know a good indicator to what this interaction of a phonon with an electron does is that, a phonon after interacting with electron does not increase the energy of the electron too much ok. But it is got, but it is it is got the potential to significantly affect the momentum of the electron ok.

In other words, the energy E, if you look at think of an E K diagram, E K diagram which we have already seen before an interaction with the phonon has got the possibility for the electron to move a large distance in K space, but it does not have the ability to transit much in the E direction. And this plays a very important role while deciding the generation and recombination mechanisms of electrons in something called as an indirect band gap semiconductor which he will come back to when we look at the generation of carriers.

(Refer Slide Time: 13:30)



So, finally, we will take one more step to try and understand and model the concept of mobility a little better. So, we have seen the way we have defined mobility so, far is that we have said that the electron response to an applied electric field by moving by drifting through the crystal and having a drift velocity v d which is equal to say some mu times the electric field. And this was true for low electric fields and this mu was defined to be the mobility. And we said that this mobility is essentially governed by microscopic phenomena such as the electron interaction with the vibrating lattice, the electron interaction with charged defects etcetera. But let us say we would let us say that we want to quantify the mobility and develop relations that are a little bit more suitable for calculations and for modelling.

So, one approach to doing that is by using a concept called as the effective mass of the electron and what I mean by this is. Let us say you have you have electron moving through the crystalline lattice ok. You have all the atoms in the lattice and the electron is accelerating because the applied electric field, then it quickly scatters through the lattice and then, it accelerates again quickly scatters and loses its velocity etcetera. So, because of this very complex movement of the electron in the lattice, it is quite impossible for us to use Newton's laws of motion directly. But let us say that we really like to use a simple model for the movement of the electrons. For example, we would really like to define the force experienced by the electron as a rate of change is the momentum of the electron. And try to use Newton's laws, then we must make some arrangements or we must come

up with some models that have got some artefacts in it. And one such artefact one such modelling parameter is something called as the effective mass.

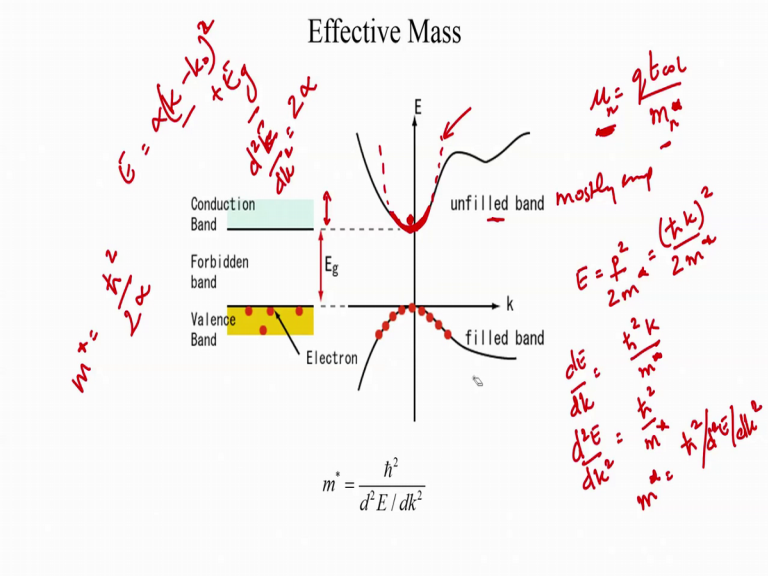
So, we say that the electron in vacuum has a mass of 9.1 into 10 power minus 31 kilograms. But the electron in a crystal does not have this mass. We will not use this mass because we want to develop a model that is very simple and that can describe the movement of an electron in a in a crystalline solid in a manner that is very close to Newtonian mechanics ok. And then we say that let q E be the force experienced by the electron and that force is equal to the mass which is now not 9.1 into 10 power minus 31 kilograms, but by this effective mass of an electron which is a modelling parameter that we define.

So, we say that the effective mass of an electron is m n star n stands for the electron and m p would stand for the hole and the star there says that it is the effective mass. So, we say that q into E which is a force is equal to the rate of change of momentum which is the mass the effective mass of the electron into the drift velocity of the electron divided by t collision. And what does t collision? It is the mean collision time of the electron.

So, we say that the electron has managed to acquire some velocity and the mean time before width which it is going to lose that velocity is something called as the mean collision time t collision. So, it is typically around these expected values of time which is a statistical estimate it is expected that the electron undergoes a scattering event after every d collision seconds. And therefore, this equation defines the movement of the electron as per our simple model ok. So, that is the kind of model that we would like to develop and if we continue using this model, we see that the drift velocity is nothing, but mu times the electric field. And therefore, this mu which is the mobility automatically becomes defined as q into t collision by m star where m is the effective mass of the electron.

Now, while performing calculations using such an equation, it is important to note that you will get the mobility in terms of meter square per volt second which is the units of mobility. It will be in meter square per volt second. So, what is this effective mass?

(Refer Slide Time: 19:00)



So, we said that we would like to define or you know create this parameter called as the effective mass of the electron, in order to write down a simple expression governing the motion of the electron in the crystal. And therefore, we defined mobility saying that the mobility is q into the mean collision time divided by the effective mass of the electron. So, that was the electron mobility.

So, we would like to define the effective mass in a more careful manner. So, how do I extract, how does one extract? What this effective mass is? What value you should be used for this effective mass? Is it a guess or is it is a why is it a wild guess that we just use in to fit any data or is there at least some starting point for this guess ? The answer is that there is a starting point for this guess and that starting point comes in from our E K diagram ok.

So, let us say let us go back to the E K diagram that we used when we define the Kronig Penney Model etcetera. And there we had these s shaped stitches and I had mentioned at that point that these s shaped stitches the edges of those could be modelled as little parabolas. So, let us come back to that point.

Now, here is an example of an E K diagram for a certain crystal. You have an clear energy band gap which is got a this crystal has got a band gape E g. So, those are the conduction band states and these energies correspond to say those conduction band states

and here are the valence band states and these energies correspond to the valence band states. So, these are all the filled states and those states are mostly empty.

So, if an electron were to jump into one of these conduction band states. Let us say an electron is to go there ok. So, which means the electron is free to conduct through the crystal and therefore, it has got a certain mobility and therefore, in order to calculate this mobility, we need to identify the effective mass or its vice versa. We know the mobility and we can therefore, calculate the effective mass. So, how does the effective mass appear in this picture ok? And the answer is given by this little relation here while the derivation for this has to be a little bit more careful. We will do something that is a bit more crude.

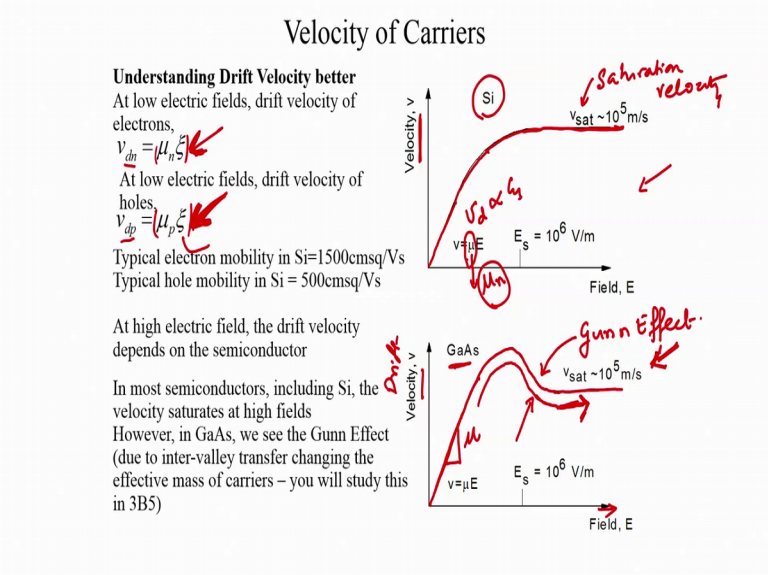
So, let us say that the energy is nothing, but p square by 2 times the effective mass and p is nothing, but our momentum which is h bar k the whole square by 2 times the effective mass. So, what is d E by d k ? The answer is it is minus it is its sorry it is h bar square into k by m star where m star is the effective mass. And what is d square E by d k square ? The second derivative we say that it is h bar square by the effective mass. And therefore, the effective mass can also be written as h bar square by d square E by d k square which is the expression given here. So, therefore, if we know if we have the energy E k diagram for a crystal, we take these energies this E k region which the electron occupies in the conduction band.

And we say that we will that we would fit the best fitting parabola to this region and let us say that parabola has a relation such as alpha k square plus say whatever coefficient let us say that is E g. So, it is got its got an E k relation that goes as alpha k minus k naught square plus E g in a very general manner. And once this fit is made the term d square E by d k square is given by 2 times alpha. And therefore, the effective mass can be said to be h bar square by 2 times alpha

Now, note what happens in the valence bands ok. So, what is important to the valence bands is that you have holes that are the charge carriers. So, let us say you have empty states in the valence band. So, let us say we have a hole here ok. Now let us for a second imagine that the hole is now going to behave like a particle and if you are going to use the same kinds of laws of motion that is a relation between the force and force and the effect the sorry the mean collision time and the mobility and the effective mass for a

hole, we can calculate the mass for a hole using a similar argument ok. But since this parabola is inverted, we find that we end up with negative values for the mass of hole. So, the hole could be imagined to be having a negative charge in response to the that moves in an opposite direction with regards in response to an applied electric field or it could be assumed to be having a negative mass which is moving in the opposite direction in response to the applied electric field.

(Refer Slide Time: 25:02)



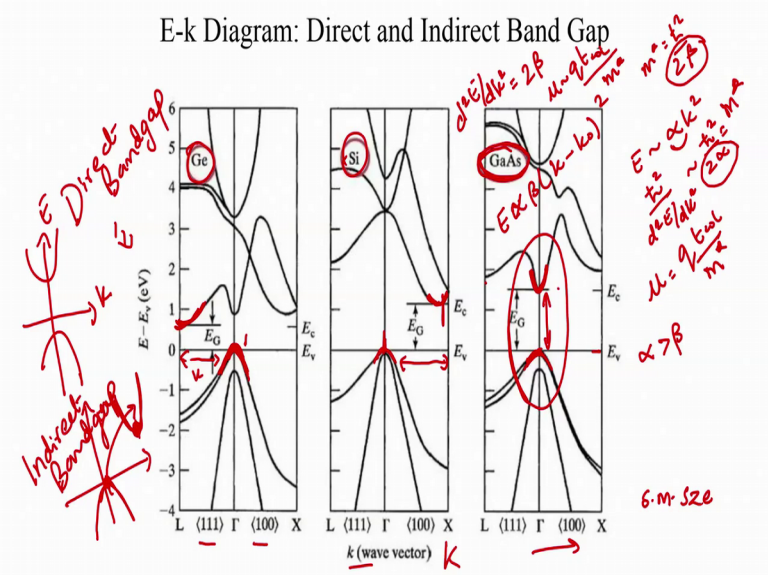
So, in order to understand the velocity of carriers in response to an electric field better, we will now look at these 2 plots. Now these plots show the rest of the velocity of the carriers in response to an electric field for a very large electric field range. So, in materials like silicon, we will see that the drift velocity of electrons typically follows this kind of a profile.

At low electric fields, we find that the drift velocity is proportional to the electric field and that constant of proportionality is the mobility. So, just like how we define an electron mobility one could you also use the same kind of a relation to define the drift velocity of holes. So, since both electrons and holes are carriers of current the holes in the valence band and the electrons in the conduction band are both carriers of current, we are we can define a drift velocity for both these species and that can be given by this relation for low electric fields.

Now, as the electric field continues to increase the drift velocity rolls off and begins to saturate ok. And the mobility and the velocity of these carriers saturates and to a point which is called as the saturation velocity. So, at very high electric fields the velocity of the carriers and materials like silicon saturate. Now other materials like say for example, gallium arsenide behave in a slightly more interesting manner. So, here is a plot of the drift velocity of carriers in gallium arsenide, drift velocity of electrons in gallium arsenide versus the electric field.

So, at low electric fields we see that the drift velocity is proportional to the applied electric field and the slope defines the mobility. But as the as the electric field begins to increase, the drift velocity tends to saturate, but then it suddenly starts to drop low and it drops down and begins to saturate at a new value. Now this interesting phenomena is something called as the gun effect and it can be explained by looking at the full E k diagram for gallium arsenide.

(Refer Slide Time: 27:37)



So, in this these plots show the complete energy versus k diagram for different semiconductors for 3 different semiconductors; germanium, silicon and gallium arsenide. And these plots have been taken from our reference textbook which is S.M. SZE book on Semiconductor Physics. And in these plots, what you see are the e k diagrams considering the fact considering the entire 3 d crystalline material and you find that depending on the different planes and the different directions you have these range of

curves which constitute the energy k diagram. And our conduction band and the valence band that we have looked at appear in this particular manner. You look at the lowest energy of all the states that are mostly empty and that defines our conduction band edge E c and you look at the highest states of all these energies that are mostly filled and that defines our energy level E v and this gap is defined as E g. So, this is the case of germanium. And here we have the case of silicon where you have the lowest point of the conduction band conduction bands and the highest point of the valence bands and their energy gap E g.

Now, the material of interest to us with regards the previous slide is gallium arsenide. So, here you have the energy k diagram for gallium arsenide and we are particularly interested in this region that I have shown in red. And therefore, here is the lowest point which is your E c and that is the highest point of the valence bands that is our E v. Now at thermal equilibrium, before we apply any voltage, we find that this is the smallest gap that is your energy gap.

And we find that the electrons from the valence band are promoted to the conduction band and they begin to occupy this valley here. And this valley has got a certain profile you have an energy k profile which is say which is say defined as alpha k square and depending on the value of alpha, we have these electrons are having a certain effective mass. And based on that effective mass; so in this case, it would be h bar square by d square E by d k square which would be of the order of h bar square by 2 alpha.

Now based on this effective mass the electrons would have a certain mobility which is given by q into t collision by m star. Now as the electric field increases the n the momentum of this electrons increases and there they reach they pick up a values of energy that allow these electrons to cross over to a different valley in the E k diagram. So, these some of these electrons move over to this valley here.

Now, this valley is along the energy axis, it lies at a higher energy as compared to E g and therefore, the electrons were normally not populating this valley. But upon the application of very high electric fields, the electrons could now migrate to this valley here. And this valley has its own parabola which could be defined as E is let us say proportional to beta into k minus k naught the whole square where k naught is given by this little translation in k space. And therefore, our d square E by d k square ends up as 2

times beta and our mobility and is defined as q into t collision by m star where m star is h bar square by 2 beta.

So, we therefore, had electrons that had an effective mass defined by alpha and now after movement into another valley their effective mass is defined by beta. Now which of these 2 is greater? This parabola is much sharper as compared to this parabola here. And therefore, alpha can be said to be much greater than your beta. So, if alpha is much greater than beta, then the effective mass of the electron sitting in this valley is lower as compared to the effective mass of the electron sitting in the valley here. Or in other words the mobility of the electron sitting in this valley is higher as compared to the mobility of electron sitting in this valley.

So, that is why we see a E k diagram that looks like this. So, the electrons initially had a high mobility and they were occupying the sharper valley, but then as the electric field increased electrons migrated to the upper and more shallow E k diagram or the upper valley that had a more shallow e k diagram. And therefore, it their effective mass increased and their mobility is came down and therefore, we see a drop in the drift velocity. So, that is the explanation for the gun effect, in the case of gallium arsenide.

Now, before we wrap up and close this topic, there are a couple of points that I would like to make with regards to this E k diagram. And these are points that we will come back to later as well, but since we are already here we might as well just discussed this aspect. Now in the case of silicon and germanium; so, we note that the valence band that is the E k diagram corresponding to the valence band and the E k diagram corresponding to the conduction band have their minima or their maxima or their point of inflection let me say at different locations in k.

So, we see that this minima and this maxima are located at different points in k. And the same is true with silicon you see that the minima of the conduction band and the maxima of the valence band are located at different points in k space. On the other hand, in the case of germanium or in the case of gallium arsenide, we find that the minima of the conduction band and the maxima of the valence band are located at almost same point in k; there is no need for any translation in k space.

So, these materials where there is the minima of the conduction band and the maxima of the valence band are located at the k same value of k, these materials are materials said

to have a direct band gap and these are called as direct band gap semiconductors. And on the other hand, materials such as silicon where the maxima of the valence band and the minima of the conduction band occur at different values in k which would require an electron making a leap from a valence band to the conduction band having to change its k. These kind of materials are called as indirect band gap semiconductors. And they play a significant role when it comes to and this different plays a significant role when it comes to the generation of electron hole pairs and the recombination of electron hole pairs which is a topic that we will discuss further down the road.