Advanced Linear Continuous Control Systems
Dr. Yogesh Vijay Hote
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture - 05
State Space Representation:
(Observable Canonical Form)

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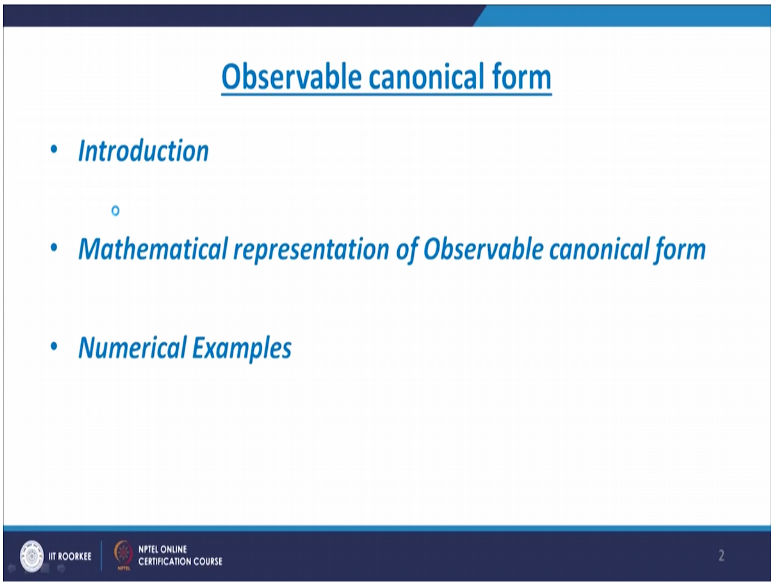

We start with Observable Canonical Form. In this, we will study introduction, mathematical representation of observable canonical form and lastly, numerical examples. Now, about introduction; if you see any state space analysis, what is the final aim? Final aim is to design a controller and the controller has been design in the state space using pole placement techniques and in the pole placement technique, there is need to check controllability and observative. So, all these issues, we will study after word.

So, in that case, particularly in case of observative, there is need an observer equations. Therefore, we require a model which is of, which is of form observable canonical form along with sometimes we required a observer design because sometimes the states cannot be measured, we have to design observer. So, in that case also we need model which is concerned with the observetive or we need observable canonical forms. Therefore, today we will going to study the Observable Canonical Form of model.

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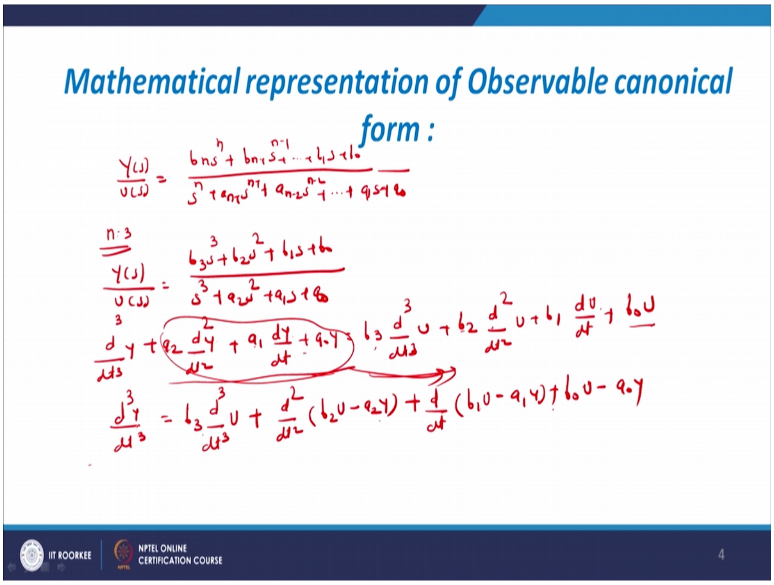

So, we start with is mathematical representations. So, we will take a same transfer function model which you are take a last time. So, Y of s equal to U of s equal to b n s n b of n minus 1 s raised to n minus 1 plus b 1 s plus b naught divide by s raised n a of n minus 1 s raised n minus 1 plus a of n minus 2 s raised n minus 2 plus plus a 1 s plus plus a naught. This is the model. So, last time when we are determined the state space model, we are written in equation in terms of x. Now, here we have concern with observative; that means, concern with the output therefore, we have to write this equation in terms of y. So, here in order to get result clearly or in understand it, I am taking a third order system. So, in this case, we will take n equals to 3.

So, when n equals to 3, your model becomes y s by u s equal to b 3 s cube b 2 s square plus b 1 s plus b naught divide by s cube plus a 2 s square plus a 1 s plus a naught. So, this transfer function s cube s cubes. So, this is we called as a proper transfer function and last time we are seen, the different between proper transfer function and strictly proper transfer function ; now either told we want observative.

So, we have to write the equation in terms of y. So, here what we write here ok. We write differential equation for this we write d 3 by d t cube of y plus this a 2 d square by d t square of y plus a 1 d y by d t plus a 0 into y plus b 3 into d cube by d t cube of u plus b 2 d square of d t square of u b 1 d u by d t plus b 0 into u. Now we want y. So, what will do where write this equation as d cube y of d t cube equal to, so, what will do all the systems, we move in this site, the system. This particular terms will move on the right hand side. So, what will get? We will get b 3 into d cube of d t cube of u plus, now we will see that this is b d square d t square and this one. So, here this will take as common.

So, we will get as d t square by d t square, we will get as b 2 into u minus a 2 into y plus differentiation of b 1 into u a 1 into y plus b 0 into u minus a 0 into y. So, we can see it. This equation this like this then these and this portion you are taken common. So, we will get b 2 into u minus a 2 into y plus differentiation of t with in bracket, we will get b one into u minus a one into y plus b 0 into u minus a 0 into y. So, now, we want output equations. So, how to get output equations?

So, therefore, what will do here? We will integrates this equation 3 times because when we integrate this 3 times, we can get y.

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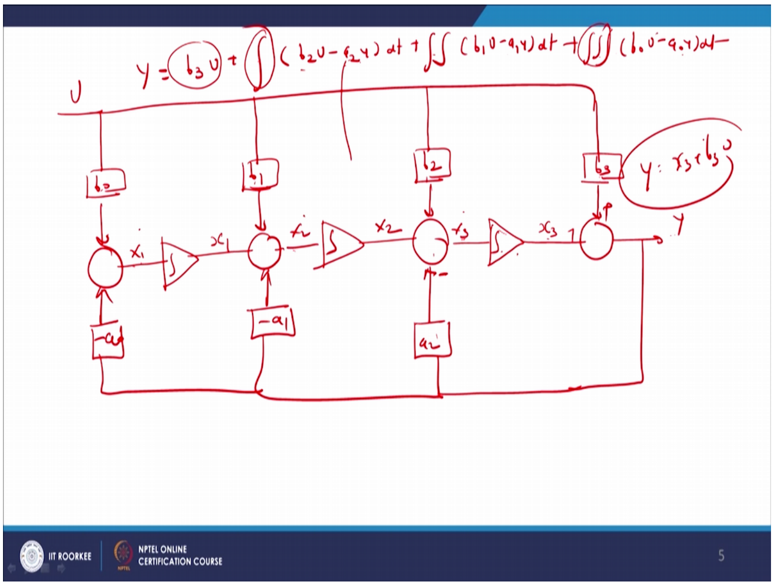

Therefore, what will write; I have write as y equals to b 3 into u plus, in bracket we write as b 2 into u minus a 2 into y d t plus double integrations, double integrations b 1 into u minus a 1 into y d t plus triple integrations b 0 into u minus a 0 y into d t. So, we will find at, so, here we are integrate 3 sign, that is why we got y. Then here d cube so, we are ingrate 3 times, we have got b 3 into u, whereas, we will find at there is two integrator or two differentiator.

So, when integrate 3 times, we will definitely get one integrator here, here there is one differentiator. When we integrator 3 times, so, we will get two integrators. So, we will find at there are two integrators. Similarly, we will find that there is no integrations integrator involved. So, no differentiator is involved. So, when we are integrating 3 times. So, we will get three integrators; so, three integrator, two integrators, one and no integrators.

Now, we want the state space model. So, prior to this one, what will do? From this equation, we try to develop a block diagram or we can says Simulink diagram. So, we start with this. So, here y, so, I can show here this is y now this y equal to b 3 into u, this is b 3 into u. So, what I will take? I will take this as b 3, this is b 3 and now this is u. So, I am writing down this u compliment like this, this is u. So, y equals to b 3 into u. Now, here after this now, what is this is.

The one integrator is involved. So, here I am showing one integrator here. So, one integrator here; this is integrator. Now this integrator, I have shown and now we take one variable. This is x 3 and this when integrate; that means, x 3 dot. So, now, all these x 3 dot equal to b 2 into u minus a 2 into y. Therefore, I am Simulink summation block here, here one b 2 b 2 and now here, this is a 2. So, here a 2; this a 2. This is minus and this is y. The you will find at for this, one we are put one integrator here, x 3 dot.

So, this x 3 dot equal to b 2 into u see is a u and now minus a 2 into y. Now, after word there is another integrators. So, there are another integrator we have to write. So, here we show another integrator. This two integrator; so, first and second; so, another integrator we added. So, in this case, we take another variable. So, here x 2 and this integrator this all, this all integrator or this one equals to b 1 into u minus a 1 into y. So, here what while I doing here x 2 dot, this is the block an x 2 dot equal to this is coming b 1. So, here is b 1 and now here, we have to show a 1; so, here minus a 1 into y.

So, we will find that in there are two integrator involves. So, integrator 2, integrators b 1 into u; so, you will shown b 1 into u and now here minus a 1 into y; we are shown like this. Now we will further, so, now, we have three integrators. So, now, 1, 2, 3, we are to add another integrators. So, here I am adding another integrator and we take another variable x 1 and what is this? This is x 1 dot and what is the x 1 dot equals to b 0 into u minus a 0 into y.

So, we write as a block like this. This is b 0 into u and here we write down as minus a 0 like this. So, here we are written the complete is Simulink diagram of plant. And now from this, we have to develop a state space model. So, in order to develop the state space model, we write the equation of x 1 dot, x 2 dot and x 3 dot.

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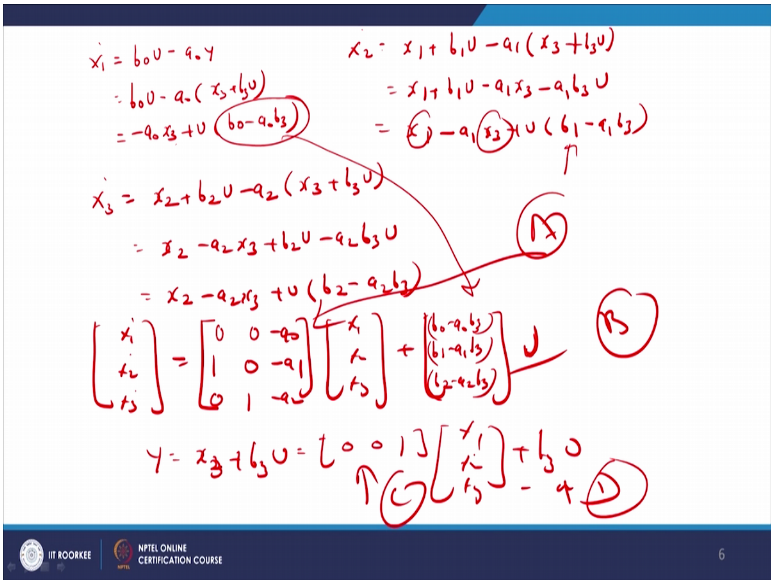

So, now x 1 dot, so, x 1 dot equal to b 0 into u minus a 0 into y. So, we will find this. So, what is the x 1 dot, equal to b 0 into u minus a 0 into y what is y; y equals to x x 3 plus b 3 into u, that is y here this plus y plus 2 x 3 plus b 3 into u. So, we have to replace this equation here. So, we will get as b 0 u minus a 0 into y x 3 plus b 3 into u. And if you after solving this equation, you will get a 0 x 3 plus u b 0 minus a 0 into b 3. This is the first equation we have got. Now, coming into the second equation; now what is x 2 dot? x 2 dot equals to b 1 into u plus bracket minus a 1 into y plus x 1.

So, we will write as x 2 dot equal to x 1 plus b 1 into u minus a 1 into y; y is x 3 plus b 3 into u. So, we will get as x 1 plus b 1 into u minus a 1 x 3 minus a 1 b 3 into u and after solving will get as x 1 minus a 1 into x 3 plus u b 1 minus a 1 into b 3. So, we have got equation of x 1 dot as well as x 2 dot. Now, coming into the equation for x 3 dot; so, now, x 3 dot you see that x 3 dot equals to b 2 into u sees here, b 2 into u plus x 2 minus a 2 into y. Just see here, this x 3 dot.

So, we are write the equation for x 3 dot as x 3 dot that is equals to x 2 plus b 2 into u minus a 2 x 3 plus b 3 into u. Now you solve it. So, what will get, x 2 minus a 2 into x 3 plus b 2 into u minus a 2 b 3 into u and therefore, we write as x 2 minus a 2 into x 3 plus u b 2 minus a 2 into b 3. So, we have got three equation. We have got x 1 dot, we have got x 2 dot and x 3 dot. Now, we have to write down this in terms of state space model.

So, we write this as. So, we write x 1 dot, x 2 dot, x 3 dot equal to, so, x 1 dot equal to this into x 1, x 2, x 3 and plus this some elements and with expect to you. So, how will write? So, x 1 dot equals to here see here x 1 dot a 0 x 3. So, elements of x 1 and x 2 are 0 in the this case. All the elements of x 3 is there. So, we write this equation as 0, 0, minus a naught. Now, coming to the equation of x 2 dot in x 2 dot? What will find at these are terms you know for x 1, this term from x 3, but there is no term for x 2. So, term for x 2 is 0.

So, we write this equation as 1 for x 1, 0 minus a 1. Now coming to the x 3, x 3 dot, in x 3 dot, we will find at there is no term for x 1. That is a x 1 is 0 where as there is term form x 2 and x 3. So, we write this equation as 0, 1, minus a 2. Now, coming into the u terms, for u's you will find at for a x 1 dot, this is term b 0 into b 0 minus a 0 into b 3. Now about this x 2 dot, this term for u is d 1 a 1 b 3, you get b 1 a 1 into b 3. And particularly, the term for x 3 dot we will get it as, what will get b 2 into a 2 into b 3. Now what is the equation for y?

So, we will find at what the equation of y? We written y equals to x 3 plus b 3 into u. So, we write output of y as x 3 plus b 3 into u. So, we write 0, 0, 0, 1. This is x 1, x 2, x 3 plus b 3 into u. So, this is the state space model of the system. So, in this state space model, this is a matrix, this is b matrix, this is c matrix and this is d matrix. Previously, we have seen a controllable canonical form. This is observable canonical form. So, we will find out is there any relationship exist between controllable canonical form and observable canonical form. Let us see it.

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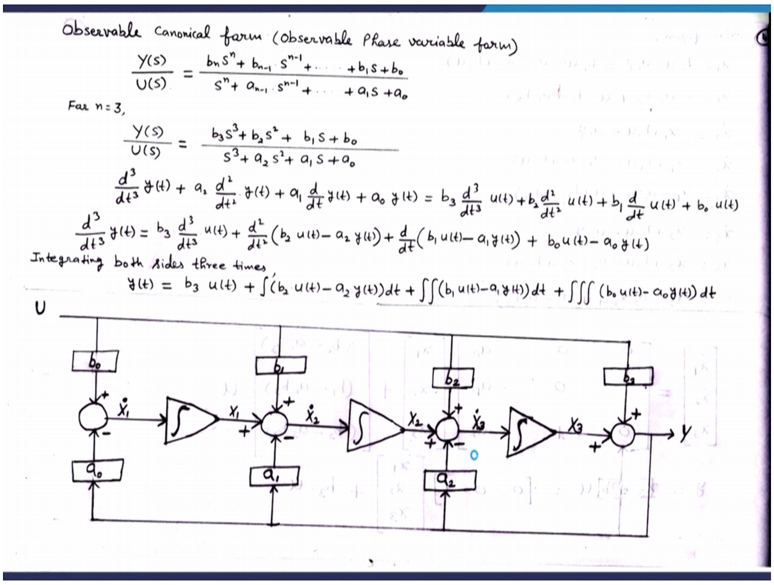

So, in order to do it, in order to do it, what will do? We will take we will take a transpose.

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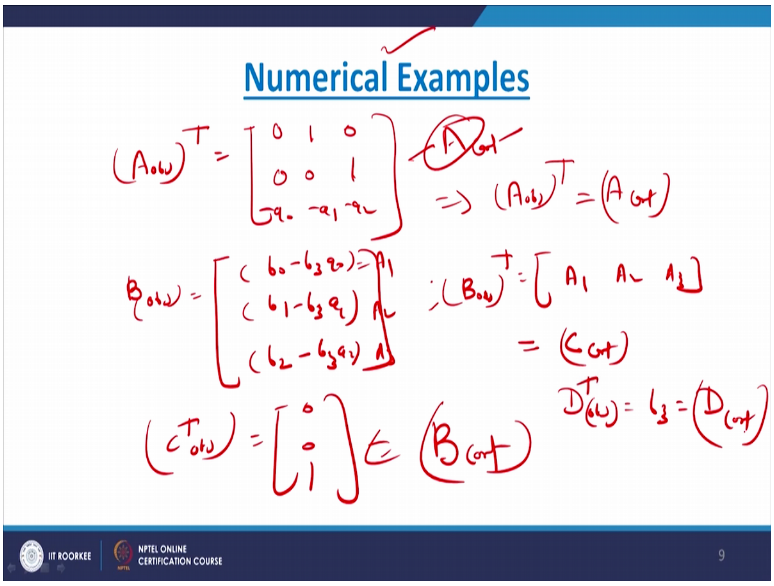

So, here, the numerical example we see later on, first of all we will see this concept. So, here what will write; we take the transpose of a observable transpose.

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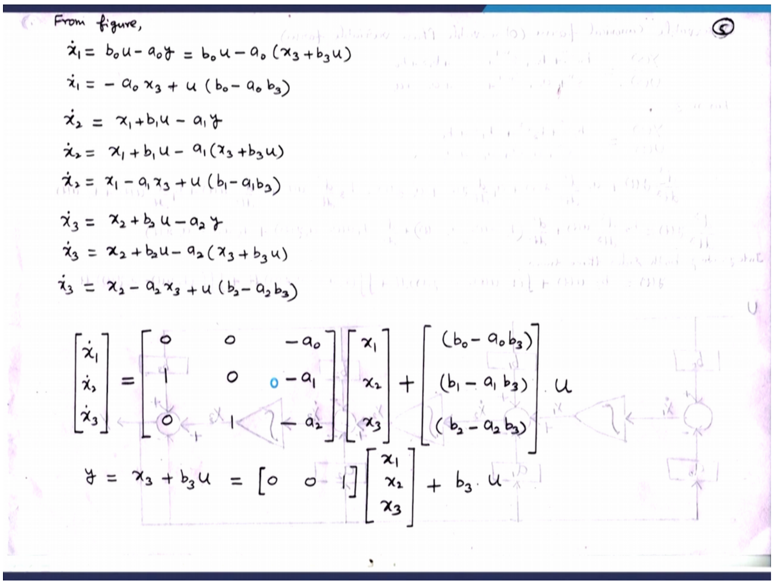

So, we will find at, so, this 0, 0, 1, a 0; 1, 0, 1 a 1; 0, 0, 1 a 2 and even we will find this is the guess 0 1 1 a 0, 1 0 a 1, 0 1 a 2. So, if we take the transpose of this. So, what we will get? Transpose of it is 0, 1, 0, 0, 0, 0, here a minus a 0, minus a 1, minus a 2. This is the transpose.

So, we will find at these a is nothing but the system actives of a controllable canonical form. That is a controllable; that means, what relationship will got A observable transpose, that is equals to a controllable. Now we will find the equation for B observable. See here this is the equation for B observable. So, we will write the equation or B observable equal to b 0 minus b 3 into a naught b 1 minus, here b 2 b 3 into a 2. Now, what will do? We make transpose of this. So, what happen if you make transpose of this one? So, B observable, B observable transpose, what will get, let us say this is a 1, this is a 2 and this is a 3, this is a 1, this is a 2 and a 3.

So, here in case of observable form, the b matrix we are coming in the original state space model where as what happen in case of the controllable canonical form, so, this elements is coming as the c matrix. Therefore, this is a nothing but C controllable matrix. Now, about the this matrix let us say we take, this is C matrix.

So, write C transpose observable equal to we get 0, 0, 1. So, this is matrix is nothing but the B matrix of a controllable canonical form that is, here B controllable, this is b controllable. And particularly, if you see the observative matrix and D observable, so, D observable in this case, we will get as b 3 here b 3 it is same as D controllable matrix. So, we will find at, there is a relationship between controllable canonical form, observable canonical form particularly, we find this entertain job elements in B and C. More detail things or whatever I have thought you, you can also seen through this one. So, it is given like this here, then this is also like this. So, you can refer this also ok.

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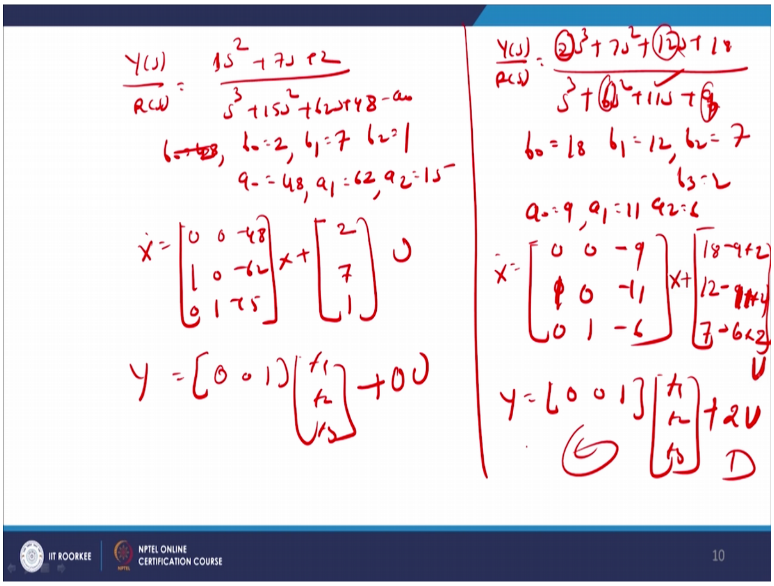

Now, we start with numerical example. So, when take example, Y s by R s, that is equal to s square plus 7 s plus 2 s cube plus 15 s square plus 62 s plus 48. Now, in this case the elements, we write here b 0 equals to 48 or b 0 equals to 2, b 1 equals to 7, b 2 equals to 1. So, this is one variables; then about this is a 0. So, a 0 equal to 48, a 1 equals to 62, then a 2 equals to 15. Now, we have to write down this into a observable canonical form. So, what is the form your x dot, all right x dot. So, you will find; so, x dot equals to 0, 0, a 0, 1, 0, a 1, 0, 0, minus a 2. So, you write as 0, 0, minus 48, 1, 0, minus 62, 0, 1, minus 15 into x plus.

So, we will find at what is the b, is b is this one; this values a b b 0, a 0, b 3, b 1, a 0, b 3, a 2, a 0, b 3, but now we will find that this transfer function is absolutely proper transfer function. So, in this case, the value of your b 3 is 0. Therefore, you will get only b 0, b 1 and b 2 therefore, write the elements as directly 2, 7, 1 u and what is the output y output y, we write down as how you write the output of y 0, 0, 1.

So, here we write down as 0, 0, 1 into x 1, x 2, x 3 plus this is know, 0 into u. Now you take the second example. So, here Y s by R s equal to 2 s cube plus 7 s square plus 12 s plus 18 divide by s cube plus 6 s square plus 11 s plus 9. This is example. Now again here b 0 equals to 18, b 1 equals to 12, b 2 equals to 7 and here b 3 equals to 2 and here a 0 equals to 9 a 1 equals to 11, a 2 equals to 6, this is 6 6. So, now, here we will find at this is a proper transfer function, degree of numerator and denominator are the same. Now, we have to write down in terms of observable canonical form.

So, you will find at these no much change in the a matrix. So, a matrix similarly is like this 0, 0, minus 9, 1, 0, minus 11, this is a 0, a 1. So, minus 11 and here we will get 0, 1, minus 6 plus, now here what will happen, here we have to do the calculation like this b 0 into b 3, a 0, b 1, b 3, a 1, b 2, b 3, a 2. So, here we write down as the elements as 18, minus 9 into 2. So, 18 this minus 9 and b 3 is 2.

So, 18 minus 9 a 2 second, 12, this 12 minus we take same 9 sorry 9, we can write 11, this is 11 11 multiplied by same 2, This is 2 and thirdly, we have to write as third element as 7 minus then is 6, 6 multiplied by same 2 and we will get output as 0, 0, 1, x 1, x 2, x 3. And particularly, we will find at you will get it two elements, two means here, the elements b 3 will get it. So, when the transfer function involvement of the proper, this always D element, this is D element, this is C element. So, D is involved where as for stifle transformation D is 0's.

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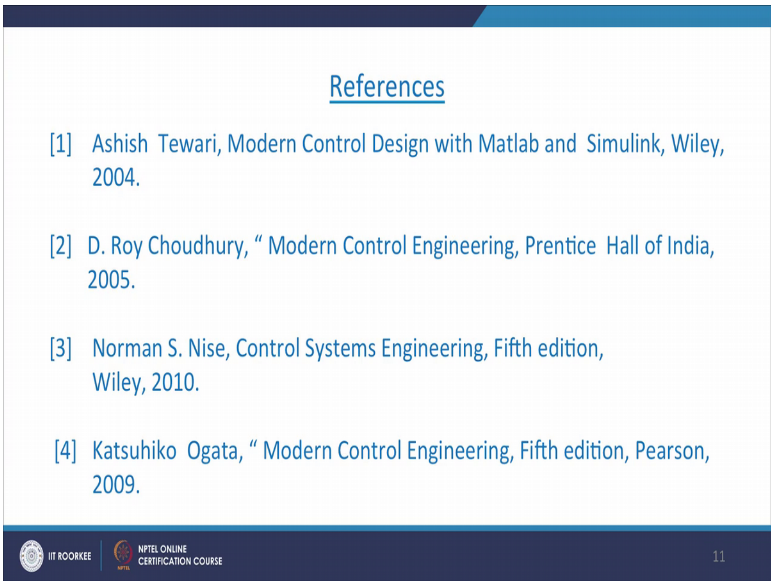

So, this is about various, various examples which I have taken for observable canonical form. So, you can also see some references, Ashish Tewari, D. Roy Choudhury, Norman Nise and Ogata.

Thank you.