Advanced Linear Continuous Control Systems
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Lecture – 09
State Space Representation: Numerical Examples on State Space Modelling (Part-I)

Now today we start with Numerical Examples on State Space Modeling. Till later we have seen some discussion on different types of modeling that is, we have seen Controllable canonical form, Observable canonical form, Diagonal canonical form and Jordon canonical form.

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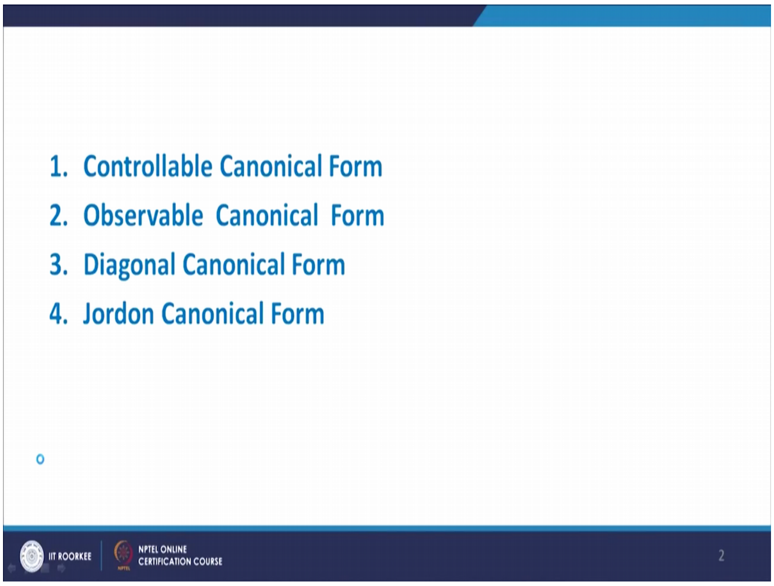

So, what we are done we have discussed the issues related with this modeling, along with that simple example you are solved.

Now, what here we have doing we are taking different examples and along with this we are comparing the results with respect to different types of modeling. So, first of all we start with controllable canonical form. And now here I am taking an example which is of strictly proper transfer function.

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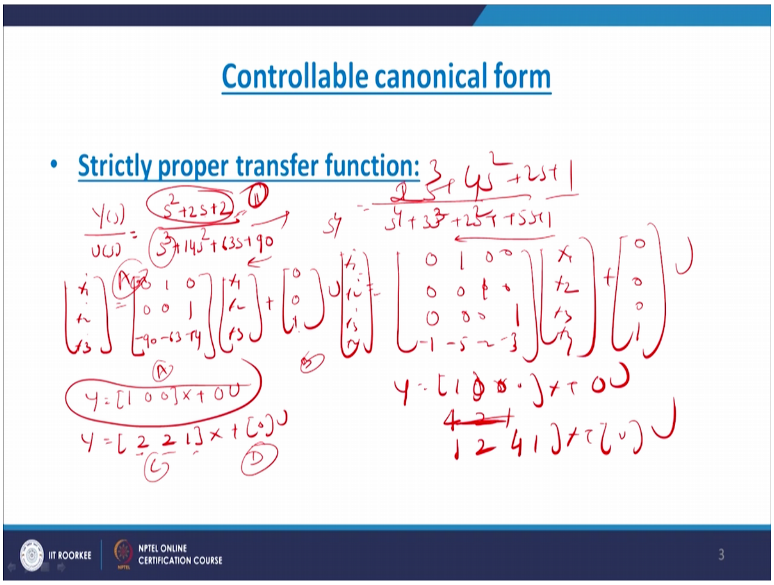

So, here I am writing down the equation as Y of s divided by U of s equal to S square plus 2 S plus 2 divided by S cube plus 14 S square plus 6 3 S plus 90. So, this is a transfer function, we call as strictly transfer strictly proper transfer function. Degree of denominator is 3 and degree of numerator is 2.

Now, we have to write down this transfer function in controllable canonical form. We have seen that in order to solve this one, what we are done? We have taken output as if first output as first variable and remaining variables, we have taken as the derivative of the output that we have done it. Now here without doing calculations we have to solve this.

So now here x cube is there; that means, the number of states involved they are 3. So, 3 states are involved. So, we will take states as x 1 x 2 and x 3. So, we are taking 3 states. So, we write down this equation as x 1 x 2 dot x 3 dot these are 3 states now x dot equal to you have x 1 x 2 x 3 and this is U. Now we have to write down the elements of A matrix and elements of B matrix without doing any mathematical analysis.

So now x 1 dot we write as 0 1 0 then x 2 dot we write as 0 0 1 and to write the elements of x 3 x 0, we start from right to left; that means, we started writing the elements on this side. So, we write this as minus 90 minus 63, 63 minus 14. So, minus 90 minus 63 minus 14 and here, 0 0 0 1 U an output Y equal to 1 0 0 x and 0 into U.

But if this is the case, when we this is the case, when here the element is 1, but now here there is involvement of S square plus 2 S 2 therefore, we write this equation as Y equal to 2 2 1 into x plus 0 into U; that means, same transfer function if it is having one element 1 you write as 1 0 0 x into 0 0 into U, but there is a involvement of S square plus 2 S plus 2. So, we will written the equation as 2 2 1 x into 0 into U. So, see this C matrix and this is D matrix.

Suppose here instead of S cube if the let us say these are S 4 that is let us say if you are trans function like this 1 upon S 4 plus 3 S square plus 2 S 2 S 2 S square plus 5 S plus 1. If this is the transfer functions 4th order. So, how will write the elements as 0 1 0 0 0 0 0 0 1 0 0 0 0 1 and write the elements in this order that is minus 1 minus 5 minus 2 minus 3 x 1 x 2 x 3 x 4 that is x 1 dot x 2 dot x 3 dot x 4 dot like this, plus 0 0 0 1 into U and output is 1 x 1 0 0 x into 0 into U. Here 1 this 1 is been put it here.

But suppose if you are having 2 S plus 4 or 2 S cube plus 4 S square plus 2 S plus 1 ok. If in this case, we have to write down the elements as 1 2 4 2 into x into 0 into U. So, this is about the a transfer function there is controllable canonical form that is strictly proper transfer function is in controllable canonical form, but as I told you that sometimes we need the transfer function in observable canonical form.

Now how to write down the transfer function in an observable canonical form?

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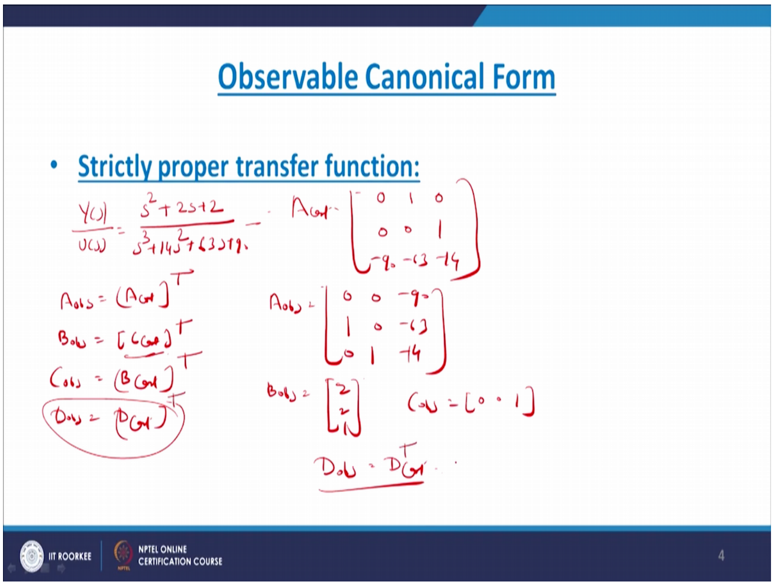

Now, we take the same example, example is Y of S equal to U of S equal to S square plus 2 S plus 2 divided by S cube plus 14 S square plus 63 S plus 19. This is a transfer function, and we have to write down in observable canonical form and the most important thing is that if you know transfer function in controllable canonical form. We can write down directly in an observable canonical form. So, what is the position?

So, we have discussed this already in the previous section, but now I am repeating it again. So, here A observable equal to A controllable transpose. If you were B observable the B observable equal to C controllable transpose. And if you want C observable equal to B controllable transpose and if you want D observable required that is where write D controllable only that is, A observable equals to A controllable transpose, B observable equal to C controllable transpose, C observable equal to B controllable transpose and D observable equal to D controllable that is both are same.

So, already we have determined the A matrix A controllable matrix like this. So, I am writing down A controllable matrix here again A controllable, what is this A controllable matrix? 0 1 0 0 0 1 minus 19 minus 16 63 minus 14 this is A controllable now what is A observable matrix? So, here A observable matrix is A observable equal to 0 0 minus 90 1 0 minus 63 0 1 minus 14 this is A observable matrix.

Now, what we want B observable equal to C controllable is see here, what is C controllable matrix? C controllable is 2 2 1. So, we write B observable equal to C controllable transpose. So, here we have we have C controllable 2 2 1. So, we take transpose 2 2 1 in column that is 2 2 1. Now about the C observable, now C observable is equal to B controllable transpose. So, what is B controllable B controllable is 0 0 1. So, make a transport that is 0 0 1 that is we write C observable equal to 0 0 1, and what is D observable? D observable equals to D controllable.

So, here as the function is strictly proper. So, will it become 0. So, in this way we have written the observable canonical form that is your function your transfer function in strictly proper form. Now we have to write down the controllable and observable canonical form for an proper transfer function.

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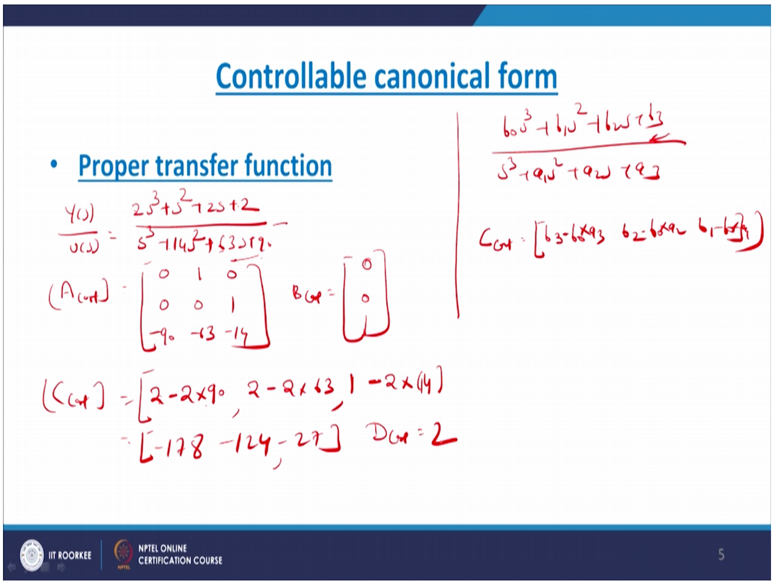

Now, we write a proper transfer function as Y of S divided by U of S, we write as 2 S cube plus S square plus 2 S plus 2 divided by S cube plus 14 S square plus 63 S plus 90.

Now here we have to write down in an controllable canonical form the function is the proper on. So, how to write down? So, first of all we write A matrix that is A controllable matrix. So, A controllable matrix in this case equal to 0 1 0 0 0 1 minus 90 minus 63 minus 14, we will find at whether the transfer function whether it is proper or non-proper there is no change in A matrix. A matrix remains same now about the B matrix, B controllable matrix 0 0 1. That is there is no change in a B matrix whether it is a proper transfer function or strictly transfer function. Now you see about the C matrix C controllable matrix so C controllable.

Now, how to represent and C controllable matrix? So, last time you have seen the basic formulae that is b 0 S cube plus b 1 S square plus b 2 S plus b 3 divided by S cube plus a 1 S square plus a 2 S plus a 3. So, how to get the values of C matrix? And that time you have seen this C matrix that can be determined as, a some logic is there what logic is there we start with writing down the b 3 elements b 3 b 2 and b 1, this order this is normal method to write down the C C matrix and now b 3 minus b 3 minus.

We write b naught minus b naught, b naught multiplied by a 3 a 2 into a that is b control means b 3 minus b naught into a 3 then this b 2 minus b naught into a 2 then b 1 minus b naught into a 1. This is the way to write the C control matrix.

So, here how write down C control matrix that is, what is the value of the we can say b 3 value? b 3 is 2 2 minus what is b naught into 2 into 90 then write b 2 as 2 minus b naught is 2 into 63 then we write b 1 as 1 minus b naught is 2 minus into 14.

So, write we C control matrix as these 2 minus 2 2 means these 2 2 is common. So, a so 2 2 1 2 2 1 where it like this then b naught 2 then these 2 naught common this 2 naught and then the write the elements the reverse order like a 90 63 and 14 like this. So, in this way without doing any of the calculations, we can write or we can say without using any of this a mathematical approach we can directly write it and C controllable matrix.

So, what is the value of this? So, C controllable matrix will get as minus 178 minus 124 minus 27, and what about D controllable? The D controllable equal to 2, this is the way to represent the any given proper transfer function in a controllable canonical form. Now what we do we have to represent the same transfer function into observable canonical form?

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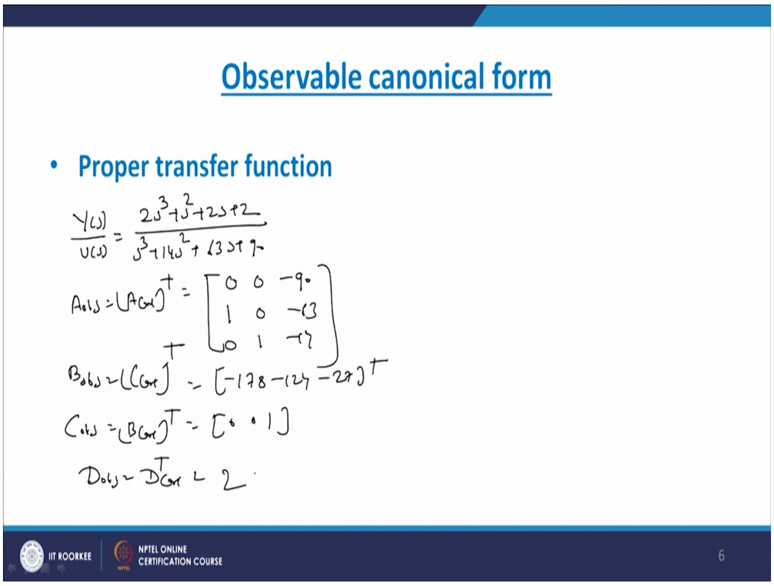

Now, we write down the strict proper transfer function in an observable canonical form. So, we I am taking the same example that is Y of S by U of S equal to 2 S cube plus S square plus 2 S plus 2 divided by S cube plus 14 S square plus 63 S plus 90. And as we have seen earlier that, what is A observable matrix? A observable matrix equal to A controllable transpose.

So, we write A controllable matrix as 0 0 minus 90 then 1 0 minus 63 and 0 minus 14. So, we got A observable matrix, then what are the B observable? B observable equal to C controllable transpose that is equal to minus 178 minus 124 minus 27 transpose, and what is C observable? C observable equal to B controllable transpose and that will get as 0 0 1 and D observable equal to D controllable transpose equal to 2.

So, as we have seen earlier earlier. So, whatever the elements of A controllable, B controllable, C controllable and D controllable we have written in an observable canonical form. Now next now we have to start with representation in a diagonal canonical form.

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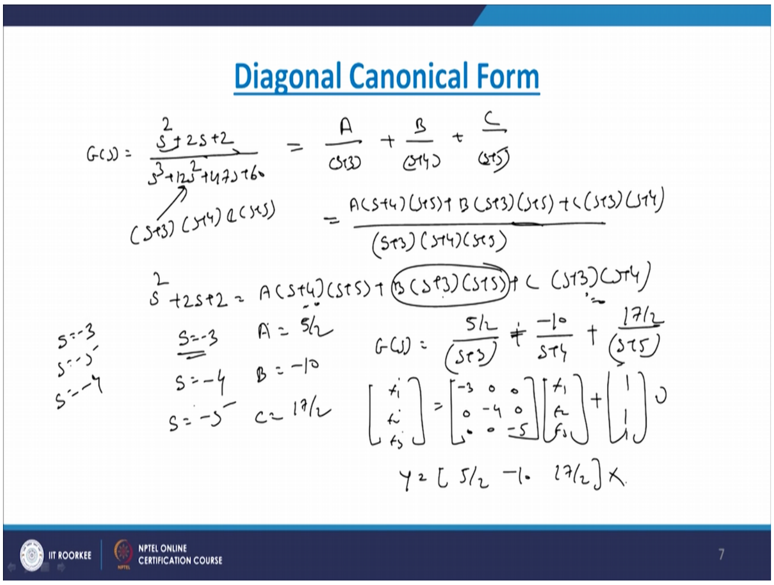

Now, we write a transfer function as G of S equal to S square plus 2 S plus 2 divided by S cube plus 12 S square plus 47 S plus 60. So, these transfer function again and strictly proper transfer function and we have to write these transfer function in an diagonal canonical form.

So, as when we write a diagonal canonical form we have to use the concept of partial fraction expansion. So, we write this equation as in terms of A is some element B plus C. So, further purpose first of all defined the roots of this equation. The roots of this equation is S plus 3 S plus 4 and S plus 5. So, write this as S plus 3 S plus 4 and S plus 5.

Now, next we have to solve these equations. Solving that is S plus 3 S plus 4 S plus 5. Now write down as A S plus 4 into S plus 5 plus B B S plus 3 S plus 5 plus C into S plus 3 into S plus 4. Now here we have determine this equation and we have this this equation. So, we can compare both this equation. So, after comparison, what will get? We write this equation as S square plus 2 S plus 2 A S plus 4 S plus 5 B S plus 3 S plus 5 plus C S plus 3 into S plus 4 this is 5.

So, here our aim is to get the values of A B C. So, how will you get the values of A B C? So, last time you have seen in order to get the values of A B C. First of all, we have to write down the equation in terms of S that is you can replace here S equals to minus 3, S equal to minus 5 or S equals to minus 4. So, when you represent or when you write S equals to minus 3 here. So, when you write S equals to minus 3 in above equation. So, what will get we will get the value of a equal to 5 by 2.

Suppose if you replace S equals to minus 4 in this equation. So, let us say it implies S equals to minus 4 that is this part becomes 0. This part ; that means, we have to deal with only S plus 3 and S plus 4 similarly as we have seen earlier we replace S equals to minus 3, this part a this part becomes 0 only we have got the value of A. So, when replace S equals to minus 4 we have got we have got the value of B equal to minus 10 and we write S equals to minus 5 you will get the values of C equal to 17 by 2.

So, here we can write down final equation as G of S equal to 5 by 2 S plus 3 minus 10 divided by S plus 4. And here 17 by 2 divided by S plus 5. Now this equation we have to represent into a diagonal canonical form. So, how many states are there? There are 3 states. So, we write the equation as x 1 dot x 2 dot x 3 dot equal to. So, here this S plus 3 S plus 4 S plus 5 they are nothing but the roots of the denominator. And is 5 by 2 minus 10 17 by 2 they are called the residues. So, what we are doing we are representing this 3 4 5 in A matrix.

So, here minus 3 minus 4 minus 5 remaining elements are 0. And this we will write as x 1 x 2 x 3 and here will get 1 1 1 into U and output is Y equal to 5 by 2 minus 10 17 by 2 into X as there is as the function is in a proper strictly proper transfer function therefore, there is no element of D. So, we can write a plus 0 into U. So, this is about the diagonal canonical form ok.

Now, we have to write down transfer function in a Jordan canonical form, as I discussed earlier Jordan canonical form is that there are repeated roots in the denominator. So, the procedure for getting the Jordan canonical form is same as diagonal canonical form.

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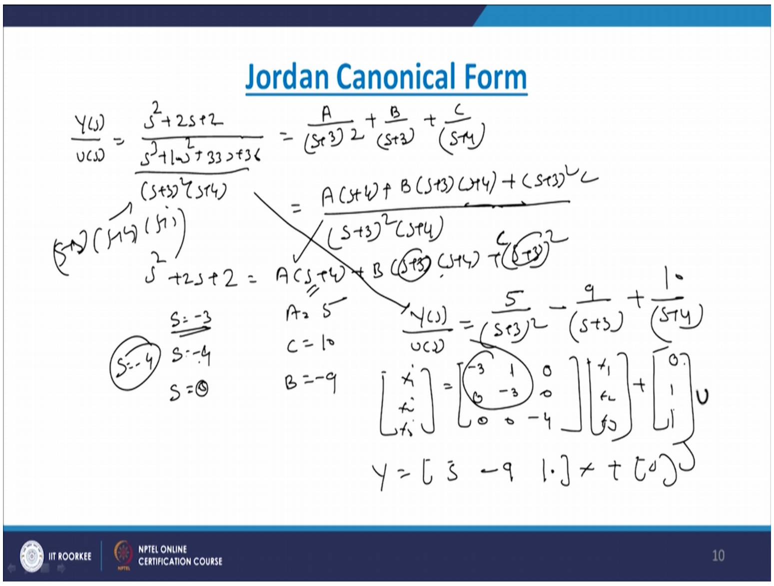

So, we write the one transfer function that is Y of S by U of S equal to S square plus 2 S plus 2 divide by S cube plus 10 S square plus 33 S plus 36, this is the transfer function numerator and denominator.

And as I told you that the roots of this denominator are repeated that is the roots are S plus 3 square S plus 4. So now, this we write this transfer function as A upon S plus 3 whole square, B S plus 3 plus, C S plus 4. And now here A is to get the values of A B and C. So, we write this as S plus 3 whole square into S plus 4. So, here if we divide this S S plus 3 whole square here. So, we will get A into S plus 4, then B when we divide S plus 3 here what will get we will get S plus 3 S plus 4 plus C when you divide S plus 4 by to this equation will get S plus 3 whole square. The next step we have to compare both these equations.

So, when you compare what you get S square plus 2 S plus 2. A S plus 4 B S plus 3 S plus 4 plus S plus 3 whole square. Now we are we are comparing here. So, here what we do we replace the equation as S equals to minus 3 and then the write equation as S equals to minus 4. So, when we replace S, S equals to minus 3 here. So, when we replace S equals to minus 3 so we will get A equal to 5.

So, as we have written replace the S equals to minus 3 we have got A equals to 5 and when we write S equals to minus 4, we are getting the equation as C equals to 10 and we write S equals to 0, we are getting B equals to minus 9. Just see here when we written S equals to minus 3 here. So, what will happen? The term of this a term of S plus 3 will that will cancel. So, you are getting the values of A. When we replace S equal to minus 4 in this equation term up A will canceled, this term will cancelled and finally, we are getting this the terms concern with this C this C S the (Refer Time: 24:22) square then when we replace the S equals to 0, when replace S equal 0 here.

So, we will get S equal 0 we get equation in terms of A B C and we get value of B equals to 9 therefore, we can write the final equation Y S by U S equals to 5 by S plus 3 whole square minus 9 by S plus 3 plus 10 by S plus 4.

So, these early just we have started with this equation and we have come across these particular equations. Taking values of a B and C, because why we have done because we want that the particular transfer function or particular testing model must be decoupled, decoupled means that we are getting the Eigen values. Eigen values only where rest of the elements have become 0. Now here we have got Y S by U S is equal to 5 by S plus 3 whole square minus 9 divided by S plus 3 10 upon S plus 4. Now we have to write down these into the state space model particularly in Jordan canonical form.

Now, after this there is no need to do much calculations. Directly looking after this we can write down the state space model in a Jordan canonical form. So, how right so here we are taking the 3 states x 1 dot x 2 dot x 3 dot that is equals 2. So, first of all to write this one, we write the diagonal elements we write the diagonal elements as minus 3 minus 3 and minus 4.

If you compare the this diagonal canonical form and Jordan canonical form, there is one difference that is, the elements of U matrix in case of Jordan canonical form elements a block concern with this Jordan block we get the elements of 0 that for example, in this case what will happening this x dot equals to x that is x 1 x 2 x 3. Here we are getting the one rest of the elements are 0s plus here we are getting 0 a rest of the elements are 1. If the same problem if you are taken care by in this way S plus 3 S plus 4 S plus 4 S plus 5 in that case, we are getting S minus 3 minus 4 minus 4 last time you have seen.

But here where minus 3 minus 3 more and here we are getting 1, but if you have S plus 3 S plus 4 S plus 5 you are getting the 0 and here we get 1, but here in this case we have got 1 and here we have got 0 elements and this is the particularly your Jordan block and finally, we can write the output as Y equal to 5 minus 9 10 x plus 0 into U. So, here D is 0 because of this if the any value of D is there we have we come across this value. So, this is particularly about the example concern with the Jordan canonical form.

So, here what I have done? I have taken few simple examples basically third order and we have shown various types of form, but there are, but in actual practice it is not a third order we are deal with the very high order system then in the higher order system how can we represent. So, all this means all these mythology for higher order system that part will see in next class.

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Now, these are some references.

Thank you.