Advanced Linear Continuous Control Systems
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Lecture – 06
State Space Representation:
Diagonal Canonical Form (Part-I)

Today we start with the Diagonal Canonical Form. In this we will study introduction to canonical form of modeling, then state space representation of canonical form of modeling; Now, about the introduction to canonical form of modeling the first point.

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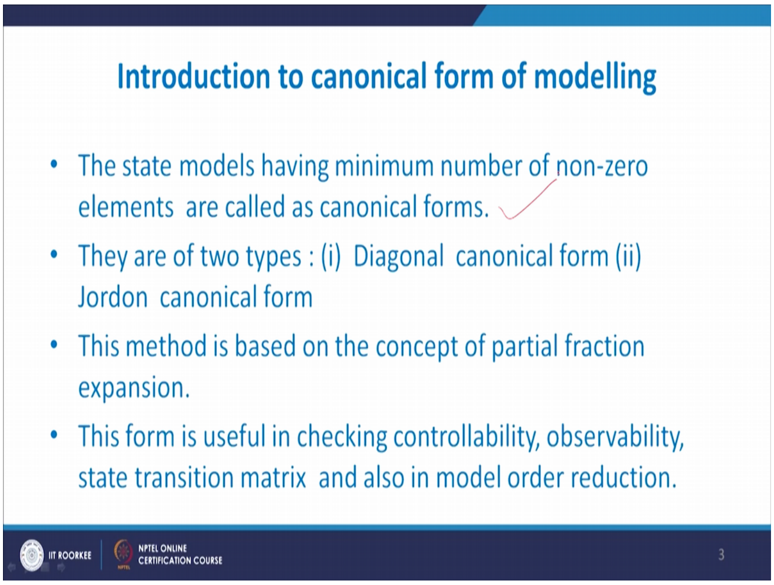

The state model having minimum number of non-zero elements are called as canonical forms. So, we will find that in this particular form, the number of non-zero elements are minimum other elements are maximum 0's. Second point there are two types, one is diagonal canonical form, and second one is Jordan canonical form. Diagonal canonical form is there are eigenvalues which are distinct they are not repeated, that is called diagonal canonical form.

A second one Jordan canonical form means there are some eigenvalues, which are repeated. This method is based on the concept of partial fraction expansion. Because when we do the model we have to apply a inverse Laplace transform.

So, if you if you take the normal model, it is difficult to determine the inverse Laplace transform. So, in this case we had to use the concept of partial fraction expansion. Last point this form is useful in checking controllability, observability, state transition matrix and also in model order reduction techniques. Controllability observability, this controllability observability is very important in controller design, particularly in state space analysis.

Therefore, when we will study controllability and observability at that time we will also means we should also get the idea about the canonical form of modeling; that means, this canonical form of modeling is required in case of controllability and observability. Then about the state transition matrix you know and model order reduction technique; Model order reduction technique means that, when system is of higher of higher order that can be reduce to low size.

So, that time also we can use this type of modeling that is canonical form of modeling. Now, here there are 2 parts; in part 1 we will see diagonal canonical form where degree of denominator is greater than degree of numerator.

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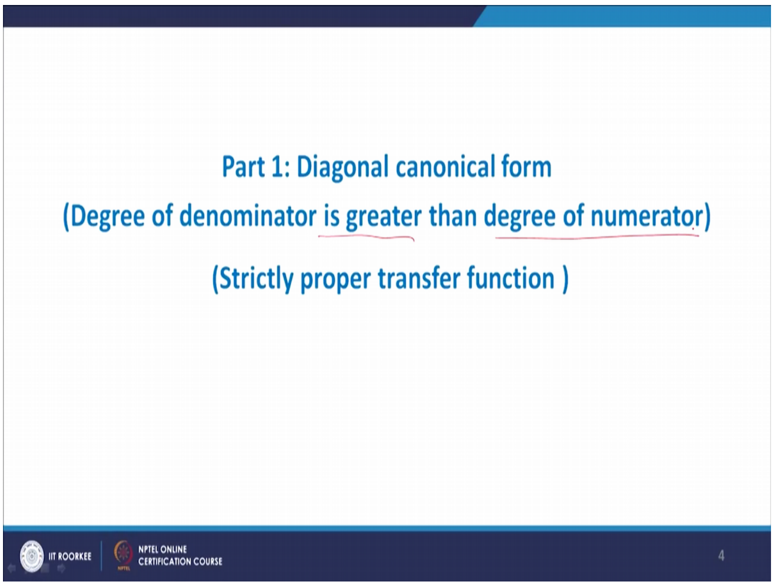

And we call these type of transformation as strictly proper transfer function. And in another part we will see right now, we will study the degree of numerator denominator is greater than the degree of numerator that is strictly proper transfer function.

Now, we start discussing on this issue. So, in order to get the canonical form of modeling, that is a diagonal canonical form we will start with a transfer function model. For simplicity we will take a third order of system. So, I am writing down the equation of transfer function.

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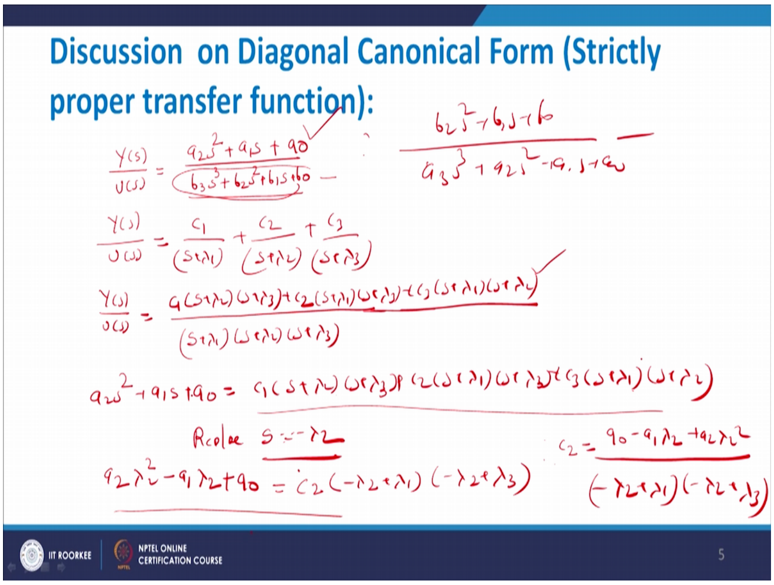

Y of s divided by U of s that is equal to a to a square plus a 1 s plus a naught. So, this is your third order of system. Now, here this has to be converted into canonical form. So, here we assume that the eigenvalues or roots of these are distinct different eigenvalues. Therefore, what we will do? Your write these transfer function as Y s by U s equal to c 1 upon s plus lambda 1 c 2 upon s plus lambda 2 and c 3 upon s plus lambda 3. So, here transfer functions.

Now, here we have written in terms of s plus lambda 1 lambda 2 lambda 3 and; that means, we should know that we should know the eigenvalues of this denominator that is lambda 1 lambda 2 and lambda 3. Now, afterward what we will do we can write these Y s by U s that is equal to so, we solve it. So, what we will get? We will get here s plus lambda 1 s plus lambda 2 plus s plus lambda 3.

Now, if you solve it what we will get here? You can c 1 s plus lambda 2 into s plus lambda 3. It is c 1 plus we get c 2 c 2 write down as s plus lambda 1 s plus lambda 3 and c 3 we write down as s plus lambda 1, s plus lambda 2. So, here we will find that we have written the equation in terms of Y s by us and like this c 1 c 2 c 3 we have written like this.

Now, what we will happen this a 2 s square a 1 s and this. So, both the equations are same. So, what we can do we can compare you compare these equation with these equation. So, if you compare this equation what we will get? We write down as a 2 a square plus a 1 s plus a naught equals to c 1 s plus lambda 2, s plus lambda 3 plus c 2 s plus lambda 1 s plus lambda 3 plus plus c 3 s plus lambda 1 s plus lambda 2. So, this equation we have written.

Now, our main purpose here is to get the values of c 1 c 2 and c 3. So, how to get this value of c 1 c 2 and c 3? So, we will find out there are two option; one option is that you can directly solve this equation, and compare the coefficient of a square then s 1 and s naught , but that will be a tedious process. So, in order to tackle this one what we can do here that we replace the different values of s.

So, initially what we will do? We can replace if you replace s equals to minus lambda 2. If you replace s equals to lambda 2. So, what we will happen? So, will get the equation as here a 2 lambda 2 square minus a 1 into lambda plus a naught what we have done? We are replace s equals to lambda 2 in this equation. So, what we will get? Lambda 2 square a 2 minus a 1 lambda 2 plus a naught that is equals to we will replace s equals to lambda 2 in this equations what we will happen? This becomes 0 then here you will find at we replace s equals to lambda 2 here this because to 0 that the remaining term is of lamb this s plus lambda 1 and s plus lambda 3. So, what we will get? We can write down as c 2 minus lambda 2 plus lambda 1 here minus lambda 2 plus lambda 3.

And from this you can easily get c 2 equal to c 2 will get as a 0 minus a 1 lambda 2, plus a 2 lambda 2 square divided by minus lambda 2 plus lambda 1 minus lambda 2 plus lambda 3. Sometimes norm if you take this equation as like this b 2 a square b 1 s plus b naught, there in that b a 3 s cube plus a 2 s square plus a 1 s plus a naught in that case this coefficient offer this a 0 a 1 it will be replaced by b 0 b 1 and b 2.

So, now we have got c 2 similarly we can also get the value of c 3 and c 1 how to get the value of c 3. So, what is the best option? Here just like we are replace s equals to lambda 2.

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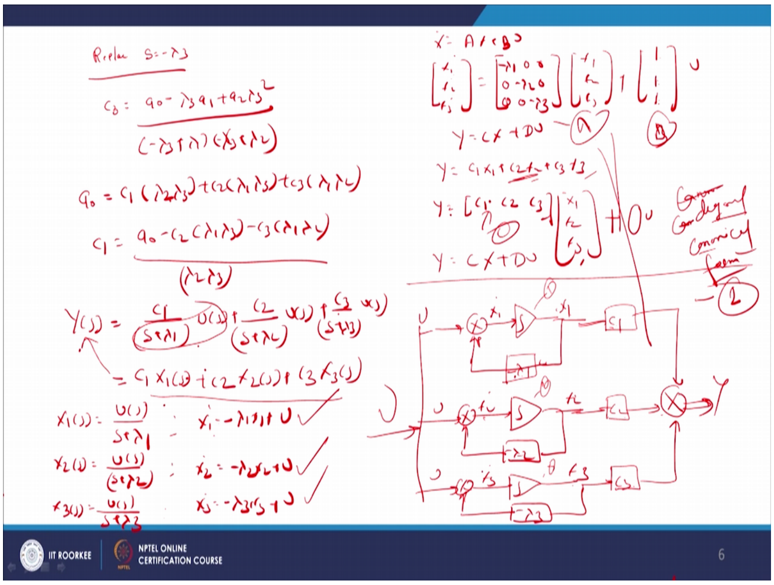

So, what we can do here we can replace s equal to lambda 3. So, if you replace s equals to lambda 3 there are 2 2 elements the 3 first part second part 3 third part 2 part will be automatically 0, and we can directly get the values of c 3. So, what we will do here? We replace S equals to minus lambda 3.

So, if you replace S equals to lambda 3. So, what we will get? We will get the equation of c 3 as a 0 minus lambda 3 into a 1 plus a 2 lambda 3 whole square divided by so, will what we will happen? Here you have got here lambda 2 plus lambda 1 lambda 2 plus lambda 3. So, here will get minus lambda 3 plus lambda 1 minus lambda 3, plus lambda 2. So, we will get the value of c 3.

Now, to get the value of c 1 so, what is option now? Because here we have taken care of lambda 3 and this lambda 2, because we initially you replace s q lambda 2 s equal to lambda 3. So, now, what we will we can we will do we replace s equals to 0. So, if you replace s equal to 0 in these particular equations.

So, what we will get we will get here a 0 c 1 equals to lambda 2 lambda 3 plus c 2 lambda 1 lambda 3, plus c 3 lambda 1 into lambda 3 lambda 1 into lambda 2 that is a 0 equals to c 1, lambda 2 lambda 3 c 2 lambda 1 lambda 3 c 3 lambda 1 lambda 2. And from this you can get the values of c 1 equals to a 0 minus c 2 lambda 1 into lambda 3, minus c 3 lambda 1 into lambda 2 lambda lambda 2 divided by lambda 2 into lambda 3.

So, v of v or we have got the values of c 1, c 2 and c 3. And as I told you earlier normally we are taking a 2 a 1 a 0 in the denominator and b into b 3 b 3 b 2 b 1 in the numerator. So, I have to like this if you change is you can also get the similar types of replace. The condition thing is that instead of a 2 a 1 a 0 you can get b 2 b 1 and b naught.

So, you have got c 0 c 1 c 2 and c 3, the main our purpose is to develop the state space model now you see the output. Output Y of s equal to c 1 plus s plus lambda 1 into u of s, plus c 2 s plus lambda 2 into u of s plus c 3 s plus lambda 3 into u of s y of s equal to s plus lambda 1 2 u of s c 2 s plus lambda 2 into u of s and c 3 equals to s plus lambda 3 into u of s you can just see it this equation.

So, what we have done? This it has been multiplied by us. So, finally, you got this. So, what we can do here we can write this u of s, s plus lambda 1 as s 1 of s. That is we can write c 1 into x 1 of s that is this particular portion; plus c 2 into x 2 of s, plus c 3 into x 3 of s. c 1 c 2, c 3, x 1, x 2, x 3 s and what is x 1 of s? So, x 1 of s equal to u of s s plus lambda 1.

Now, what to do further? Because it is in s domain and we required in the time domain. So, here what we will do? We will apply inverse Laplace transform. So, when you apply inverse Laplace transform, what we will happened? You will get s into x 1 s that is equal to x 1 dot that is equal to minus lambda 1 into x 1 plus u. Now, next we have got x 1 s similarly we can go for the x 2 s. So, what is x 2 s? X 2 s equals to u of s plus s plus lambda 2. So, how remove? So, here we have got s into x 2 that is x 2 dot equals to minus lambda 2 n 2, x 2 plus u.

Similarly, we can write x 3 s; x 3 is equal to u of s s plus lambda 3; so, what we will get? X 3 dot equals to minus lambda 3 into x 3 into u. So, we have got x 1 dot, x 2 dot and x 3 dot. State space model is nothing, but x dot equals to Ax plus Bu that is your state space model is x dot equal to Ax plus Bu. So, how many states? There are 3 states therefore, we write this equation as x 1 dot x 2 dot x 3 dot that is and now what is the A matrix; A matrix is a system matrix and there is state x 1 x 2 x 3 plus this is u.

So, first step x 1 dot x 1 dot equals to minus lambda 1 x 1 plus u, but there are no terms of x 2 and s 3 therefore, what we will do? We can write this is as minus lambda 1 0 0 x 1, and is there term for u s this is 1 term of u that is 1 into u. So, we can write 1; second x 2 dot what is x 2 dot x 2 dot equals to you just see here minus lambda x 2 plus u , but there is there are no terms for x 1 and x 3 terms for x 1 and x 3 are 0.

So, what we will; how we will write? So, x 2 dot equals to 0, minus lambda 2 0 and here this is also term for u that is 1 and now about the x 3 dot. So, x 3 dot equals to what to write? 0 0 minus lambda 3; So, lambda 1 lambda 2 lambda 3 because here also x 3 dot there are no terms for x 1 and let see our lams 3 dot equals to, lambda 3 x 3 equal to u and whereas, this is a term for u.

So, this is a state space model , but again what we want? We required the model of output output equals to y x; Y equals to cx plus du that is you want model Y equals to cx plus du c is a c is the output matrix and d is the compliment that we have seen last time now we have to write the equation for c. So, how will write down you will you will find at you find a y of s equals to c 1 x 1 of s c 2 x 2 of a c 3 x 3 of this is y of s. So, here what we will do, we can also apply inverse Laplace to these equations. So, what we will get? We can write Y equals to c 1 x 1 c 2 x 2 c 3 x 3 we can write down into some Y of t also. Y of t c 1 x 1 of t, c 2 x 2 of t or c 3 x 3 of t also.

Now we write the equation for Y equals to c 1 c 2 c 3 here x 1 x 1 x 1 x 2 x 3 plus is (Refer Time: 17:16) d no you will find that , there is no term of t is present. So, we can write this as 0 into u therefore, your final equation is y equals to Cx plus D u. So, we got a matrix, a b matrix, this is a c matrix and what was c 1 c 2 c 3? C 1 c 2 c 3 are called residues, they are called the residues. And again we will find that the most of the elements in this matrix a r 0 we defined as 0 0 0. Therefore, in introduction I have mentioned that number of terms non-zero terms are very less that is here lambda 1 lambda 2 and lambda 3 and remaining terms are 0. And again 1 thing is that in the output of u there are terms of only 1 1 1 and Y is corresponding to c 1 c 2 and c 3.

So, these type of modeling is called diagonal canonical form canonical form diagonal canonical form and this is a part 1. Why I am seeing part 1? Because the something is involved in this part 2 what is the problem here? The problem is that here we have taken a degree of denominator is greater than the degree of numerator. So, that part we select on, but now here what is the thing because we have developed the model. And now whatever the model we develop we need to do the simulation because we have to see the result therefore, what we will do here, that is we have to develop the block diagram of a given state space model.

So, how to develop the block diagram? See you see there are 3 state x 1 dot, x 2 dot and x 3 dot. So, in this case there are 3 states then, how many integrators are there? There are requirement of the 3 integrator. So, we will go for the first integrator. So, we will draw this is first integrator. So, you see here I have shown an integrator. So, what is the state? First state is x 1 and how will it come it form x 1 dot? See a this equation x 1 dot equals to minus lambda 1 x 1; that means, here we have to use minus lambda 1 this positive; plus u that is u is there.

So, x 1 dot equals to lambda 1 x 1 plus u similarly we can do for the x 2 dot and x 3 dot. So, how will do it? So, we will write another integrator, we can draw another integrator now here is the block, here we can show minus lambda 2 u here is a x 2, here x 2 dot. So, similarly there are another integrator required because we have x 3 dot. So, we can show another integrator here x 3 dot here is x 3, here is u here is minus lambda 3 and we are connected it. So, x 1 dot x 2 dot and x 3 dot. So, we have taken 3 integrate integrator 1 integrator 2 integrator 3. So, u is common. So, we can connected it. So, we can finally, case of this is u.

So, here we have drawn the block diagram of a 16 equations that is x 1 dot, x 2 dot and x 3 dot, but in the model we have output is y equals to Cx plus Du. So, here we have to show the block diagram of the block diagram concern with the output and the y, that is output is nothing, but the y. So, here we want to show output. So, I have shown here 1 block summing block and now here Y equals to c 1 x 1. So, here x 1 I can you will take c 1 and here coming is 1. So, this is Y. So, y equals to c 1 into x 1 similarly we want this is c 2 x 2 that x 2 this is c 2. So, y equals to c 2 into x 2 and lastly we have c 3 into x 3.

So, this is why c 1 x 1, c 2 x 2, c 3 c and now other x 1 dot equals to lambda 1 x 1 into u, x dot equals to lambda 2, x 2 into u and x 3 dot equals to lambda 3 into lambda 2 x 3 into u. So, here we will find at we have develop a block diagram of a diagonal canonical form. So, all state equations and this is state equations and this is a block diagram. And here 1 important thing is that, here the degree of denominator is more than a degree of numerator.

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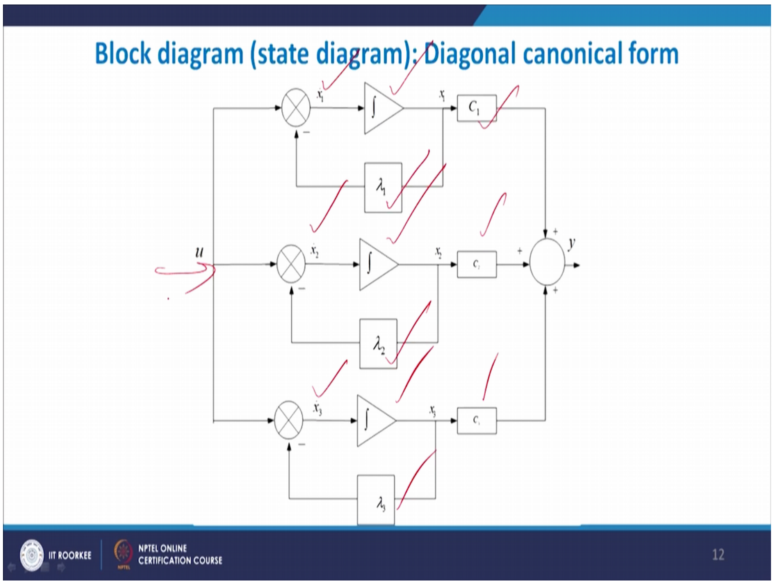

And now here we have develop the block diagram? So, your x 1 dot x 2 dot x 3 dot 3 integrator integrated 1 integrate 2, integrator 3 then this is lambda 1 lambda 1, lambda 2, lambda 3 these are eigenvalues c 1, c 2, c 3 are the output elements of the output matrix and this is input u.

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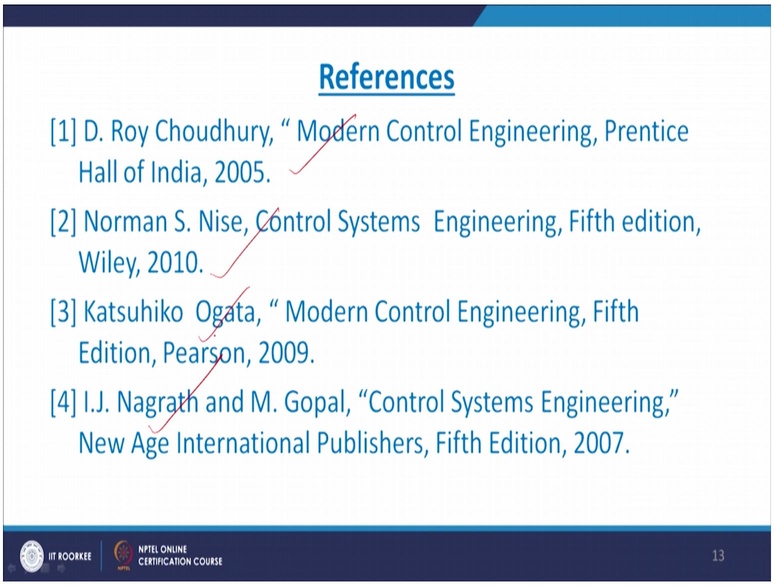

Now, these are some references you can use these references Norman Nise, Katsuhiko Ogata, I. J. Nagarath.

Thank you.