Advanced Linear Continuous Control Systems
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Lecture – 07
State Space Representation:
Diagonal Canonical Form (Part-II)

Now, we start with Diagonal Canonical Form Part 2. In this we will study modeling of diagonal canonical form part 2. Part 2 means here degree of numerator is same as degree of denominator that is part 2, numerical example and finally, advantages and disadvantages of canonical form of modeling.

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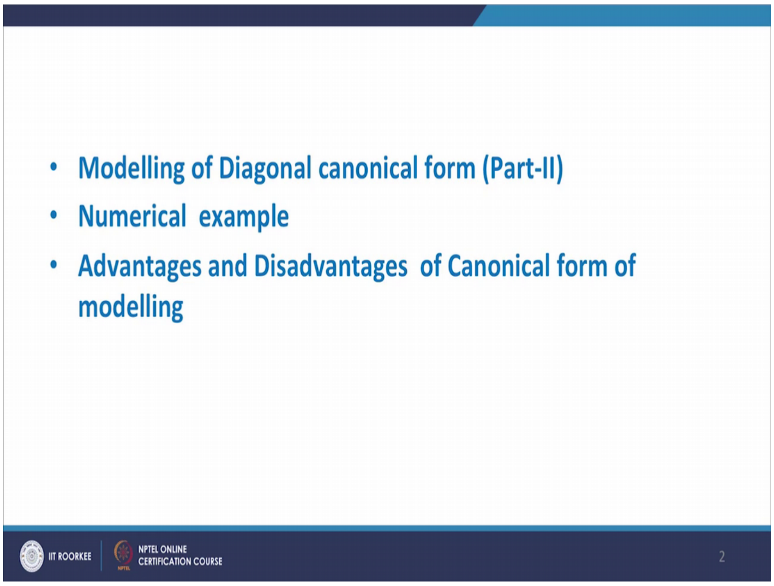

The part 2 diagonal canonical form, here degree of numerator degree of degree of numerator is same as degree of denominator that is proper transfer function. In last part we have seen a model a canonical diagonal canonical form of model, where degree of denominator is greater than degree of numerator. But sometimes it may possible that in practical system degree of denominator is same as degree of numerator. So, in that case how you develop the model? So, now, here what we will do? We have to develop the model when the case is like this.

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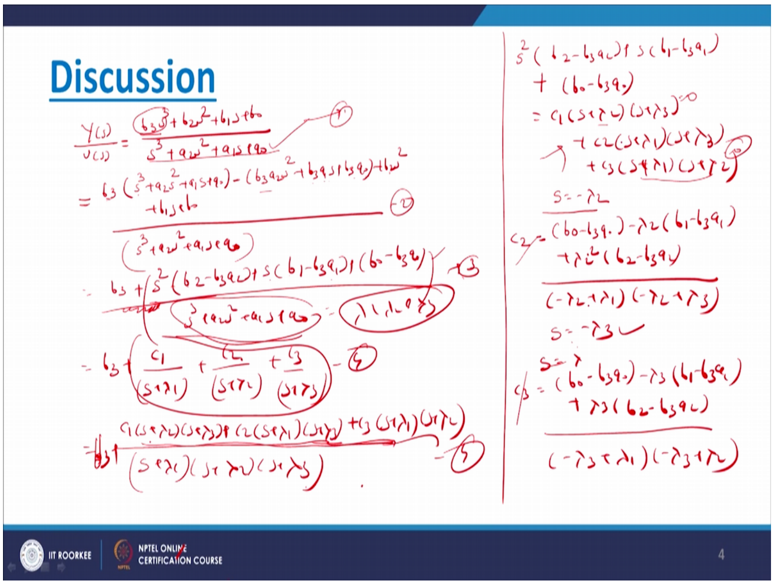

So, for that purpose we will take one system a transfer function system that is Y s by U s equal to b 3 s cube b 2 a square plus b 1 s plus b naught divided by s cube, plus a 2 s square plus a 1 s plus a naught. So, in this transfer function b 0 b 1 b 2 b 3 are the coefficients of the numerator and a 2 a 1 a 0 are the coefficients of the denominator.

And in this case what we are assuming that eigenvalues of the denominator are distinct, they are different they are not repeated the how to develop the model? And problem is that in this model here the degree of numerator and denominator are same, it is difficult to apply a partial fraction expansions because in order to apply partial fraction expansion, degree of denominator should bigger than degree of numerator.

Now, how to handle these issues? So, here what we do? We will convert this proper transfer function into strictly proper transfer function. So, what procedure we apply, what we will do we write this b 3 s cube as b 3 in bracket we have write down s cube, plus a 2 s square plus a 1 s plus a naught. So, we what we want? b 3 s cube. So, b 3 s cube is there. So, this additional term we added therefore, we will subtract this terms a b 3 a 2 a square plus b 3 a 1 s plus b 3 a naught. So, what we will happen? If you make subtraction of this 1 we will get same b 3 s cube plus b 2 a square plus b 1 s plus b naught divided by s cube plus a 2 s square plus a 1 s plus a naught.

So, here little mathematical adjustments we have made, but finally, we will find out the result of this equation let us say this is equation number 1 and equation number 2 both are same. These terms will automatically cancel this b 3 a 2 a square b a b 3 a 1 s b 3 a 0 with this one finally, we will get the same b 3 s cube b 2 s square b 1 s b naught divided by s cube a 2 s square a 1 s plus a naught Now, what we will do? We divide both equations. So, up division of this what you will get? You will get b 3 plus plus s square b 2 minus b 3 into a 2 plus s b 1 minus b 3 into a 1 plus b naught minus b 3 into a naught you just see here.

So, you will find that s square terms equal to this b 2 minus b 3 into a 2, then s terms this s b 1 minus b 3 into a 1 and plus b 0 minus b 3 into a naught divided by divided by this divided by s 3, a 2 s square plus a 1 s plus a naught. So, we will find that original transfer function is proper whereas, after doing some mathematical adjustment these transfer function has been converted into a strictly proper transfer function why we are done it? It is because we have to take we have to take inverse Laplace, and inverse Laplace it is better to we have to that we should have strictly proper transfer function

Now, if you follow the same process in which we have follow earlier now, that is b 3 plus we can write down c 1 s plus lambda 1, c 2 s plus lambda 2 and c 3 s plus lambda 3; that means, we assume that this denominator has 3 eigenvalues lambda 1 lambda 2 and lambda 3 they are distinct. So, b 3 c 1 s plus lambda 1 c 2 s plus lambda 3 c 3 s plus lambda 3. Now, what we do? Now we solve this. So, after solving with s plus lambda 1, s plus lambda 2, and s plus lambda 3 what we will do we will take this part only; this part and this part.

So, equation number 3 and we take equation number 4. So, what will happened? S plus lambda 1 s plus lambda 2 s plus lambda 3; So, after solving what we will get we will get c 1 s plus lambda 2 s plus lambda 3 plus c 2 s plus lambda 1 s plus lambda 3 plus c 3 s plus lambda 1 s plus lambda 2. So, now we have got this equation and this equation and we have taken this b 3 separately. So, let us say b 3 b 3 plus this is, but now we are comparing this equation 3 and 4. So, when you compare the equation 3 and 4 what we will get? Now we can write down as s square b 2 minus b 3 into a 2 plus s b 1 minus b 3 into a 1 plus b naught minus b 3 into a naught.

So, what we have done? We have taken these particular equations this equation and we have to compare with this. So, we write down this as c 1 s plus lambda 2 s plus lambda 3 plus c 2, s plus lambda 1 s plus lambda 3 plus c 3 s plus lambda 1 s plus lambda 2 that is this particular part this part, you have taken like this here and this is a particular part we have taken.

Now, we have to our aim is to determine the values of c 1, c 2 and c 3 as I told last time also there are two ways; we can solve this equation and compare the coefficient of s square s 1 and s 0, second method is that we can replace the different values of s let us say first of all we replace s equals to lambda 2. If you replace s equals to minus lambda 2 in this equation. So, what result we will get? If you replace s equals to lambda 2. So, this becomes 0 then this part is also become 0.

So, remaining will be the value of c 2. So, finally, if you replace s equals to lambda 2 here, we get the value of c 2 as we can get the value of c 2 as c 2 equals to b naught minus b 3 into a naught minus lambda 2 b 1 minus b 3 into a 1 plus lambda 2 square, b 2 minus b 3 into a 2 divided by minus lambda 2 plus lambda 1 bracket minus lambda 2 plus lambda 3 2. So, in this way we have got s c 2

Now, how to get the value of c 3? So, let us say if you replace s equals to minus lambda 3. If you replace s equals to lambda 3 in this equation what will happen this c 1 will be removed then again we replace s question lambda 3 here c 2 will be removed and finally, here lambda 1 lambda two. So, we will get the value of c 3. So, therefore, in this equation; So, you will get c 3 as b 0 minus b 3 into a naught, minus lambda 3 b 1 minus b 3 into a 1 plus lambda 3 b 2 minus b 3 into a 2 divided by minus lambda 3 plus lambda 1 minus lambda 3 plus lambda 2.

Now, to get the value of c 1 so, there are two option or a last time what I told that you can replace s equals to 0. So, here we can have two option; one option is that we replace s equal to minus lambda 1 you can get the equation in terms of c 2 and c 3 other thing is that, we can replace the values of s equal 0. So, when you replace the value of s equals to 0, you can get the c 1 which is in terms of c 2 and c 3. So, in this that equation you have to replace the values of c 2 and c 3 and you can get the value of c 1.

So, you can try any of the option, but now here what we do we are replacing s equal 0. So, if you replacing s equal to 0 s equals to 0 in this equation. So, if you replace s equal to 0 in this equation. So, we will get c 1 in terms of lambda 2 lambda 3 c 2 in terms of lambda 1 lambda 3, and c 3 in terms of lambda 1 and lambda 2.

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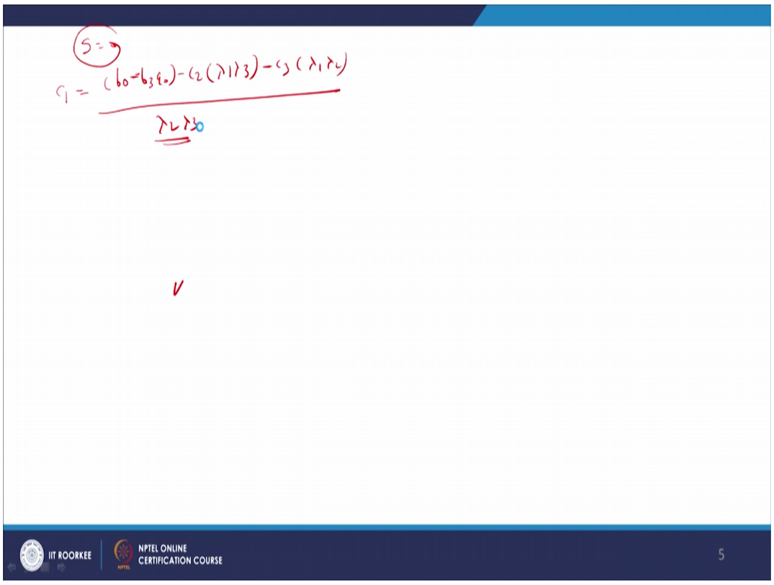

So, finally, we can write the values of c 1 as c 1 equals to b 0 minus minus b 3 into a naught; minus c 2 lambda 1 into lambda 3 minus c 3 lambda 1 into lambda 2 divided by lambda 2 and lambda 3. So, in this way we can have got c 1 c 2 and c 3. So, to the procedure which I had told we will again see it systematically. So, here consider a third order system with proper transfer function that is degree of denominator is same as degree of numerator.

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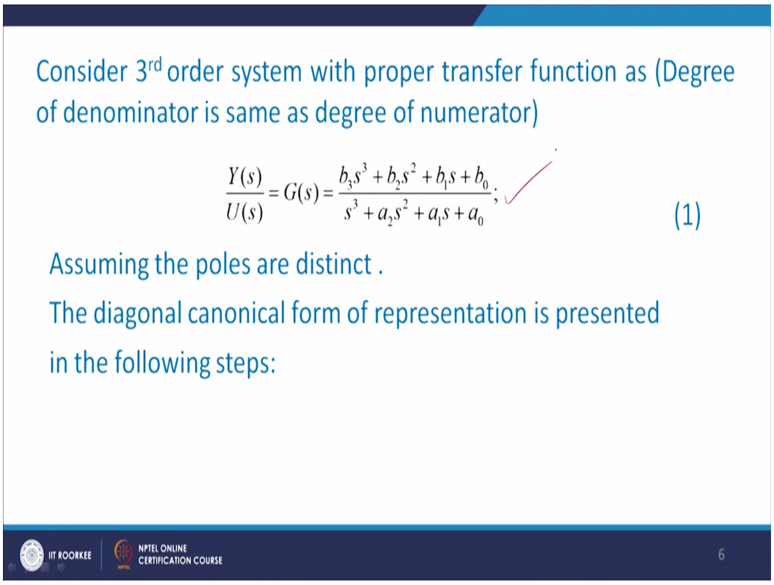

So, we will take b 3 s cube b 2 s square b 1 s b naught s cube a 2 s square a 1 s a naught assume poles are distinct. The diagonal form of representation is represented in the following steps as.

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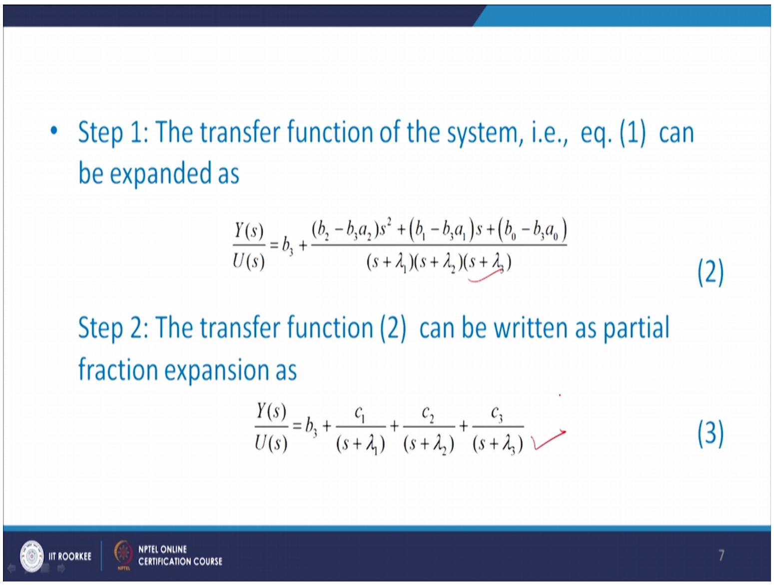

First steps the transfer function of system that is equation 1, can be expanded as b 3 b 2 b b 2 b 3 a 2 s square b 1 b 3. a 1 s. b 0. b 3 a 0 s plus lambda 1 s plus lambda 2 s plus lambda 3 that means, here you can see this is the part this we are shown here. Now, the transfer function of 2 can be written in a partial fraction expansion. So, here b 3 c 1 s plus lambda 1 c 2, s plus lambda 2 c 3 s plus lambda 3.

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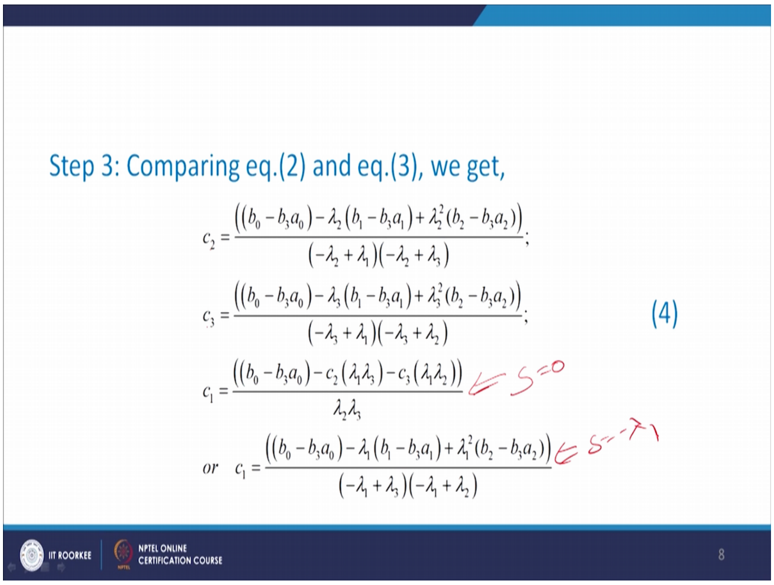

Now, comparing equation 2 and 3 so, what happened? Comparing equation 2 and equation 3 so; that means, here earlier we have shown here this is this equation we are comparing with this and here this is the same thing equation 2 and equation 3. So, what we will get? c 2 c 3 and c 1 c 1 is there are 2 ways to determine as I told you that either you can replace s equal to 0 or you can replace s equals to minus lambda 1.

So, these particular equations we have got replacing s equal 0 and this equation we have got replacing s equals to minus lambda 1. So, both the way you can do it; that means, here we for getting c 1 you have to replace the value of c 2 and c 3 in these equations.

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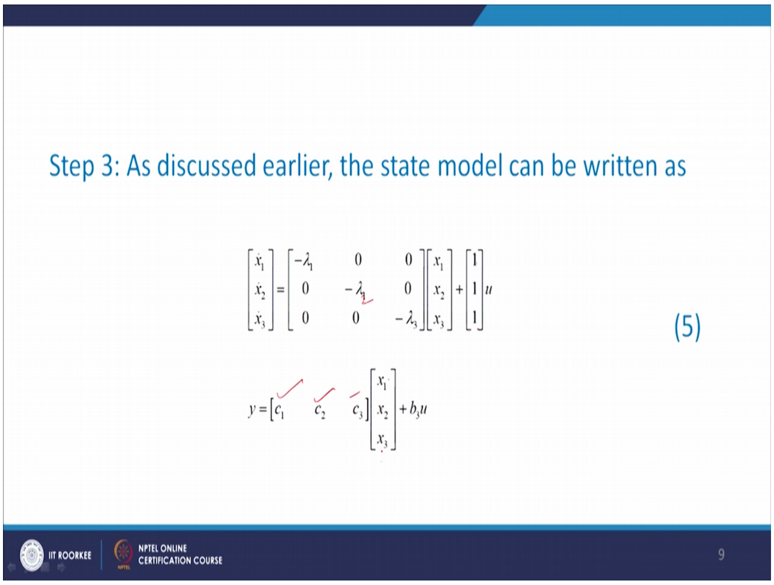

As discussed earlier the state model can be written as x 1 dot equals to lambda 1 x 1 to you, the x 2 dot equals to minus lambda 1 lambda 2 into x 2 and once the x 3 dot lambda 3 into x 3 1 1 into u. And what is the output is c 1, c 2, c 3, x 1, x 2, x 3 now what is the addition thing is happened, here we have got b 3 into u. That means, this b 3 this y we write down in terms that is b into u of s. So, here in the output we will get along with c 1 c 2 c 3 we are getting b 3 into u. So, we will get b 3 into u and a block diagram of the system you will see that here.

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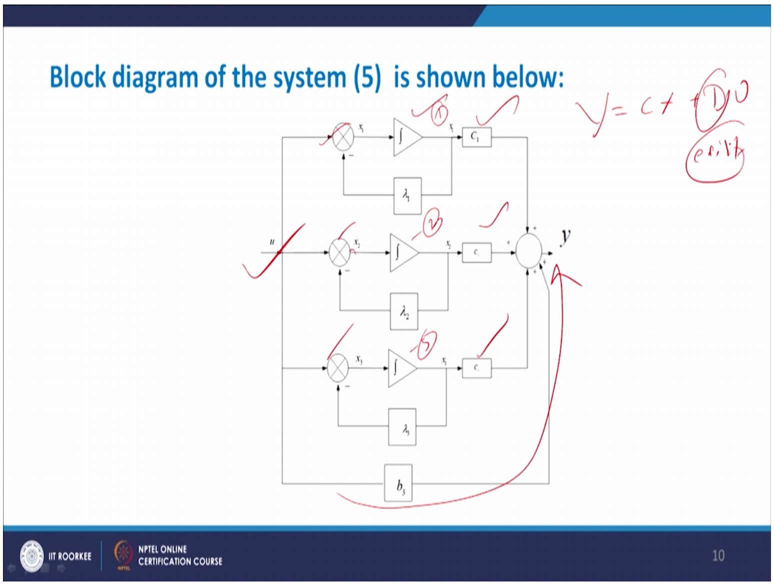

3 integrators integrator 1 integrator 2 and 3 y 3 integrator because you have 3 states x 1 dot x 2 dot and x 3 dot these summation blocks now this is the input. This c 1 c 2 c 3 which we have determined here c 1 c 2 c 3 and here specifically see b 3 into u

So, in this equation your Y equals to cx plus D u d is exist in the part 1 we have find that D is not exist, but here a D is exist. So, when the D will exist where the degree of numerator is same as degree of denominator then this D is exist. So, we have shown b 3 into u.

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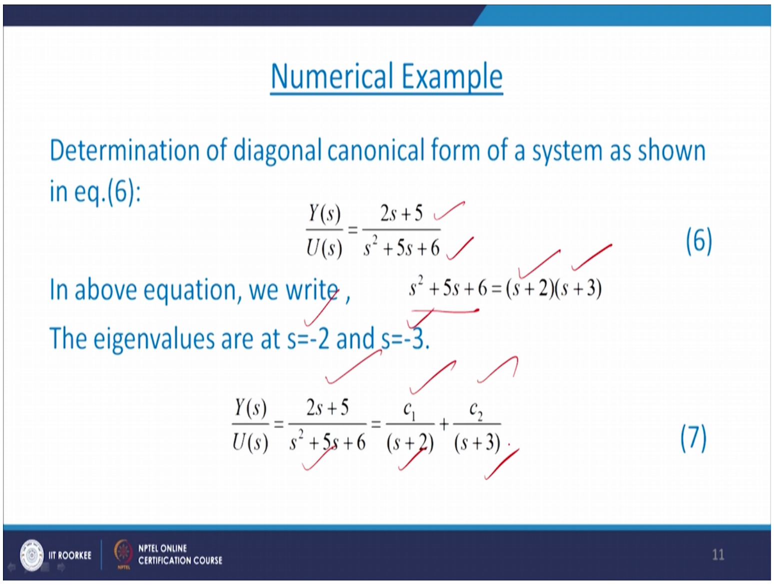

Now, we will study this with a numerical example, but here for the numerical example what we are doing here we are taking a degree of denominator is greater than a degree of numerator. So, for understanding purposes I have taken only second order system. That is 2 s plus 5 s square plus 5 s plus 6, now s square plus 5 s plus 2. So, second order. So, how many eigenvalues or we can say roots the roots are s equal to minus 2 and another at s equals to minus 3. So, you will find that s square plus 5 s plus 6 s plus 2 s plus 3.

Now, here we what we want we want? Diagonal canonical form therefore, we had to use partial fraction expansion. So, here 2 x plus 5 s square plus 5 s plus 6 it has been written as c 1 s plus 2, c 2 s plus 3 now after solving.

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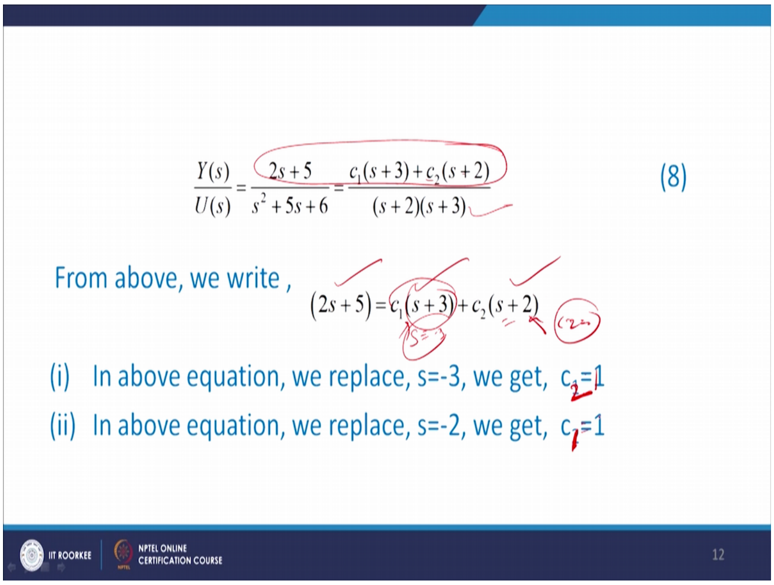

So, here we have solve it s plus 2 into s plus 3, c 1 s plus 3 c 2 s plus 2. So, now what we do? We compare these equations. So, after comparing you have 2 s plus 5 here, c 1 s plus 3 c 2 s plus 2. So, in this case what we do what we are done? We are replace s equals to minus 3 in this equation. So, what happens this becomes 0 ok.

So, we will get the values of c 2 and finally, we will get the values of c 1; that means, if you replace here s equals to minus 3. So, these becomes these becomes 0. So, what is c 2? c 2 is you replace s equals to minus 3. So, we will get c 2 equals to c 2 equal to 1 then in the above equation we replace s equals to minus 2. So, s equals to minus we replace this c 2 becomes equals to 0. So, we will get the values of this is c 1 not

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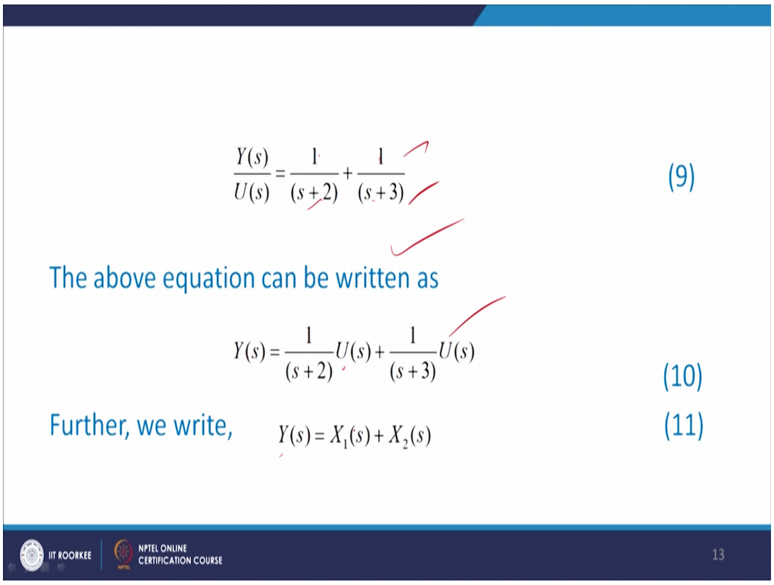

So, we get c 2 and c 1 and finally, we have got y is s equals to 1 upon s plus 2, 1 upon s plus 3 the above equation can be written as 1 upon s plus 2 into u of s, 1 upon s plus 3 into U of s finally, y of s equals to this is x 1 of s and this is this is x 2 of s.

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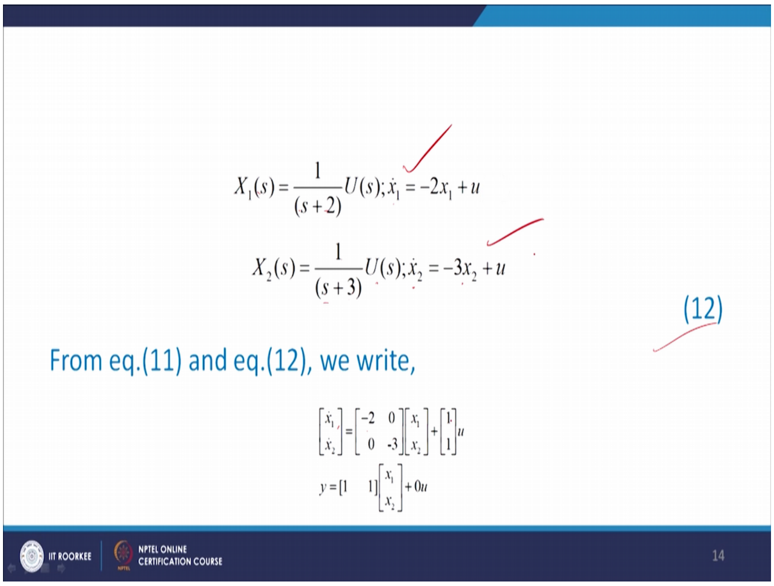

And we have written x 1 of s equals to s plus 2 that is sx x 1 equals to x 1 dot equals to minus 2 x 1 plus 2 that is here what we have done we have use inverse Laplace transform and because of that we have got this.

Similarly, and this is X 2 of s; X 2 of s equals to U of s divided by x plus 3. So, X 2 of s equal to 1 upon s plus 3 into U of s. So, what we will get? S x 2 x that is x 2 dot equals to minus 3 x 2 plus u. So, from using equations this 11 and equation 12,we have got x 1 dot x 2 dot equals to minus 2 x 1 into u x 2 dot equals to 0 minus 3 into u and here 1 1 u and y equals to here 1 1 x 1 x 2 0 into u.

So, you will find that this system we have taken that is strictly proper, and because of that is 0 has not 0 has come that is d 0. But in the same problem; if I have taken 5 s square 5 s square plus 2 s plus 5 s square plus 5 s plus 6.

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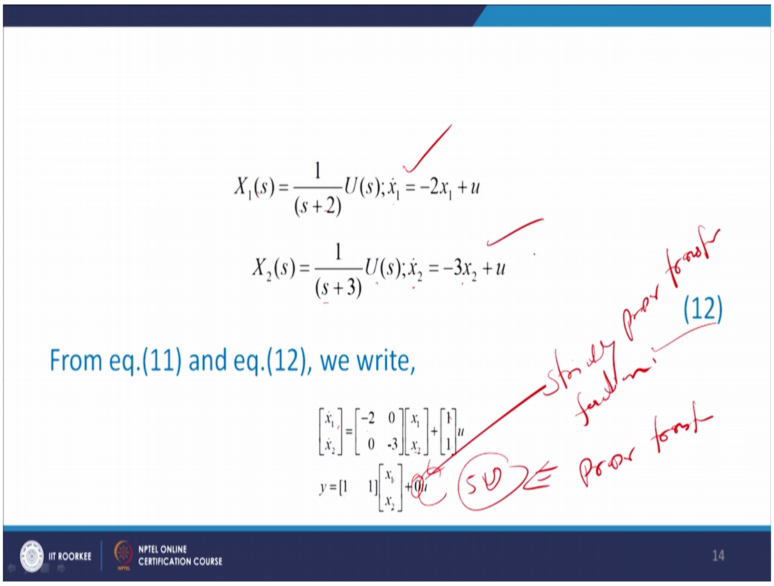

So, in that case this 5 may be reappeared here this is 5 into u; 5 into that is the case when it is only proper transfer function and this 0 has come because of strictly proper nature, proper transfer function. So, strictly transfer function you are get 0, but the proper we will get here 5 if you are taken the problem of this nature. Now, you see some advantages of this approach.

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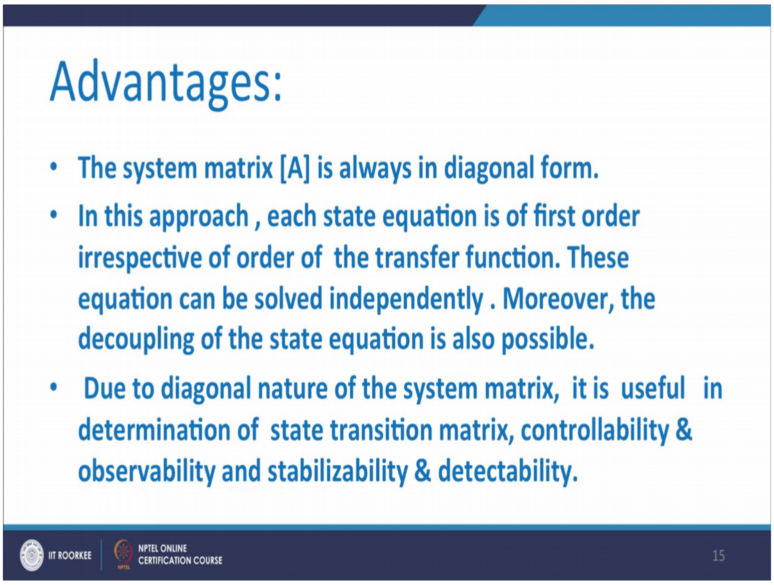

System matrix A is always in diagonal form, that is you will find that the in diagonal, elements we will get the eigenvalues and the elements are very less. Therefore, for analysis purpose this type of modeling is very useful. Second point, in this approach each state equation is of first order irrespective of the order of the transfer function. These equation can be solved independently. Moreover, the decoupling of the state equation is also possible.

It means that what are may be the order of the transfer function the state equation is always a first order. And moreover, this they can be solved independently and therefore, decoupling of the states is possible. And therefore, we find out this type of decoupling concept has been used in checking the controllability and observability. And in state space the main important is to design a controller for the control design also this type of model is very much useful.

Now, third due to diagonal nature of the system matrix, it is useful in determination of state transition matrix controllability, observability and stabilizability and detectability. So, along with the controllability and observability, there are other methodologies for control design that is stabilizability and detectability. So, that can get can also possible using this type of modeling.

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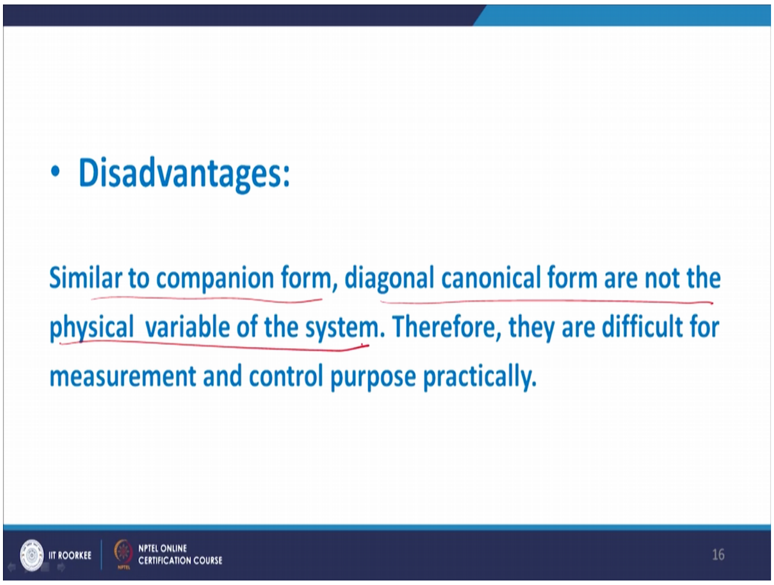

Now, there are some disadvantages. Similar to companion form, diagonal canonical form are not the physical variable of the system, that is similar to companion form, diagonal canonical form are not the physical variable of the system. Therefore, they are difficult for measurement and control purpose practically.

See here what are model we have taken, we are assume the variable variables state variables, but they are not actually variable they are not a physical variables. Therefore, these type of analysis or this type of model cannot be useful for measurement and compute proper practically. But as per as analysis part is concerned these type of modeling is very much useful; that means, if you are having a system you are convert into some differential equation form. And in that case what should be the result, what should be the parity result, that can be possible by means of this type of canonical form of modeling

So, although we find it there are some advantage, similarly for every side there is a two side advantage and disadvantage. So, there are little disadvantage, but still this type of modeling is very much useful in control applications. There are some these are some references Choudhury D. Roy Choudhury's, Modern Control Engineering, Norman Nise, Ogata's, Nagrath and Gopal. So, you can use these references.

Thank you.