

**Power Quality Improvement Technique**  
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**Lecture – 14**  
**PWM Rectifier – II**

Welcome to our NPTEL courses on Power Factor Improvement Technique. We are discussing the PWM Rectifier. This was going to be our second lecture on the PWM Rectifier. So, we were discussing about the boost rectifier and we are also discussing about continuous and the discontinuous operations of the boost rectifier.

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**Realization of a near-ideal rectifier (Cont...)**

**Open-loop DCM approach**

Advantage: simple control

Disadvantages: higher peak currents, larger input current EMI

Like other DCM applications, this approach is usually restricted to low power (< 200W)

The boost converter can also be operated in DCM as a low harmonic rectifier. Input characteristic is

$$\langle i_d(t) \rangle_i = \frac{v_d(t)}{R_s} \left( \frac{v_d(t)}{R_s} - v_d(t) \right)$$

Input current contains harmonics. If  $v_d$  is sufficiently greater than  $v_r$ , then harmonics are small.

**Other similar approaches**

- Use of other converters (in CCM) that are capable of increasing the voltage.
  - SEPIC, Cuk, buck-boost
  - Flyback, isolated versions of boost, SEPIC, Cuk, etc.
- Boundary or critical conduction mode: operation of boost or other converter at the boundary between CCM and DCM
- Buck converter: distortion occurs but stresses are low
- Resonant converter such as parallel resonant converter or some quasi-resonant converters
- Converters that combine the functions of rectification, energy storage, and dc-dc conversion

Now, the advantage of this is that we can have an open loop approach also. But open loop approach although it is simple because you do not take any feedback. Once you do not bother to take feedback, your system may be simple, but your accuracy will be a question mark.

So, for this reason there can be disadvantages like there can be the higher peak current and the higher the peak current is higher will be the rate of change of the voltage  $L di/dt$ . Thus this leads to the higher EMI EMC noises. So, large current with large EMI. Like other DCM applications, the approach is usually restricted to only for the low power rating up to 200 Watt and where for sake of the rating or for the reduction of the cost you just allow to operate it in the open loop.

But converter can also operate in DCM as a low harmonic rectifier and the input characteristics will be given by  $v_g(t)$  that is Re of this part plus there will be an extra term. But problem lies in the fact that this input current contains harmonic. If this  $v_g$  is significantly greater than  $v$  then this harmonic will be present. Because this part is a harmonic contributor.

Ultimately, if this  $v$  is more than  $v_g$  then harmonic content will be less and there, we can also use it for the power factor correction technique.

And there are other approaches as well. Use other converts in CCM that are applicable for the SEPIC converter, Cuk converter, buck-boost converter and also there can be an isolated DC to DC converter that is the flyback converter, isolated version of the boost converter, SEPIC and the Cuk converter.

The boundary or the critical condition. The boundary or the critical condition mode of operations of boost operation or of other converter will be almost same converter between the CCM and the DCM. But in case of the buck converter, there is an issue because voltage stress will be higher. Because problem lies in the boost converter, if you see the topology you know this is the inductor there after this and this.

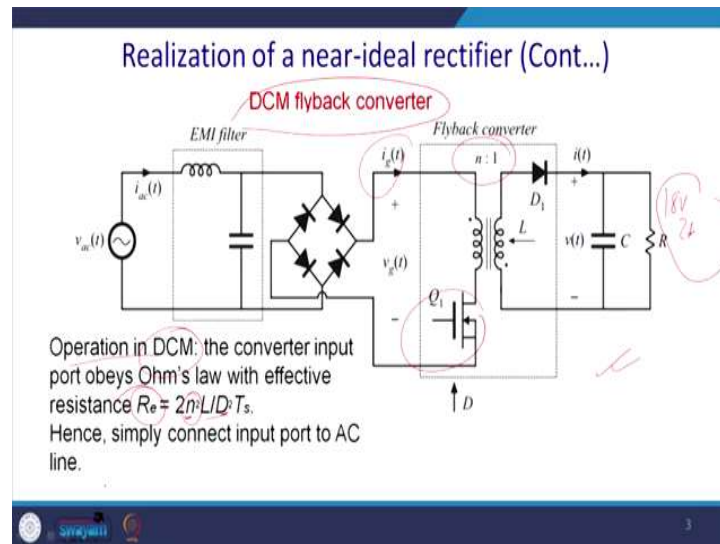
So, it is connected to the ground. So, once you switch it on, the voltage at this point goes to the ground. But in case of the buck converter, if you switch it on, generally it connects to supply. So, you got a higher voltage stress. So, you have to see this problem.

Thus, what happens? For buck converters, this is a disadvantage of it. The distortion occurs but as stresses are low because current will flow depending on difference of  $v_{in}$  and  $v_0$ . On the other hand, the resonant converters such as parallel resonant converter or some quasi resonant converters can also be used. So, that it can push. There is a resonant in tank and it will throw the frequency quite high because of the soft switching features.

Converter that combines the functions of rectifications energy, can be a total package. Converter, it combines the function of the rectification, energy storage and DC to DC conversion because you might be having some applications or the load that may not be fit for your rectified voltage and thus DC to DC conversation is required. Sometime, you may require for the designing so you specify that this much of the voltage interruption the circuit should tolerate.

Thus, you may have an energy storage and also you perform the function of rectification and that comes as a single package sometime. So, let us see that. This is what you have seen previously. It was the non-isolated DC to DC converter that is a boost rectifier topology.

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And what you will have it here? It is again a DCM. This is essentially a flyback converter, while energy will flyback. I request you to go through my advance power electronics courses of the DC to DC converter to understand this topological application, if you are finding it difficult.

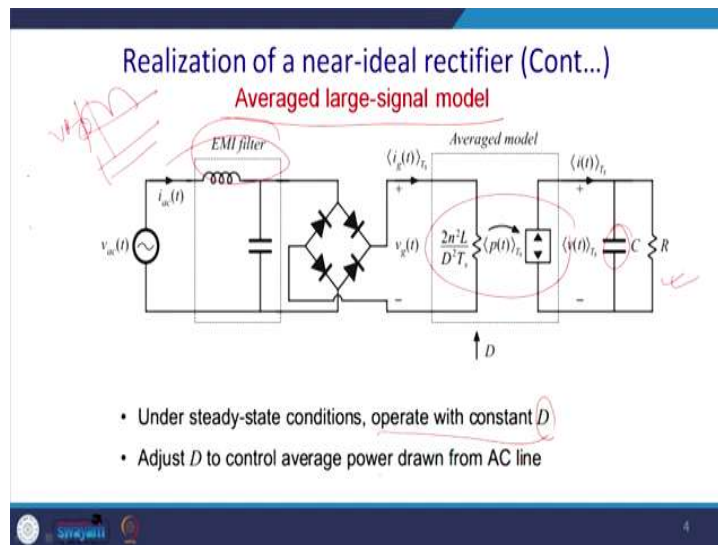
But this kind application is used for isolated DC to DC converter and it is the circuit that generally uses in your laptop charger, your mobile charger. All those charges up to wattage rating from the 500 Watt maximum. Your musical instruments also may be this 18 Volt, 2 Ampere something like this kind of rating. Your keyboard charger.

So, the operation ensures that voltage. This is the EMI EMC filters and we also ensure the same thing here  $v_g$  and  $i_g$  is there.  $v_g$  and  $i_g$  will be sinusoidal thus it is being reflected to the AC side and thus, you have a low THD and high-power factor. The operation of this discontinuous conduction mode of converter input obeys the Ohm's law. With the effective resistance here that will be '2' where n square, n square is a ratio of this turns from primary to secondary.

Generally, it is bucking most of the cases based on this application and this bucking is achieved by both switching as well as this turns ratio. So, you can have a heavy bucking. Generally, if the duty cycle is less than 0.1, you have seen there is a huge discontinuous mode of conduction. Thus generally one of the problems of the discontinuous mode of conduction is that the control is quite complex.

But here, you know your bucking can be done through this number of turns as well as with this duty ratios and thus, bucking is more effective. Hence, simply connect the input port to the AC line and we can assume that it has got a conversion to AC to DC. More specifically ideal AC to DC conversion.

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So, this is the same thing. We are going for the average model for this transfer function analysis and the stability. Here, we have an EMI EMC filter and apart from that you got a diode bridge rectifier. Then, you got a switching and this switching will be modeled as an average value and then we have same here that is effective Re in this case is  $\frac{2n^2L}{D^2 T_s}$ .

So, it will be replaced by this and ultimately energy will go to the secondary and this will be  $\frac{v T_s}{i T_s}$  and it will store some amount of the energy and thus energy will flow there. Under steady state condition, it operates with the constant D. Adjust D to control average power drawn from AC line. As we have seen that voltage in  $v_g$  is this and you require a constant

$v_g$ . So, here you required a least duty cycle because voltage is more and here you require more duty cycle as per the AC line.

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**Realization of a near-ideal rectifier (Cont...)**

**Converter design**

Select  $L$  small enough that DCM operation occurs throughout AC line cycle. DCM occurs provided that  $d_3 > 0$ , or

$$d_2(t) < 1 - D$$

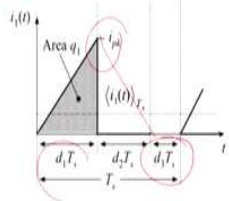
But

$$d_2(t) = D \frac{v_g(t)}{nV}$$

Substitute and solve for  $D$ :

$$D < \frac{1}{\left(1 + \frac{v_g(t)}{nV}\right)}$$

Converter operates in DCM in every switching period where above inequality is satisfied.



To obtain DCM at all points on input AC sinusoid: worst case is at maximum  $v_g(t) = V_M$ :

$$D < \frac{1}{\left(1 + \frac{V_M}{nV}\right)}$$

So, it is also a challenge to select the value of the inductor. So, it is inside it. Ultimately, there is a leakage inductance. A flyback converter is also called the inductor transformer. Because you have to put little bit of inductance to store energy and that energy will fly off once switch is off.

So, for this reason select 'L' enough that it can be operated a DCM mode. That DCM operation occurs through the AC line cycles and DCM occurs provided that  $d_3 > 0$  and this is the case where,  $d_2(t) = 1 - D$ . But  $d_2(t) = D \frac{v_g(t)}{nV}$ . Substitute this and solve for

D which is  $D < \frac{1}{\left(1 + \frac{v_g(t)}{nV}\right)}$  and converter operates in discontinuous conduction mode every switching period and where this inequality is satisfied.

And ultimately, this is a zone you know, because this is the area in the current once the switch is on and current will go up. This is the  $i_{pk}$  of peak and ultimately, till the duration it will flyback. That is the resetting of the flux. Thereafter, till this time  $d_3$ , there is a no current through the inductor and we say that it is a discontinuous conduction mode.

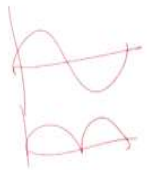
And one of the advantages of discontinuous conduction mode is size of the inductor is less, to obtain the DCM at all points on the AC sinusoidal. Worst case is that at  $v_g$ . So, you

really going to have a huge duty cycle and thus, you can come into the continuous conduction mode and  $v_g = V_M$  and ultimately, this is the condition of the worst duty cycle that we required to maintain for the operation of the flyback converter.

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**Realization of a near-ideal rectifier (Cont...)**

Choice of L to obtain DCM everywhere along AC sinusoid



We have:  $D < \frac{1}{\left(1 + \frac{V_M}{nV}\right)}$  with  $\frac{V_{rms}}{V_{ac,rms}} = \sqrt{\frac{R}{R_e}}$

Substitute expression for  $R_e$  to obtain

$$D = \frac{2nV}{V_M} \sqrt{\frac{L}{RT_s}}$$

Solve for L:

$$L < L_{crit} = \frac{RT_s}{4\left(1 + \frac{nV}{V_M}\right)^2}$$

**Worst-case design**

For variations in load resistance and ac input voltage, the worst case occurs at maximum load power and minimum ac input voltage. The inductance should be chosen as follows:

$$L < L_{crit-min} = \frac{R_{min}T_s}{4\left(1 + \frac{nV}{V_{M-min}}\right)^2}$$

Now, choice of L. Obtain in the DCM, where every of the sinusoidal voltage in AC side it is this and in DC side this. So, duty cycle has to be maintained within that. So, we have

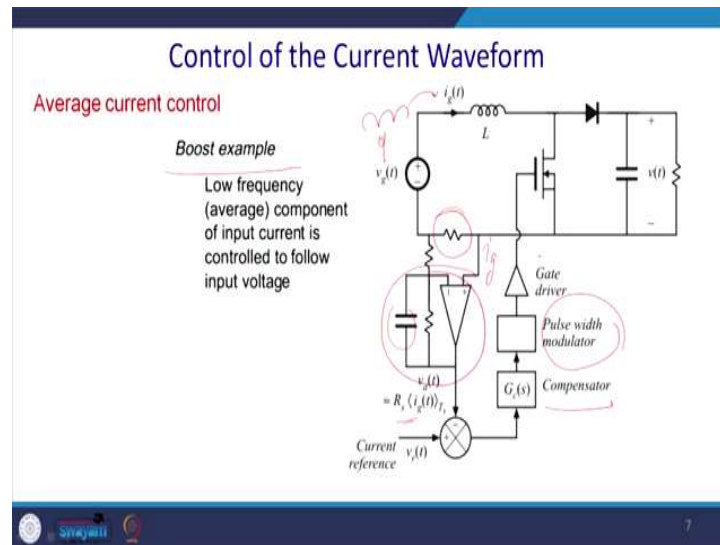
$$D < \frac{1}{\left(1 + \frac{V_M}{nV}\right)} \text{ with } \frac{V_{rms}}{V_{ac,rms}} = \sqrt{\frac{R}{R_e}} \text{ and thus substituting this expression, we can have the value}$$

$$\text{of } D = \frac{2nV}{V_M} \sqrt{\frac{L}{RT_s}}$$

So, we can solve for the value of the L. That is quite important for designing the value of the transformer in case of this isolated DC to DC converter. So, L will be less than  $L_{critical}$  that value will be given by  $\frac{R_{min}T_s}{4\left(1 + \frac{nV}{V_M}\right)^2}$ . So, what is the worst-case design?

For variation in the load resistance and the AC input voltage, the worst case occurs at maximum load power and the minimum ac input voltage. That is a challenge. The inductance should be chosen follows that  $L_{critical}$  equal to  $L_{minimum}$  and considering this, critical case has been satisfied.

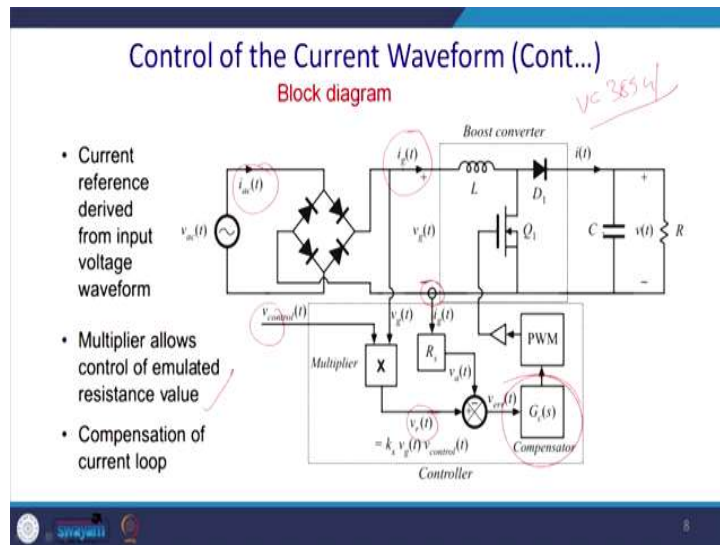
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So, how you do that? Let us first take the example of the average model for the boost rectifier. So, it is a low frequency average model and you know you got this kind of  $v_g$  and  $i_g$  which will have a same profile. Just it is a change in magnitude. So, you will be sensing the current of  $v_g$  and ultimately you can see that this is the op-amp and of this measure this current is essentially  $i_g$  and generally you can see that is a feedback path.

So, it is essential a PI controller. So, this current has been converted to this entity. That is  $R_s$  ultimately  $R_s i_g(t)$ . So, this PI controller convert this and ultimately you have the reference  $v_r$  which is coming from the  $v_g$ . So, you subtract it and you just have a compensator. Thereafter you got a pulse width modulator, then you have a gate driver that will switch on the gate and we will maintain that  $i_g$  and  $v_g$  sinusoidal. So, this is the overall block diagram.

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And you see this is. This is a block diagram of mostly UC. I have to change little bit of the circuit because of this Texas instrument patent.

So, UC3854 you can refer or 5556. So, this is the control logic generally they follow and there is a little bit change from that. So, current reference derived from the input voltage waveform. That is all it is and the multiplier allows to control the emulator resistance value  $R_e$  and compensation loop is been introduced.

So, this is  $i_{ac}$  and thus at this point you got  $i_g$ .  $i_g$  generally instead of sensing through the resistance, as we have seen previously. we generally sense it by the hall effect current and thus, this will be multiplied with the value of the  $R_s$ .  $R_s$  is the synthetic resistance. So, which will be giving you the value of  $v_a$ .

Similarly, you have  $i_g$  and you have  $v_{control}$ . You will multiply. Multiplication gives you the power. This power you have will be a  $v_r$  by  $t$ . So, that will be  $v_r$  and you will compare with  $v_a$  and then, you will put to, generally it is a PI controller and essentially a compensator. So, there after you will fit to the PWM and we will run this boost rectifier. So, the current sensor has a gain of  $R_s$  that you will be tuning.



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### Control of the Current Waveform (Cont...)

- Current sensor has gain  $R_s$  :  
 $v_a(t) = R_s [i_g(t)]_{T_s}$
- If loop is well designed, then:  
 $v_a(t) = v_r(t)$
- Multiplier:  
 $v_g(t) = k_x v_r(t) v_{control}(t)$
- Hence the emulated resistance is:  

$$R_e = \frac{v_g(t)}{i_g(t)} = \frac{\left( \frac{v_r(t)}{k_x v_{control}(t)} \right)}{\left( \frac{v_g(t)}{R_s} \right)}$$

The emulated resistance

which can be simplified to

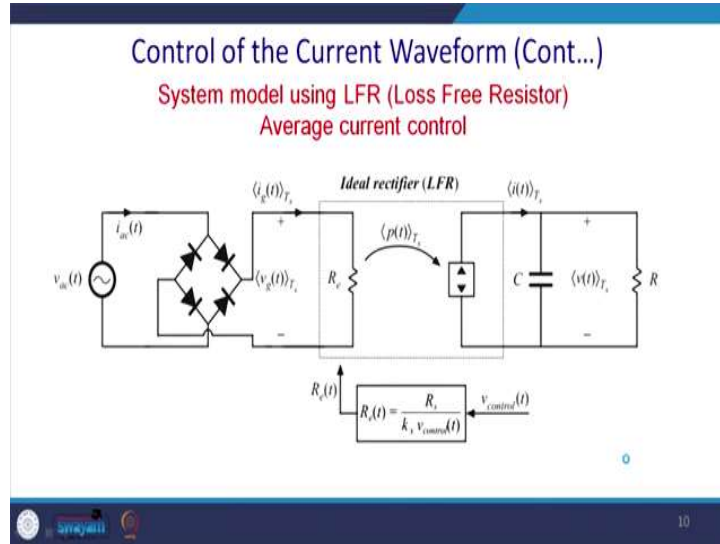
$$R_e [v_{control}(t)] = \frac{R_s}{k_x v_{control}(t)}$$

So,  $v_a(t) = R_s [i_g(t)]_{T_s}$  and the emulator resistance is the  $R_s$ . The loop is well designed when  $v_a(t) = v_r(t)$ . So, proper compensation can be achieved. You can see that this is  $v_{ac}$  this is  $i_{ac}$ . Thereafter rectification. From there it gets  $i_g$  and this is the  $v(t)_{control}$  and multiplied with the  $v_g$ . So, ultimately you got  $v_r$  and this point it becomes  $k_x v_g(t) v_{control}(t)$ .

Similarly, you got a current sensed and that has a gain of  $R_s$  that makes it  $v_a$ . Ultimately,  $v_r$  should be equal to  $v(t)$  and that will be feed to the PI controller and it will run these switches so that you can get a sinusoidal voltage and current at the input of this power factor corrector.

Hence, you have to calculate the resistance. That is emulated resistance. This is very important. In this configuration, there is  $R_e v_g$  that is the function of  $t$ , by  $i_g$  function of  $t$  and which can be further simplified to  $R_e(v_{control}(t)) = \frac{R_s}{k_x v_{control}(t)}$ . So, this is the way to control the voltage.

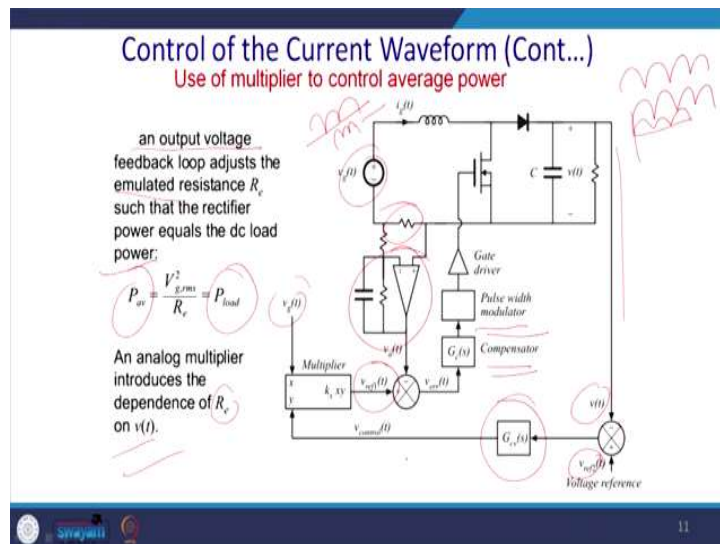
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Now, the control of the current wave form. That is one of the features and we will assume that this is ideal rectifier which we have said in the previous class. What is ideal rectifier? The system model using LFR, that stands for the Loss Free Resistance and the average current required to be sinusoidal.

Ultimately, this is  $v_{ac}$ . This is  $i_{ac}$ . This is rectification and at this point, this is DC  $i_g$  that is the ripple, 100 hertz's ripple and thus voltage is also 100Hz ripple. Thereafter you got effective  $R_e$  and so on, you calculate this.

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Now, see that how you can achieve actual current control mode. An output voltage feedback loop adjusts the emulated resistance  $R_e$  such that the rectifier power equal to the DC load power and  $P$  average equal to  $P_{load}$ . An analogue rectifier introduces the dependence of  $R_e$  and  $v(t)$ .

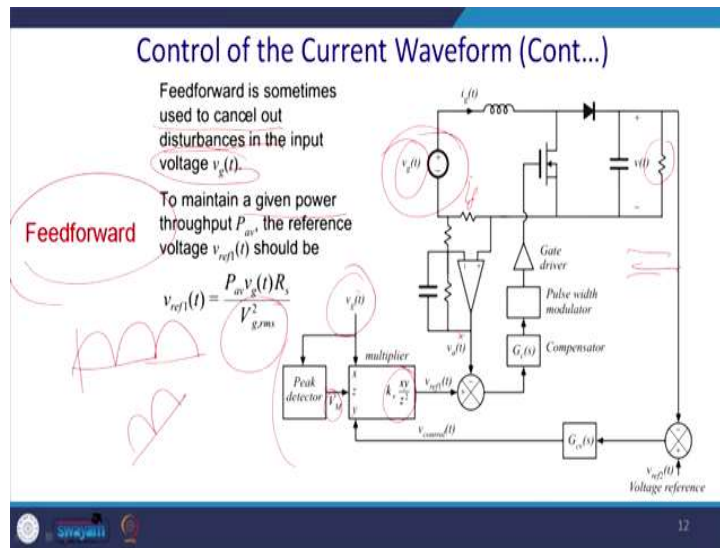
So, you can see that how does it work. That is your  $v_g$  and thereafter that is in this kind of form please mind it. Ultimately you got  $i_g$ ,  $i_g$  you will have this kind of form. You will be sensing the resistance. There is a sensor resistance and this is nothing but a PI controller. Here, we got  $v_e$  and from  $v_g$ , you get the feedback. This is the outer voltage control loop.

So, this is  $v(t)$  and this is your reference voltage. That is voltage you wanted to keep and ultimately, you required to process the gain because it is the DC and here you put it and this  $v_g$  and this error has been multiplied.

And thus, from there with the transfer function, we generate the  $V$  reference. It means that if that is an error it will be multiplied and you will be keeping this error value more. So, essentially you are changing the reference value of  $i_g$ . If the error is less,  $i_g$  will come down here. If reference value is more  $i_g$  will come down like this.

So, this is the way you are changing. So, essentially you are shifting the value of the envelop. So, that is the task of this job and thus, you have an error. Thereafter, you have feeding to the compensator. Thereafter, you got a pulse width modulator and thus your designing or switching the switches.

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Now, this is the feedback combination. We can have a feedforward for the faster action and that also sometime helps to cancel out the noises. Feedforward is something we use to cancel out the disturbance voltage  $v_g$  to maintain a given power throughout  $P$  average.

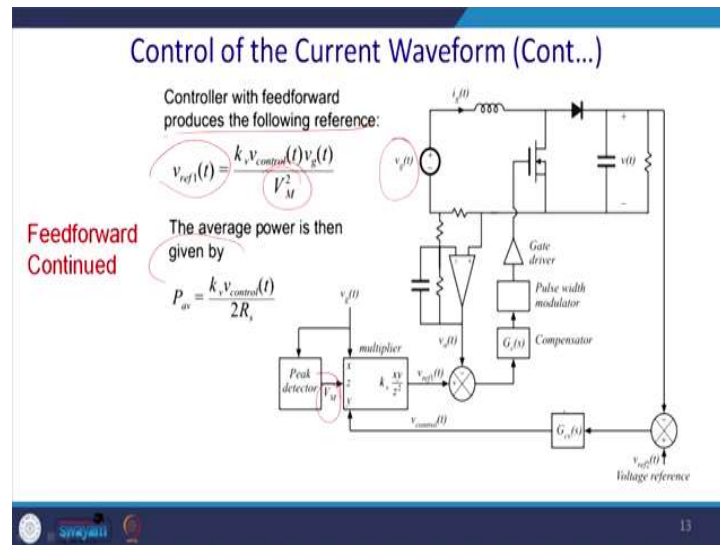
The reference voltage  $v_{ref1}(t) = \frac{P_{av} v_g(t) R_s}{V^2_{g,rms}}$  and from there you got a  $v_g$ . Thus, you got  $i_g$  and this is the rectified boost topology and you sense the value of  $i_g$  and from there you put to the PI controller and thus you get the  $v_a$  star.

Here, you generate the  $v_g$  and you generate the peak of it that also ensure that whether there is a sag in the voltage or not this value will come and that is value of the  $V_M$ . So, this gives you a DC value and this also gives a DC value if there is a voltage is maintained or not. From there you will calculate the value of the current envelop and thus, you multiply with this.

So, it will take to input whether there is a sag. If there is a sag, then voltage duration required to be increase. So, for this reason that input has been taken care of and also if there is an output error that also taken care of and accordingly, you generate this  $v_{ref}$  profile and that will be compared and this will be then switch it on.

This is the feedforward mode and this part is the feedforward and that will ensure that. The disturbances here because of the sag or swell has been eliminated. So, this the purpose and you get a constant DC voltage here.

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So, we shall continue with the feedforward operation. So, control of the current waveform. The controller with feedforward produces the following reference. You can see that. Here it is  $v_{ref1}(t) = \frac{k_v v_{control}(t) v_g(t)}{V_M^2}$ , where  $V_M$  is coming from the peak detector. So, that gives the information whether voltage is healthy or not.

And thus, the average power is given by  $P_{av} = \frac{k_v v_{control}(t)}{2R_s}$ . Because if there is a voltage sag average power may come down. So, in that case  $v_g$  is the power, otherwise it is same. Same feedforward logic has been used here.

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### Control of the Current Waveform (Cont...)

Averaged (but not linearized) boost converter model:

Modeling the inner wide-bandwidth average current controller

$$\langle v_g(t) \rangle_{T_s} = V_g + \hat{v}_g(t)$$

$$d(t) = D + \hat{d}(t) \Rightarrow d'(t) = D' - \hat{d}(t)$$

$$\langle i(t) \rangle_{T_s} = \langle i_1(t) \rangle_{T_s} = I + \hat{i}(t)$$

$$\langle v(t) \rangle_{T_s} = \langle v_1(t) \rangle_{T_s} = V + \hat{v}(t)$$

$$\langle v_1(t) \rangle_{T_s} = V_1 + \hat{v}_1(t)$$

$$\langle i_1(t) \rangle_{T_s} = I_1 + \hat{i}_1(t)$$

Problem: variations in  $v_g$ ,  $i_1$ , and  $d$  are not small.

So we are faced with the design of a control system that exhibits significant nonlinear time-varying behavior.

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Now, based on that we can go for the small signal analysis. This is quite important aspect while controlling or designing this DC to DC converter or AC to DC converter followed by a DC to DC converter. So, we have this perturbation.

So, this is the perturb term and this is the average value for  $v_g$ . Same way duty ratio has been perturbed and we can rewrite it like this and  $i(t)$  can be perturbed, it can be rewritten like this.  $v$  can be perturbed,  $v_1$  can be perturb,  $i_2$  can be perturbed.

So, this is the  $i_1$  in the AC side and  $i_2$  is in DC side. So, what we can say here is the variation of the  $v_g$  and  $v_g$ ,  $v_g$  and  $i_1$  and  $D$  which are not small. So, we are faced with the designing a control system that exhibits significant non-linear variable. So, if you try to solve by the linear control, you may not achieve a satisfactory result for designing these DC to DC converter. We can say that.

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**Control of the Current Waveform (Cont...)**

When the rectifier operates near steady-state, it is true that

$$\langle v(t) \rangle_{T_s} = V + \hat{v}(t)$$

with

$$|\hat{v}(t)| \ll |V|$$

In the special case of the boost rectifier, this is sufficient to linearize the equations of the average current controller.

The boost converter average inductor voltage is

$$L \frac{d\langle i_L(t) \rangle_{T_s}}{dt} = \langle v_s(t) \rangle_{T_s} - d'(t)V - d'(t)\hat{v}(t)$$

substitute:

$$L \frac{d\langle i_L(t) \rangle_{T_s}}{dt} = \langle v_s(t) \rangle_{T_s} - d'(t)V - d'(t)\hat{v}(t)$$

Linearizing the equations of the boost rectifier

When the rectifier operates near steady state then, we can say that it is  $v$  of  $t$ , this is average term and this is a perturb termed or the disturbance and in that case, the average value is much greater than the perturb value. So, you are operating at very close neighborhood of it and thus, you can apply the small signal analysis. In the special cases of the boost rectifier, it is sufficient to linearize the equations of the average current mode controller.

And thus, you have an equations of  $L \frac{d[i_L(t)]_{T_s}}{dt} = [v(t)]_{T_s} - d'(t)V - d'(t)\hat{v}(t)$ . Thereafter, we can substitute this value with the perturbed and then average term and thus, it will take this kind of values.

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### Control of the Current Waveform (Cont...)

Linearized boost rectifier model

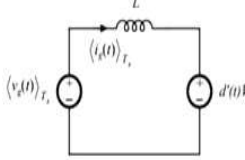
$$L \frac{d\langle i_g(t) \rangle_{T_s}}{dt} = \langle v_g(t) \rangle_{T_s} - d'(t)V - d'(t)\hat{v}(t)$$

The nonlinear term is much smaller than the linear ac term. Hence, it can be discarded to obtain

$$L \frac{d\langle i_g(t) \rangle_{T_s}}{dt} = \langle v_g(t) \rangle_{T_s} - d'(t)V$$

Equivalent circuit:

$$\frac{i_g(s)}{d(s)} = \frac{V}{sL}$$



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And what you can see that this term is a non-linear term, but it is a multiplication of the two small quantities and thus, we can neglect the higher degree small order.

So, non-linear term is much smaller than the linear term and hence, it has been discarded and thus, we can obtain that  $L \frac{d[i_g(t)]_{T_s}}{dt} = [v_g(t)]_{T_s} - d'(t)V - d'(t)V$  and from there, this becomes the equivalent circuit of the small signal model where  $\frac{i_g(s)}{d(s)} = \frac{V}{sL}$  and thus, this is the approximations of the quasi-static approximation.

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### Control of the Current Waveform (Cont...)

The quasi-static approximation

- The previous approach is not sufficient to linearize the equations needed to design the rectifier averaged current controllers of buck-boost, Cuk, SEPIC, and other converter topologies. These are truly nonlinear time varying systems.
- An approximate approach that is sometimes used in these cases: the quasi-static approximation.
- Assume that the ac line variations are much slower than the converter dynamics, so that the rectifier always operates near equilibrium. The quiescent operating point changes slowly along the input sinusoid, and we can find the slowly-varying "equilibrium" duty ratio.

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And what we can say that in a previous approach, not sufficient to linearize the equation needed to design the rectifier average current controller for buck, Cuk, SEPIC and other converter topologies. An approximate approach is sometime used in cases like quasi-static approximations

And in quasi static approximations, we will linearize a system in the neighborhood of it and assume that AC line variations is much slower because it is occurring at a 10 millisecond. Ultimately, this is a cycle for the rectified side and if you take to the AC cycle, it is the 20 millisecond. So, that is what we can say. We assume that the AC line variations are much slower than the converter dynamics.

So, that rectifier always operates near at the equilibrium. The quasi resonant operating point changes slowly along with the sinusoidal and we can find the slowly equilibrium duty ratios. So, this is called the quasi-static approximations and that we shall apply to our boost, buck and buck-boost converter.

Thank you for your attention. We shall continue to discuss about its control technique not only up to the understanding of this boost rectifier and the PWM technique, but also how can you design and implement it in practical life that also we shall discuss but not in detail because of the lack of time.

Thank you so much for your attention.