Advanced Linear Continuous Control Systems
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Lecture - 40
State Observer Design (Part-III)

Today we start with State Observer Designing part 3. In this we will study introduction to minimum order observer and design of minimum order state observer matrix.

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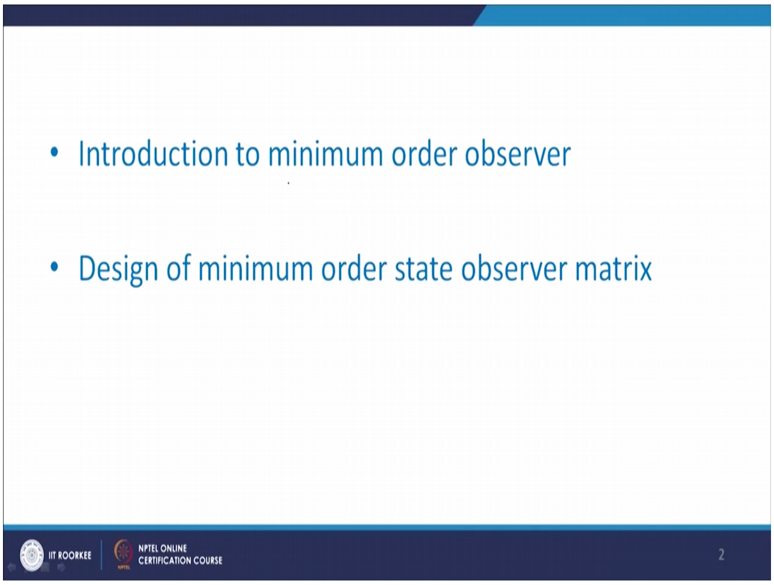

Now, about the introduction to minimum order observer, as we have seen that the state needs to be measured, but sometimes it is not possible to measure the states. So, in that case instead of going for unmeasured state we are estimating all the states of the system; that means, even though there are some states which can be measured which cannot be measured. So, what we are doing we are estimating all the states that is called full order observer. Whereas, in whereas some of the states which cannot measured that only estimating remaining states which we have measured we keep as it is then, that type of technique is called minimum order observer or we can say that type of design is called minimum order observer. That means, we had to estimate the states which are not possible to measure only.

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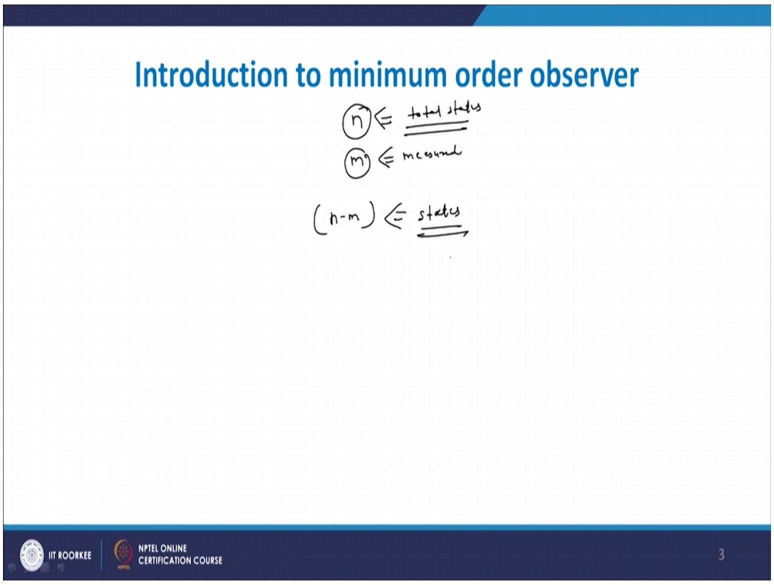

That means suppose this system has n state and there are say m states which are say measured and this is n that is called total states. So, n is called total states and m which can be measured states; that means we had to only estimates n minus m states. So, when we are estimating only n minus m states that is called minimum order observer.

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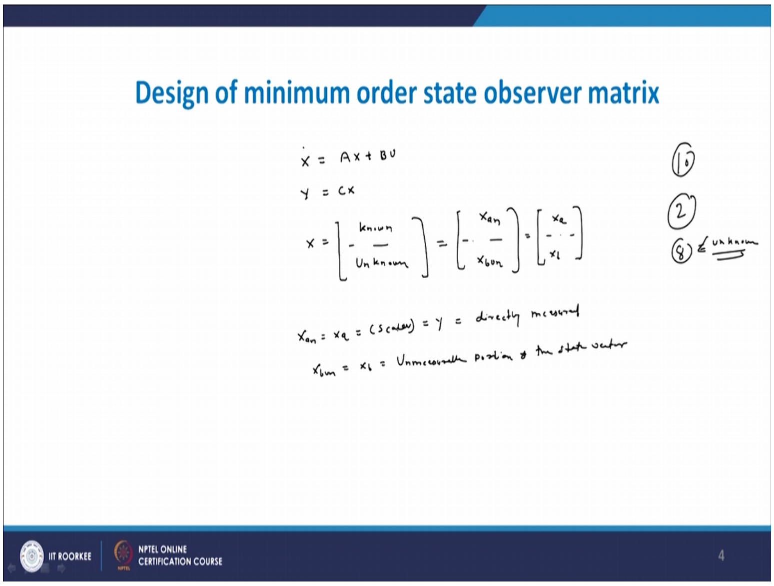

Now, we will see how to design this minimum order state observer matrix as here there are some states which had to keep at is, that is the states which we have we have already measured. That actually we have to do measurement and the states which cannot be measured that only to be estimated; that means, in a total states there must be a some segregations division. So; that means, if you see the mathematical part of it is somewhat complicated in comparison to the full order observer, although we derive it we find it simple, but the process is somewhat laborious.

So, now we start with the designing of minimum order observer. So, we have as per our original convention we have plant X dot equal to Ax plus Bu and Y equal to Cx, now in this case X X is the state. So, this X involve both the thing, one is measured as well as unmeasured therefore, we write this X involvement of known state, this X involved known state and also unknown state.

So, this can be represent as X an and X b un that is unknown. So, this is known and unknown; that means the state matrix has been divided into two, two parts known and unknown. So, now, this can be represent for simplicity say X a and say X b, now we had to do the analysis of the system. So, let us say there are say 10 states, we can say that the 2 states are known and 8 are say unknown, but now for simplicity purpose what we are doing we are assume that we knows only 1 state, that is 1 state is known and other state need to be estimated.

Therefore, we can write like this, this X an equal to X a is the scalar quantity equal to Y that is directly measured and is X b un equal to X b that is un measureable portion of state vector. So, it is measured, it is unmeasured where as we are taking a measure state only one state that is scalar. So, X a equal to Y which that is directly measured.

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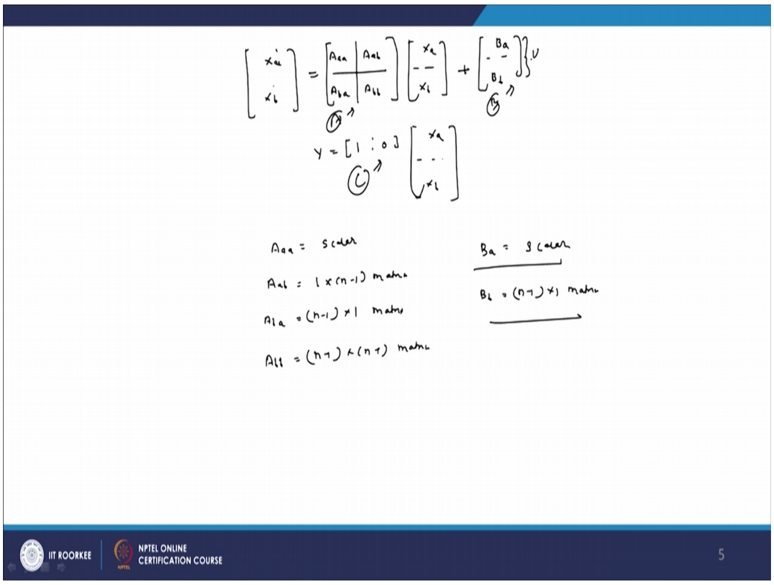

Now, we write this matrix as x a dot, x b dot that is equal to, now this matrix that is x dot equal to x. So, in x dot we have x a dot x b dot, where x a dot is measured, x b dot is unmeasured. So, now, this has to written in terms of matrix form that is a matrix can be written as A aa, A suffix ab, A suffix ba, A suffix bb. And now here state as x a, x b plus this is B a and B b into u and output Y can be written as 1, 0 and here x a and here is x b.

So, in this particular system, we have, what we have done we known Y equals 1 that is X a output and here U matrix, let it U matrix we have B, B has B a part and b part and and here these x a, x b and this is a matrix. Now, in this equation this aa is a scalar quantity and here A suffix ab this one. Now, this a quantity of 1 so it can be written as 1 multiplied by n minus matrix, that is 1 row n minus 1 column then A ba is written as n minus 1 into 1 matrix, then here A bb is written as n minus 1 multiplied by n minus 1 matrix.

And about this portion so this portion B a is a scalar, it is because we are assuming that X a involve 1 value, 1 quantity one only measure remaining are unmeasured. Therefore, B a written as scalar and here B suffix b is written as n minus 1 multiplied by 1 matrix. So, here dimension of all the elements in the given matrix A, B, C we have written; that means, here this is A this corresponds to B, and this corresponds to C. So, all the information we have written.

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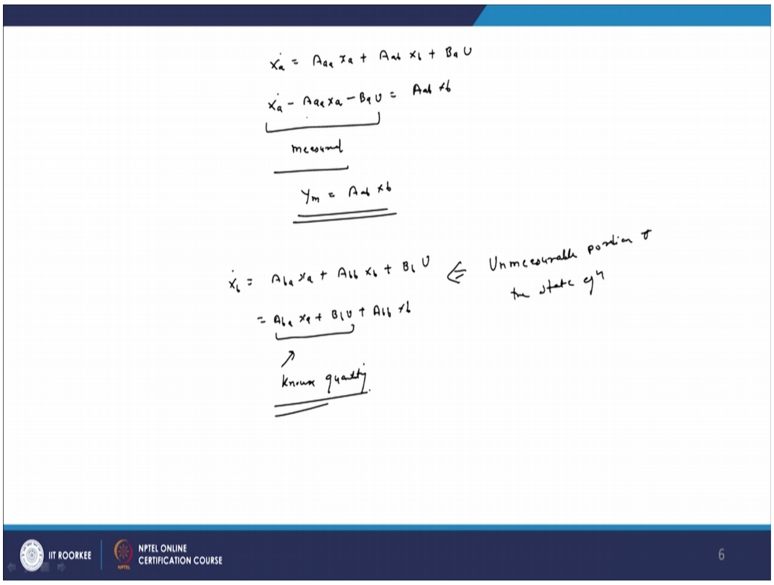

Now, from this part X a dot equal to A suffix aa, x a, a b x b plus B into u. So, that we can write down as X a dot equal to aa X a plus ab x b plus B a into U. So, now, this equations we have written, so up to look your after this equation. So, we observe that here this X a dot minus this A aa into x a minus B a into U into A a b into X b. So, what we have seen that in this particular equation this X a dot is there a a into X a and B a into U these are the known quantity. That means, in a given state space equations we have taken one particular state, which we known and from that state we observe that that this particular portions is a measure portion and this can be written as Y m equal to A ab into x b. That means, this shows the relationship between the measure part and unmeasured part.

So, this particular part we can use as a output equations, now from this equations X b dot equal to A ba into X a, A bb into X b plus b b the B suffix b into U that indicate the the unmeasured portion of the state equation. Therefore, we can write this as there is unmeasured portion can be written as X b dot equal to A ba into X a plus A bb into X b plus B b into U. Now here also will find at these A ba X a plus B b into U, A bb into X b. Now, these portion A ba X bb into U, that is the known quantity in the unmeasured state equations. That means, these particular portion or this particular part is called unmeasured, this portion is called un measureable portion of the state equation and this particular part is the known part, this is the known quantity.

So, now here we have written the equation for measurable state as well as the un measurable state. Now, we have to compare the full order observer state equations as well as the reduced order observer or we can say the minimum order observer state equations.

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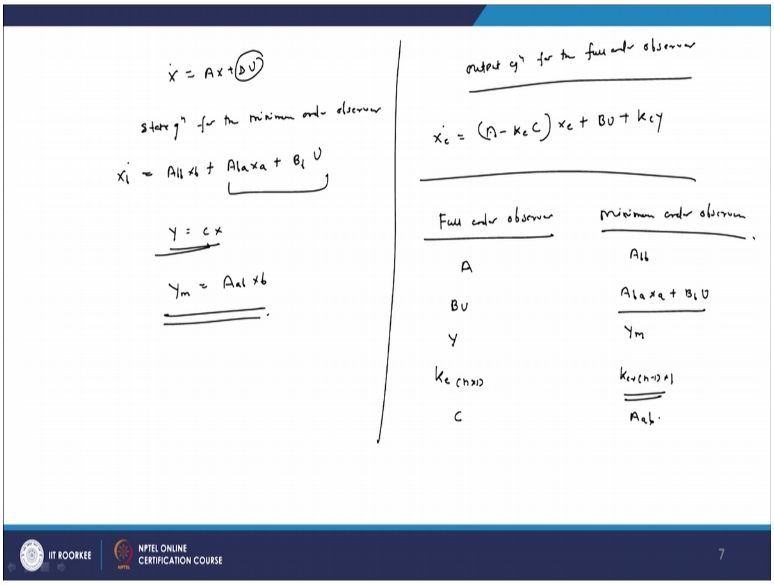

So, here we know that the full order observe equations can be written as a X dot equal to A x plus B. So, here we know that this X that is the state which we can directly estimated. So, all things are we know, now the state equation for the minimum order observer that is X b dot equal to A bb into X b plus A ba into X a plus B b into U and here is the known quantity. Now, the output equation for full order observer we can write down as Y equal to c x, but here the output equation for minimum order observer, it can be written as Y m equal to A ab into x b.

In last part we have derived the output equation of the full order observer. So, the output equation for the full order observer can be written as, that is output equation for the full order observer can be written as X e dot equal to A minus K e into C into X e plus Bu into k e into y. So, similarly, here also we have to written we have to write the output equation for the reduced order observer or we can say minimum order observer. Now, in order to get this we have to compare the elements of the full order observer as well as the minimum order observer.

Now, let us take the elements of A matrix. So, now, further purpose first of all we write a table full order observer and here minimum order observer. Now, in full order observer we have A matrix that is we have X dot equal to Ax plus Bu these are a matrix. Now, coming to the full order observer in a full order observer oh sorry, in in case of in case of minimum order observer we have X b dot. That means, corresponding to the X b dot this is A bb this is the state and this is the in terms of input because we are knowing. So, here for minimum order observer we write A suffix bb.

Now, coming to the B into U matrix that is x dot equal to ax plus B this is Bu. So, Bu in a full order observer it corresponds to here A ba into x a plus B suffix b into U. So, now, we have written for the state equations, now about the output, now in case of output in case of full order observer we have written as Y and where a minimum order observer we are written as Y m, Y m and this Y m equal to X a dot minus A a x a minus B into u. So, we can write down these as Y m.

Now, the in full order observer we have k e matrix and the order of this k e matrix is n cross 1 and here in case of minimum order observer we can write, write down as this k er n minus 1 cross 1 matrix. This k e and here k er and now about the C matrix, the C matrix in case of full order observer we can write down as C. Now here we find at this Y m output equal to A b into x b that is, but the c therefore, we can write down as this as A into ab..

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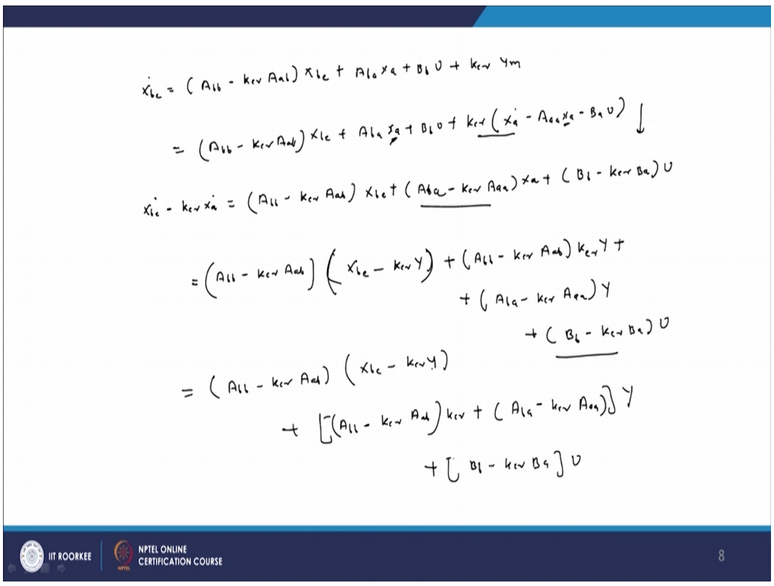

So, all these elements we have written, now, from this we write the output equation for minimum order observer as here (Refer Time: 17:47) here x b dot thus equal to A bb minus k e into r into a b into x be plus this b into u. So, b into u here A ba x a plus b b into u plus here instead of k e here we can write k e r into y this is as y m; that means, this can be written as A bb minus k e r A ab into x be here Aa ba into x a here B b into u and this here k e r this y m is x a dot minus a a x a minus b a into u.

So, now this equations we have solved, now we have to see the point now here we have got the state equation for the minimum order observer x b dot, but now here x a dot is there that is y equals to x a and in the state equation we get x a dot. That means noise is there it will amplify and that create problems therefore, we have to remove the effect of x a dot; that means, this your equation they should not be terms like x a dot. So, in order to handle this, so what we can do here to write the above equation as X b dot minus this part k into x a dot we can move the left side. So, we write k e r into x a dot now here A bb minus k e r into a b into X b e and here a b a now X a is common. So, here k e r into A aa into x a plus B b minus k e r B a into u.

So, what we have done in this equations that is from these to these, this k e r into X a dot we have move on the left side. So, X b dot dot minus k e r into X a dot in bracket this A bb k e r into a b into x x be that remains same what we have done here this x a is here, any x a, that we have to take common. So, we have taken x a common and the elements we have written as A ba minus k e r into a plus B b, B suffix b minus k e r this k e r into b into U. Now, again these equation need to be simplify. So, here we can write down as A b b minus k e r into A b. So, now, what we write, we have the equations state equation in terms of k b e dot minus k e r into x a dot therefore, in output we have to write, represent equation in terms of x b e minus k e r into x a. So, what we have done, A bb minus k e r into a b multiplied by X b e minus k e r into y plus A bb minus k e r into A into a b multiplied by k e r plus Y; that means, here A bb minus k e r into a b into x b e.

So, this equation is this one, this part and now this k e r minus ab we have to make return additional term minus k e r into y and now this term need to be relify. Therefore, here what we have written A bb minus k e r into a b that is k e r into Y plus, now this to be written, here A ba minus k e r into A aa into Y this X is Y plus B b k e r into B a into U, now this equation need to be simplified. So, in order to simplification, simplification what we write A bb minus k e r into A ab. Now, here X be minus ke r into Y plus, now here what will do we have to take this y common and therefore, what we can write this as A bb minus k e r into A ab to k e r plus A ba minus k e r into a a plus Y plus and now this portion will write like this B b minus k e r into B a into U.

That means after creating this we are coming with this particular step, now we again simplify it. So, in order to simplification what we write this x b e minus k e r Y write in terms of new variable.

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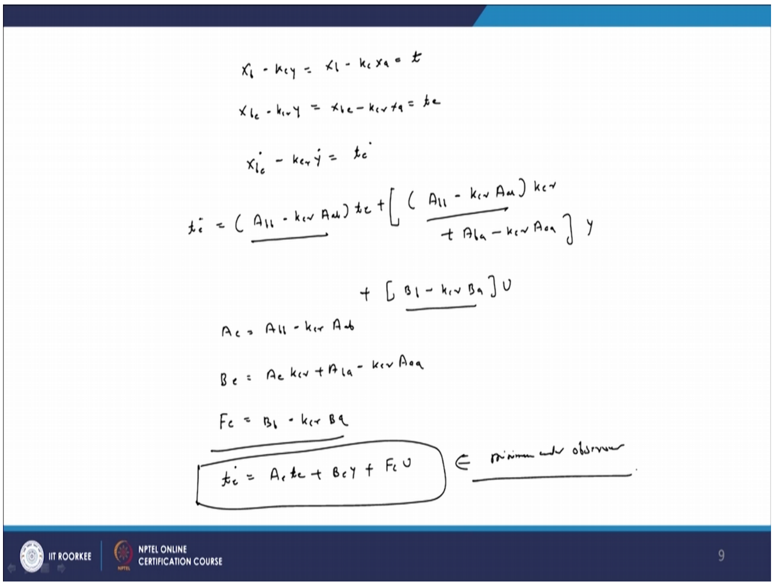

That is X b minus k e into y, we can write down as x b minus k e into x a let us say it is s t, but now in our equation we have k v e minus k e r into y equal to X be minus k e r into x a equal to t e and therefore, here x b dot minus k e r Y dot equal to t dot and therefore, this equation X b dot minus k k e r into x a dot that is t e dot equal to A bb minus k e r A ab into t e plus A bb minus k e r A ab into k e r plus A ba minus k e r into A suffix a into y plus B b minus k e r into B a into U.

So, now what we do? Now, again we simplify it just looking after this one we find at this working of the minimum order observer is somewhat complicated or mathematically laborious, but still as we have to do the analysis we have to process proceed further. So, there are this portion we can write down as A e equal to A bb minus k e r into a b and now here B e can be written as this portion, this is this part is a e into k e r plus a b a minus k e r into A suffix a and here F e can be written as this portion B b minus k er into

B a. And therefore, the final equation is t e dot equal to A e into t e plus B e into y and F e into U this is the equation for a minimum order observer.

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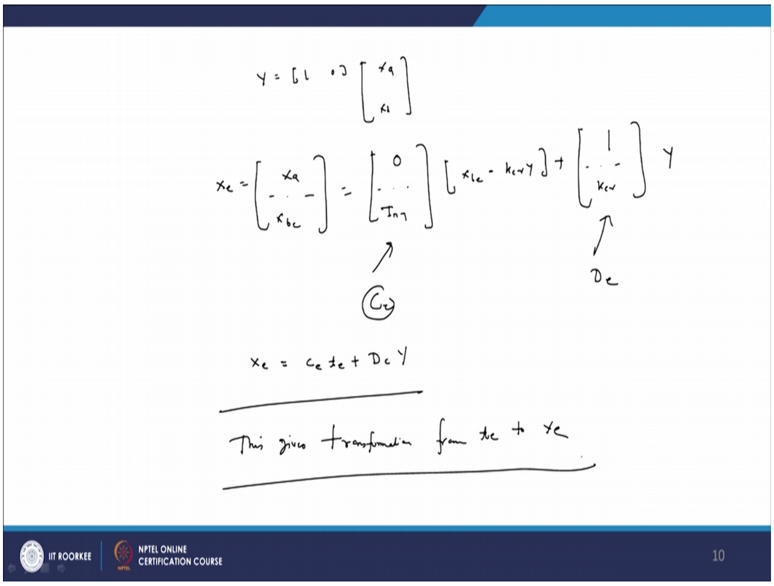

Now, about the output equation, for output equation this Y equal to 1, 0 this x a and x b and now here this x e can be written as. Now, here x a is in known and x b e is unknown and we have to write the this equation in terms of this state that is in terms of this t t e. Therefore, this equation can be written as 0 I n minus 1 here, x b e minus k e r y plus 1 by k e r into Y and here this portion above that is this part is represent as C e and this part is represent by D e. Therefore, we have this x e can be written as c e into t and here d e into y. So, this gives transformation from t e to x e.

So, now all the mathematical part we have derived. So, will find that there the in this case there is a a design of minimum order of observer then, there is a transformation and then the state will transfer to equal to U therefore, base on these we will now draw the block diagram of minimum order observer.

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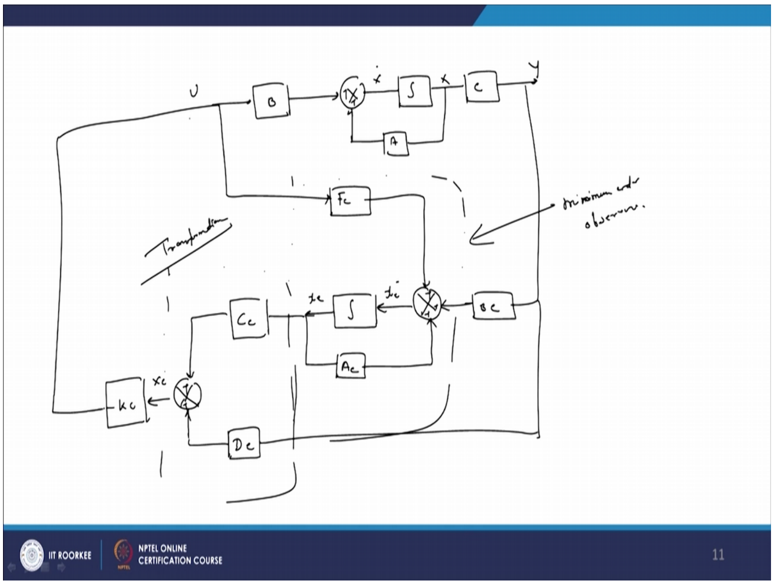

So, in this case this original plant this state space plant, now here is c here is output y. Now, here is matrix B and here is A matrix now, we have state x dot and here is x and now here is control input u. Now this is the original part.

Now, we have to draw the block diagram of a minimum order observer. So, in this case we have state which is coming that is here t e dot and here it is coming with insulator. Now, here is t e, now here the matrix A e and from this u we have matrix f e is coming here and here is a matrix b e and it is coming to output y and up toward there is a transformation that is in terms of C e and here is D e and from this will get x c and this nothing, but your k c and it goes to input u.

So, now this particular portion, we call as a transformation and this particular part this part is called minimum order observer. That means, here will find that this is original plant this is the minimum order observer and finally, we had to transfer to the controller the complete state there is a involvement, involvement of the measure state as well as unmeasured and therefore, this transformation is needed. And finally, the complete state transferred to x c which is involvement of our measured and non measured and this is the actual control k c and it will moves here.

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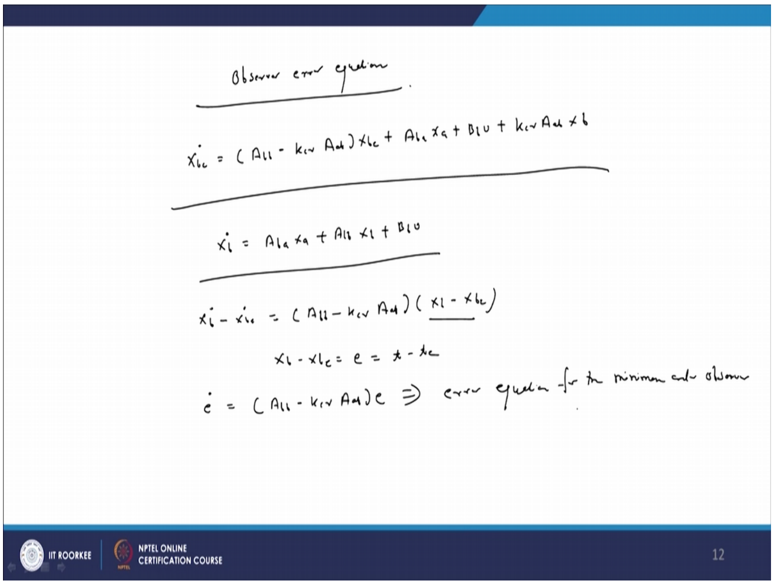

Now, about the observer equations, observer error equation now, here observer error equation is X b dot equal to A bb minus k e r into A ab into x b e plus A b a into x a here B b into u plus k e r into a b into x b. So, earlier we have seen this equation, now the now the original equation state equation for X b dot is given as A ba into x a plus A bb into x b plus B b into u. So, this is the unmeasured portion and is measured estimated portion. So, here we write this as x b dot minus x b dot. So, if you subtract this from this, what will get you will get the equation as A bb minus k er into A e b into x b minus x b e.

And now this X b x b e we can write down as a error equations and it is in terms of t minus t e therefore, this e dot equal to A bb minus k er into A b into e. So, this particular called the error equation for the minimum order observer.

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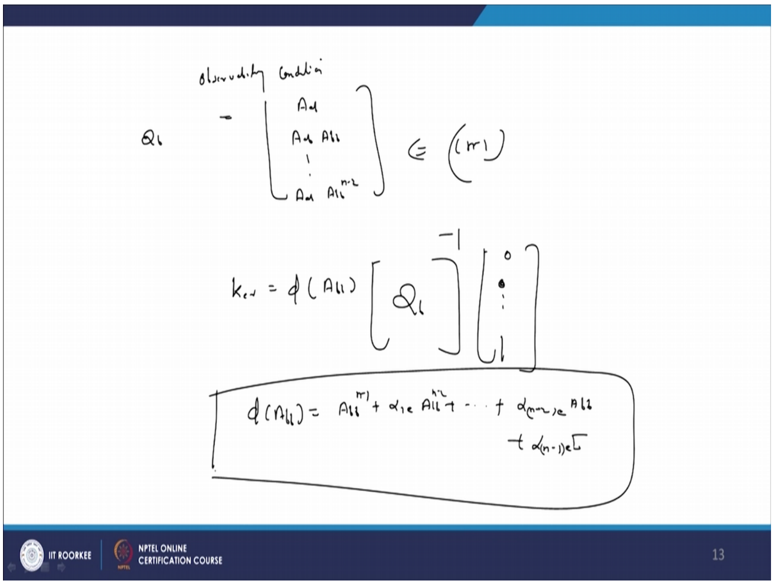

And from this we can get observability condition as that is, A ab A a b into A bb and this process repeated will get A ab into A bb into n minus 2 and it must have rank of n minus 1. If the rank is n minus one then observatory condition is satisfied and from this we can design controller this k er as phi A bb into this is a observability conditions Q b. So, this write Q b inverse and here we write down 0 0 1 and here phi A bb is equal to A bb n minus 1 alpha 1 e A bb n minus 2 plus alpha n minus 2 e into A b b plus alpha n minus 1 into I.

So, this k e r gives you the the reduce order controller. So, we have seen that the process for estimating the minimum order observer. So, full order observer as well as reduced order observer or we can see minimum order observer both can be applicable, but as far as the analysis part is concerned will find that the analysis of this minimum order observer it is quite laborious.

And main purpose is to means main purpose of all these thing is to design the controller, but now here the controller which we design base on the desired pole placement as well as the observer we are designed that is also the base on the on the some trial and error procedures. But we need a some techniques were we can get the optimal control design. So, that is the part of the l q r.

So, now till date whatever we have studied with that is concerned with the pole placement which is based on the trial error procedure, in future we can also try for the design of controller by means of optimal pole placement.

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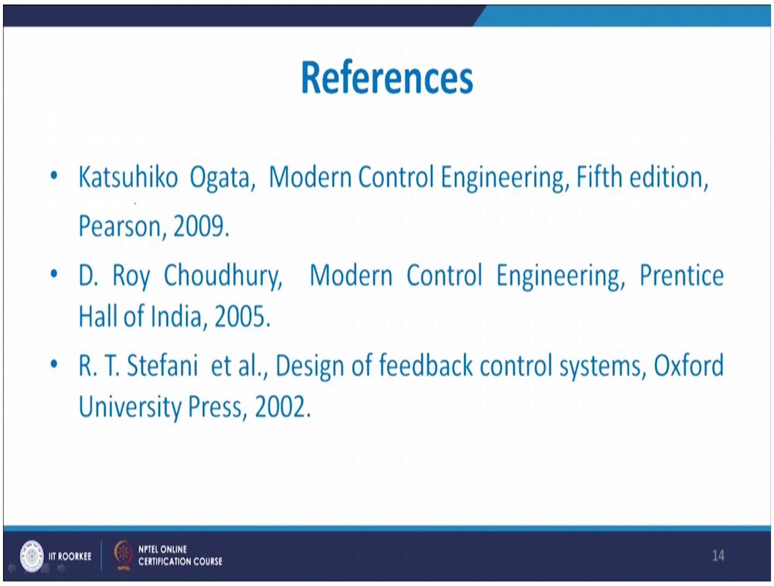

Now, these are some references Ogata, D Roy Choudhury, Stefani.

Thank you.