

Power Quality Improvement Technique
Prof. Avik Bhattacharya
Department of Electrical Engineering

Indian Institute of Technology, Roorkee

Lecture – 13
PWM Rectifier

Welcome to our NPTEL lectures on Power Quality Improvement Technique. Today, we shall discuss about the PWM Rectifier. This is one of the entrants of ac to dc conversion. So, we know that we require to rectify it, once you rectify it for the diode bridge rectifier or the control rectifier, then we inject the harmonics as well. But we shall see that this is the property, let me allow me to change the color of the ink.

(Refer Slide Time: 01:02)

Pulse Width Modulated Rectifiers

Properties of Ideal Rectifier

- It is desired that the rectifier present a resistive load to the ac power system. This leads to
 - unity power factor
 - ac line current has same waveshape as voltage

$i_{ac}(t) = v_{ac}(t) / R_e$
 R_e is called the emulated resistance

The diagram shows an AC voltage source $v_{ac}(t)$ connected to a resistor R_e . The current $i_{ac}(t)$ flows from the positive terminal of the source through the resistor.

So, properties of this ideal rectifier. So, it is desired that a rectifier presents a resistive load to the ac power system that leads to firstly unity power factor operation, secondly is that ac line current has a same wave shape as a voltage since. We know that i_{ac} equal to v_{ac} by R_e . So, where R_e can be said to be the emulated resistance. So, this is something what we want while you convert. You have the same equivalent. Though here you got an ac, this is a voltage and this is a current depending on the magnitude of the resistance you just have some kind of a scaling here. You will have this value and this value. So, this has to have some kind of an analogy. This is the voltage and this is the current.

(Refer Slide Time: 02:17)

Pulse Width Modulated Rectifiers (Cont...)

Control of power throughput

$P_{av} = V_{ac,rms}^2 / R_e(v_{control})$

Power apparently "consumed" by R_e is actually transferred to rectifier dc output port. To control the amount of output power, it must be possible to adjust the value of R_e .

3

Now, so we required to establish this and since in ac average value is '0', we would talk about the rms. So, average dc power should be equal $\frac{V_{ac,rms}^2}{R_e(v_{control})}$, and the power apparently consumed by R_e is actually transferred to the rectifier dc output port. So, there after you got a dc voltage. To control of the amount of output power, it must be possible to adjust the value of R_e . So that you can transfer the maximum power.

(Refer Slide Time: 03:25)

Pulse Width Modulated Rectifiers (Cont...)

Output port model

- The ideal rectifier is lossless and contains no internal energy storage. Hence, the instantaneous input power equals the instantaneous output power.
- Since the instantaneous power is independent of the dc load characteristics, the output port obeys a power source characteristic.

$$p(t) = \frac{v_{ac}^2(t)}{R_e(v_{control}(t))} \quad v(t)i(t) = p(t) = \frac{v_{ac}^2(t)}{R_e}$$

4

So, this is the ac current and the ac voltage and this one is the equivalent resistance and ultimately you are transferring power which is equivalent to $V_{ac}^2 R_e$ and here in this port it

has a transformation. You got a dc output and the dc current. This is the output port model of rectifier or ac to dc conversion system.

Ideal rectify is lossless. We assume that switches are lossless and contains no internal energy storage. Hence the instantaneous input ac power is equal to the instantaneous output power. But there is a problem. Here generally it does not match, then it will not be then ac because if you multiply in phase component of the voltage and current. Then what will happen? At the '0' crossing, your instantaneous ac power is 0 and thus you require to have an instantaneous dc power which is also 0 and thus what essentially you will get is this not a pure dc, the ripple dc.

The instantaneous power is independent of the dc load. So, we are not talking about since you know this is the expression of the power and there, we do not talk about the loading of the dc portion. So, we can say that. Since the instantaneous power is independent of the dc load characteristics the output port obeys the power flows characteristics and ultimately this power is given by $p(t) = \frac{v_{ac}^2(t)}{R_e(v_{control}(t))} = \frac{v_{ac}^2(t)}{R_e}$, where R_e is a voltage control resistance. So, equations of the ideal rectifier.

(Refer Slide Time: 05:57)

Pulse Width Modulated Rectifiers (Cont...)

Equations of the ideal rectifier

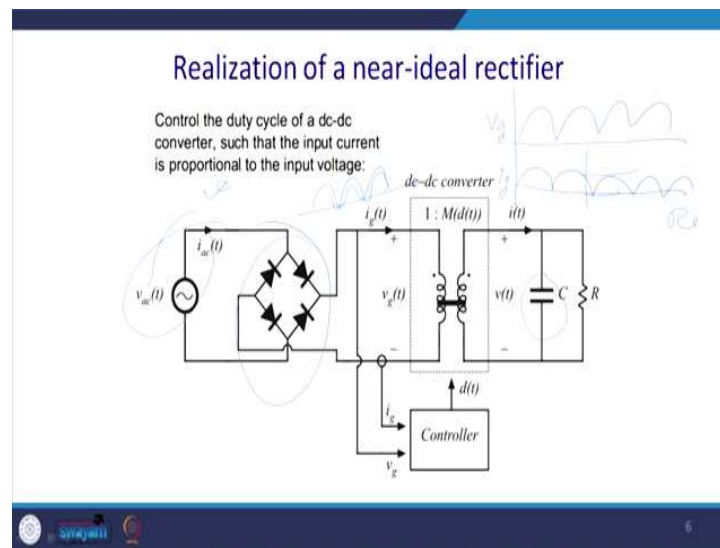
<p>Defining equations of the ideal rectifier:</p> $i_{ac}(t) = \frac{v_{ac}(t)}{R_e(v_{control})}$ $v(t)i(t) = p(t)$ $p(t) = \frac{v_{ac}^2(t)}{R_e(v_{control}(t))}$	<p>When connected to a resistive load of value R, the input and output rms voltages and currents are related as follows:</p> $\frac{V_{rms}}{V_{ac,rms}} = \sqrt{\frac{R}{R_e}}$ $\frac{I_{ac,rms}}{I_{rms}} = \sqrt{\frac{R}{R_e}}$
---	---

Now, why you require it? Of course, we have discussed about the 6 pulse 12 pulse converter there we have seen the problem and we want an ideal rectification and see that how it can be achieved by the PWM rectifier. So, defining the equations of the ideal

rectifier is that is $i_{ac}(t) = \frac{v_{ac}(t)}{R_e(v_{control})}$, similarly $v(t)i(t) = P(t)$ or $p(t)$ equal to this instantaneous value.

Whereas, you know when connected with the resistive value, the input and the output rms voltage and current are related as follows $\frac{V_{rms}}{V_{ac,rms}} = \sqrt{\frac{R}{R_e}}$. So, $\frac{I_{ac,rms}}{I_{rms}} = \sqrt{\frac{R}{R_e}}$. This is expressions. It is something like you know like transformer kind of entities.

(Refer Slide Time: 07:10)



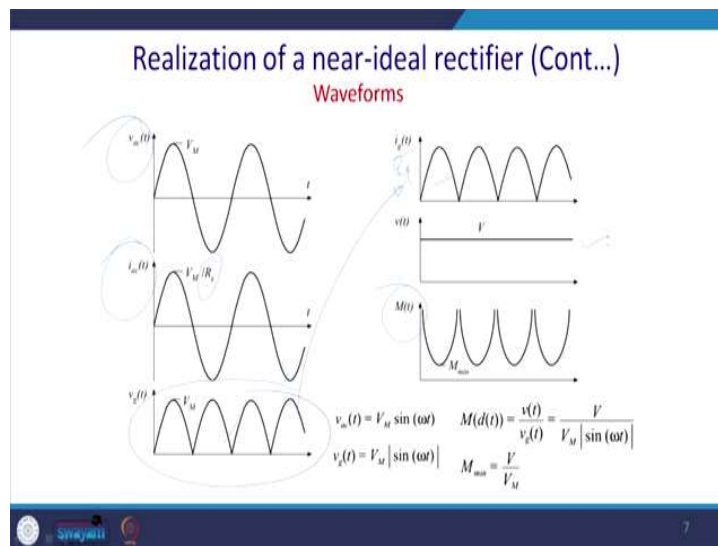
So, we require to rectify. This is the rectifier. We are showing you a single-phase model, but you know this is the effective dc to dc converter and thus it gives you the isolation for the sensitive load and also it ensures that voltage and current are within the phase. So, it will ultimately take the v_g . So, v_g seems. With the absence of the capacitor it will have a profile like this and similarly i_g also. We will try to maintain within the range of the value of the v_g .

And thus, this instantaneous multiplication of it matches and ultimately here in this side you get a ripple dc, but with the dc to dc converter you get a constant dc. But there is a power factor problem which is solved because since voltage and current are in phase, so there is no displacement power factor and all those other nasty elements and it is also free from the harmonics.

So, in that way you can mitigate it by using a real rectifier, the problem of the power quality arises because of the rectification process. So, that is what we say. The control of the duty cycle of dc to dc converter such that output current is proportional to the input voltage, as if it sees the resistive load.

So, this is the voltage at this point and i_g will follow this voltage just having the same thing with less or more magnitude depending on the value of the R_e . So, this is your v_g and this will be the i_g . So, this dc to dc converter will force. So that, i_g here will be following the envelop of v_g and thus in here you do not have any problem in power quality.

(Refer Slide Time: 09:48)



So, this is the few waveforms. So, this is the input voltage which we have applied. And we want that ac current should be like this V_M by effective resistance and for this reason I have drawn v_g in previous slide. This will be the value of the v_g and this will be the value of the i_g just by R_e . R_e can be more than 1 or less than 1 depending on this magnitude of the i_g which will vary, but output voltage here will be a dc value and this will be your i_{dc} . That is also constant.

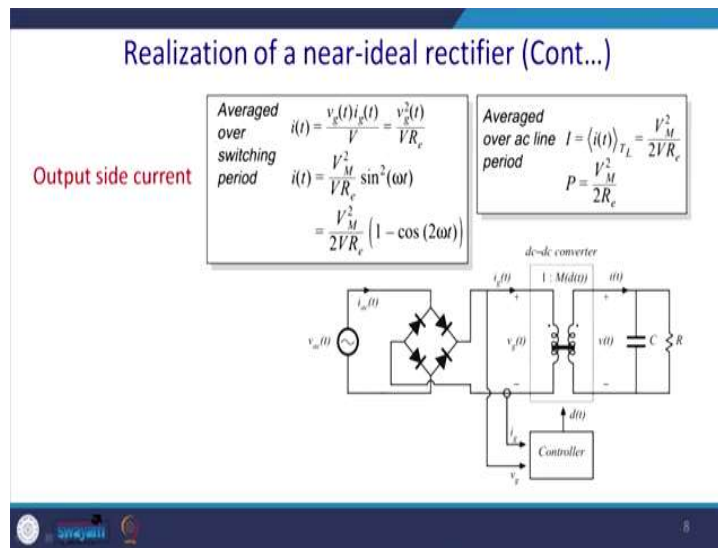
So, what you require to do here? We have to maintain this duty cycle just inverse to the voltage and current. So, that it follows this kind of pattern of voltage and current. This will automatically come. You need not have to do anything for the rectification. But dc to dc converter will force this current to be like this and ultimately capacitor will ensure that

your dc voltage. After isolated dc to dc converter is this and for this reason your dc to dc converter will have this kind of modulation index.

So, what is modulation index? $M(d(t)) = \frac{v(t)}{v_g(t)}$, essentially this is constant and this is variable. So, it is quite clear that once you are here you require to generate this voltage. You require to have a more on time whereas, when you are here to generate this voltage you require less on time.

So, in that way this modulation index itself will be varying to give you this dc voltage and ultimately you will find that i_g is also following and thus this reflection will come into the input waveform and this will be also a sinusoidal waveform. In that way, we can eliminate the problem of power factor as well as the THD in case of this ac to dc conversion.

(Refer Slide Time: 12:23)



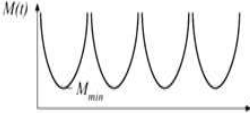
Thus, what we can say? This is the averaged over switching period $i(t) = \frac{v_g(t)i_g(t)}{V}$. This is the unit template of the current. So, we can rewrite it as $\frac{v_g(t)^2}{VR_e}$. Similarly, $i(t)$ equal to we can write in terms of that. we can multiply this through. So, $\frac{V^2 M}{VR_e} \sin^2(\omega t)$ and thus we can split it. So, this part is dc that is $\frac{V^2 M}{2VR_e} (1 - \cos(2\omega t))$, that will have a double frequency ripple. An average over the ac line period I will be $\frac{V^2 M}{2VR_e}$. So, $P = \frac{V^2 M}{2R_e}$.

So, ultimately this is your rectifier portion. Mind it there will not be any capacitor. Capacitor will be at this and i_g and v_g has to follow that particular envelope which has been shown to you in our previous slide and this capacitor has to maintain the dc bus voltage and in that way operation follows.

(Refer Slide Time: 13:40)

Realization of a near-ideal rectifier (Cont...)

Choice of converter

$$M(d(t)) = \frac{v(t)}{v_g(t)} = \frac{V}{V_M |\sin(\omega t)|}$$


- To avoid distortion near line voltage zero crossings, converter should be capable of producing $M(d(t))$ approaching infinity
- Above expression neglects converter dynamics
- Boost, buck-boost, Cuk, SEPIC, and other converters with similar conversion ratios are suitable
- We will see that the boost converter exhibits lowest transistor stresses. For this reason, it is most often chosen

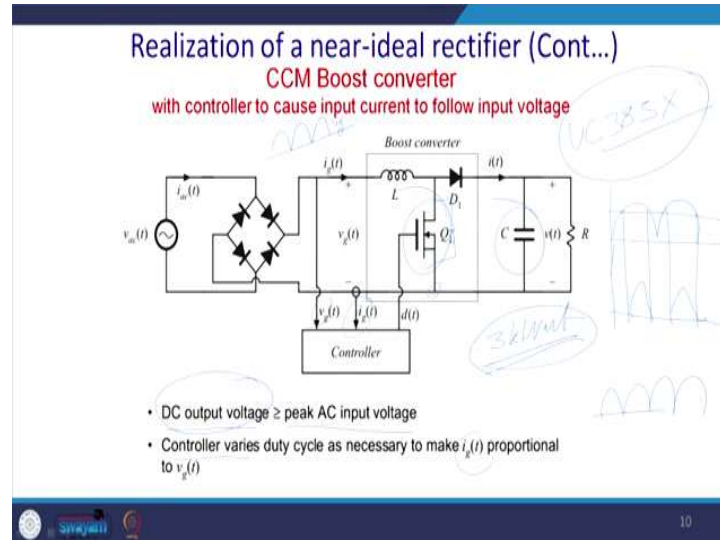
So, $M(d(t)) = \frac{v(t)}{v_g(t)} = \frac{V}{V_M \sin \omega t}$ and we will have this kind of modulation index. And thus, these are the few issues to avoid distortion near the line voltage zero crossing. The converter should be capable to produce $M(d(t))$ approaching infinity. So, this is something we require to see. So, for this reason we will restrict. These are the logical restriction we require to put while designing.

This above expression neglects converter dynamics because there will be a switching loss. There are filters, there will be losses, conduction losses. Those are not featured in to these calculations. We can use this dc to dc converter isolated as well as non-isolated. Though I have shown the picture of the isolated, it can be boost, buck-boost, Cuk, SEPIC and other converter with similar conversion ratios and that can be implemented or it can be used for this purpose.

We will see that and this is the next topic of it. In next slide we will see that the boost converter exhibits lowest transistor stresses and thus it is preferred. For this reason, it is

most chosen, but it is not that it is not possible to introduce this way of operations. So that your power factor gets corrected for buck, buck-boost or the Cuk converter.

(Refer Slide Time: 15:54)



So, ultimately this is the overall circuit. I request the student to go through the datasheet of the Texas instrument, UC 385X, X can be 5556 so on. This is the datasheet for the chip of this controller that will ensure this power factor correction technique as well as the low THD.

So, here operation is same. We require to generate the v_g . v_g will be this and we required to force i_g to be stained to the envelope and just magnitude will change. Ultimately, we will sense this v_g and i_g and get all those calculation done which has been shown into the previous slide. Then accordingly dc duty cycle of this MOSFETs has been calculated, but please note that the MOSFET should have a body diode.

So, another issue is for the higher power rating we require IGBT and generally as you know that it is a single-phase application. So, it can be up to 3 kilo watt. So, this is the way of doing it because 220 volt in our cases (India), it is 220 volt and you got a 15 ampere line. So, you multiply straight away and you get 3 kilo Watt. So, this power factor technique can be extended up to 3 kilo Watt and for this reason for the lower rating we can go for this MOSFETs and for more than 2 kilowatt we generally prefer to go for the IGBT.

So, dc motor voltage. The peak of the input voltage. The controller varies the duty cycle. So, this is the case. Dc output voltage in this case, this one is more than the peak input voltage. Why it does not work? Why you require a boost phenomenon? We required to generate the rectified voltage.

Generally, what happens? To get a after rectification you will find that there is a capacitor. So, the current will not flow unless capacitor voltage is crossing. So, this is your rectified voltage. Current will flow only in this duration and for this reason you have a problem of the displacement power factor and the THD comes out.

But in this case what happens? You are shorting this switch. So, instantaneously your voltage, this voltage gets to '0' and accordingly you build up the voltage and you ensure that you are keeping these values within the range of this simulated value of the v_g and i_g . Thus, the controller varies the duty cycle necessary to make i_g proportional to v_g .

(Refer Slide Time: 20:16)

Realization of a near-ideal rectifier (Cont...)

Variation of duty cycle in boost rectifier

$$M(d(t)) = \frac{v(t)}{v_g(t)} = \frac{V}{V_M |\sin(\omega t)|}$$

Since $M \geq 1$ in the boost converter, it is required that $V \geq V_M$.
If the converter operates in CCM, then

$$M(d(t)) = \frac{1}{1-d(t)}$$

The duty ratio should therefore follow

$$d(t) = 1 - \frac{v_g(t)}{V} \quad \text{in CCM}$$

CCM/DCM boundary, boost rectifier

Inductor current ripple is

$$\Delta i_L(t) = \frac{v_g(t)d(t)T_s}{2L}$$

Low-frequency (average) component of inductor current waveform is

$$\langle i_L(t) \rangle_{T_s} = \frac{v_g(t)}{R_s}$$

The converter operates in CCM when

$$\langle i_L(t) \rangle_{T_s} > \Delta i_L(t) \Rightarrow d(t) < \frac{2L}{R_s T_s}$$

Substitute CCM expression for $d(t)$.

$$R_s < \frac{2L}{T_s \left(1 - \frac{v_g(t)}{V}\right)} \quad \text{for CCM}$$

Now, so there is a way to achieve it. The variation of the duty cycle in case of the boost rectifier. This topology is called the boost rectifier. So, it is $M(d(t)) = \frac{v(t)}{v_g(t)} = \frac{V}{V_M \sin \omega t}$. Since, M is greater than 1 in boost converter it is required that V is greater than V_M if the converter operates in CCM that is in a continuous conduction mode, so that is what we want. That is also a challenge. But if the load is light then it does not work. So, in case of the boost converter $M(d(t)) = \frac{1}{1-d(t)}$ is the $\frac{v_o}{v_n} = \frac{1}{1-D}$. So, this is been replaced here.

And thus, the duty ratio will follow is that $d(t) = 1 - \frac{v_g(t)}{V}$, but it is for only continuous conduction mode and control is easy. And accordingly, we required to design the rectifier and also this inductor, so that it remains in a continuous conduction mode. So, this is the way we can calculate it. So, the inductor ripple is $\Delta i_g(t) = \frac{v_g(t)d(t)T_s}{2L}$, this is the current ripple that you will allow inside the inductor. You know that there is a boost apology, thus current will go up, go up, go up, come down.

The ripple that you have allowed assuming that it is a critical conduction mode, will always touch '0' and for that assumption we have this current ripple and for the low frequency this is called average model. The component of the inductor waveform which is v_g is definitely. It is given by v_g by R_e . That is the average value of 'I'. This is the straight line that I have drawn. This is the average v_{ag} .

So, converter operates in CCM when $i_g(t)_{T_s} > \Delta i_g(t)$ and then $d(t) < \frac{2L}{R_e T_s}$ and that is the limitations, where T_s is the switching inverse of the switching frequency or you can take f in numerator. So, substituting this value for the condition to be in this converter into the continuous conduction mode will be $R_e < \frac{2L}{T_s(1 - \frac{v_g(t)}{V})}$ and that is for the continuous conduction mode or if it is equal then it is a boundary condition.

(Refer Slide Time: 24:14)

Realization of a near-ideal rectifier (Cont...)

CCM/DCM boundary

$20 < \frac{2L}{T_s} < 100$

$$R_e < \frac{2L}{T_s \left(1 - \frac{v_g(t)}{V}\right)} \text{ for CCM}$$

Note that $v_g(t)$ varies with time, between 0 and V_M . Hence, this equation may be satisfied at some points on the ac line cycle, and not at others. The converter always operates in CCM provided that

$$R_e < \frac{2L}{T_s}$$

The converter always operates in DCM provided that

$$R_e > \frac{2L}{T_s \left(1 - \frac{V_M}{V}\right)}$$

For R_e between these limits, the converter operates in DCM when $v_g(t)$ is near zero, and in CCM when $v_g(t)$ approaches V_M .

12

So, this is what we are now going to discuss. Realizations of a near ideal rectifier. So, $R_e < \frac{2L}{T_s(1-\frac{v_g}{V})}$ for the CCM mode. Note that v_g of t varies with time between 0 and V_M .

Hence this equation may be satisfied at some point of the ac line. So, ac line is basically you got a v and i . So, you can draw the effective value of this R_e .

So, the converter operates in the CCM provided that this logic R_e is equivalent which is a load essentially. You know ultimately you should be knowing that load. So, ultimately you know it is 5 kilo Watt or 3 kilo Watt and what is the current rating of it. From there you can calculate R_e . So, this R_e value should be less than that ' $2L/T_s$ '.

So, value of the inductor has a strong relation with the switching frequency. So, ultimately if you want 20 ohm and you know let us consider that it should be 2 milli Henry. So, you can calculate. This should be the minimum switching frequency required to operate whether which device will be suitable MOSFET or IGBT.

And on the other hand, the converter always operates in DCM mode that is discontinuous conduction mode if R_e is more than this, and that is what happened for this R_e . Between these limits the converter operates in DCM when v_g near zero and in the CCM when v_g approaches through V_M . So, it will have a transition from continuous conduction mode to the discontinuous conduction mode.

(Refer Slide Time: 27:01)

Realization of a near-ideal rectifier (Cont...)
Static input characteristics of the boost converter

A plot of input current $i_g(t)$ vs input voltage $v_g(t)$, for various duty cycles $d(t)$. In CCM, the boost converter equilibrium equation is

$$\frac{v_g(t)}{V} = 1 - d(t)$$

Now simplify DCM current expression, to obtain

$$\frac{2L}{VT_s} i_g(t) \left(1 - \frac{v_g(t)}{V}\right) = d^2(t) \frac{v_g(t)}{V}$$

The input characteristic in DCM is found by solution of the averaged DCM model (Fig. 11.12(b)):

Solve for input current:

$$i_g(t) = \frac{v_g(t)}{\left(\frac{2L}{d^2T_s}\right)} + \frac{\rho(t)}{V - v_g(t)}$$

with $\rho(t) = \frac{v_g(t)}{\left(\frac{2L}{d^2T_s}\right)}$

CCM/DCM mode boundary, in terms of $v_g(t)$ and $i_g(t)$:

$$\frac{2L}{VT_s} i_g(t) > \left(\frac{v_g(t)}{V}\right) \left(1 - \frac{v_g(t)}{V}\right)$$

Beware! This DCM $R_e(d)$ from Chapter 11 is not the same as the rectifier emulated resistance $R_e = v_g/i_g$

13

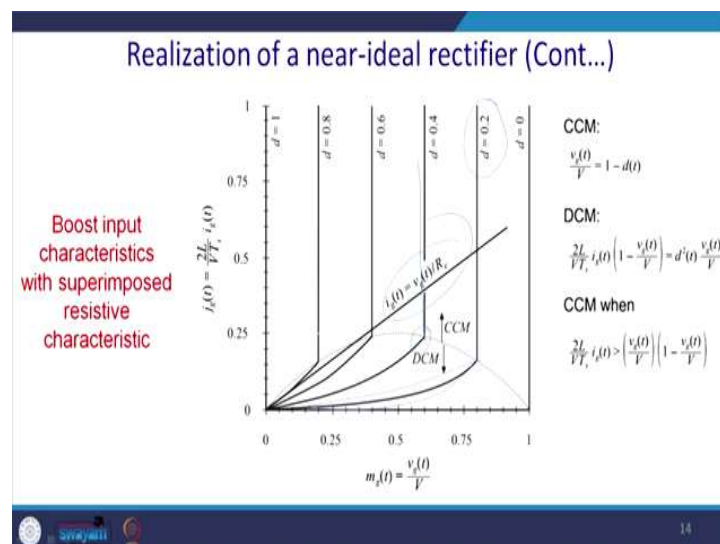
Thus, that transition does not occur because you know that whatever the expressions of the duty cycle you write for this buck-boost, Cuk, SEPIC, all are valid for the continuous conduction mode. The moment you were in a discontinuous conduction mode analysis become complicated.

So, let us think about the realization of the nearly ideal rectifier with the power factor correction. A plot of the input current i_g versus the input voltage v_g for various cycle $d(t)$ in continuous conduction mode. The boost converter equilibrium is $\frac{v_g(t)}{V} = 1 - d(t)$.

The input characteristics in DCM is found by the average DCM model that is shown in this figure and this is the v_g and ultimately this resistance has been modeled that is $\frac{2L}{d^2 T_s}$ and this is the power available to you that is $\frac{p(t)}{V - v_g(t)}$ and effectively since this a voltage is available to you, the control voltage and this is your DCM. Thus, the in DCM $R_e(d)$ from the previous is same as emulated by $R_e = \frac{v_g}{i_g}$.

So, now, simplify this DCM in expression to obtain $\frac{2L}{V T_s} i_s(t) \left(1 - \frac{v_g(t)}{V}\right) = d^2(t) \frac{v_g(t)}{V}$ and thus DCM mode of the boundary in terms of v_g and i_g it will be $\frac{2L}{V T_s} i_s(t) > \left(\frac{v_g(t)}{V}\right) \left(1 - \frac{v_g(t)}{V}\right)$, this will be the total expressions in case of the discontinuous conduction mode.

(Refer Slide Time: 29:48)



So, this is the overall pictures to analyze the boost rectifier. So, ultimately this is the circle after this you can see that discontinuous conduction mode starts and this is your load line.

That was what I was trying to draw there. So, that is the $i_g(t) = \frac{v_g(t)}{R_e}$. You can see that if duty cycle is 0. So, no question. If duty cycle is 0.2, this is the boundary for the continuous conduction and the discontinuous mode. In this mode it is a discontinuous mode and where all those conditions exist. Till this mod you are in a discontinuous conduction mode and here this is your $m_g(t)$ that value will cross over.

Similarly, if your duty cycle is 0.4 your crossover will occur faster. Since the value is light load. Because this is a value of R_e . So, for this portion of it, is challenge is maintaining this ideal characteristic of this rectifier at the light load. So, from there you will take the continuous mode of conduction. And when duty cycle is 0.6 you can almost follow the load line. And this is the above load line. So, you will reach to discontinuous to conduction mode faster.

So, ultimately in DCM expression is pretty simple for the boost converter. So, it will be $\frac{v_g(t)}{V} = 1 - d(t)$. But in case of the discontinuous conduction mode expressions is quite complex. So, it will be $\frac{2L}{VT_s} i_s(t) \left(1 - \frac{v_g(t)}{V}\right) = d^2(t) \frac{v_g(t)}{V}$ and once it comes into the domain of the continuous conduction mode then $\frac{2L}{VT_s} i_s(t) > \left(\frac{v_g(t)}{V}\right) \left(1 - \frac{v_g(t)}{V}\right)$. This is the way it should operate.

Thank you for your attention. I will continue my discussions on this rectification technique for single phase as well as 3 phase in our next class.