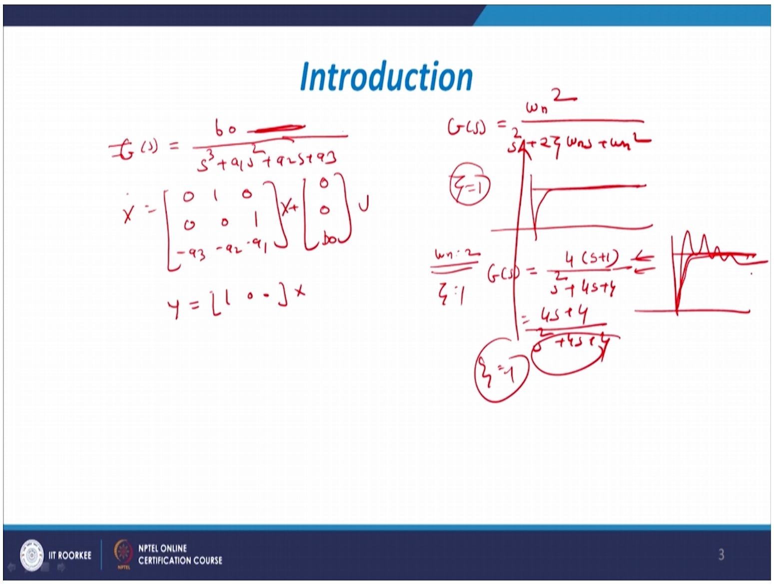
Advanced Linear Continuous Control Systems
Dr. Yogesh Vijay Hote
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture - 04
State Space Representation:
(Extended Contollable Canonical Form)

Today we start with Extended Controllable Canonical Form. In this we will study introduction, mathematical representation of extended controllable canonical form and numerical examples.

(Refer Slide Time: 00:44)



Last time what we have studied if you having G of S equal to b 0 upon S cube plus a 1 S square plus a 2 S plus a 3, this we have convert into controllable canonical form. So, what we have done that last time we have written X dot equals to 0 1 0 0 0 1 minus a 3 minus a 2 minus a 1 plus 0 0 1 U Y equal to this is b naught Y equal to 1 0 0 into X, this we have done last time. So, what is happened the b 0 is constant there is no another term involved or what we have done last time the derivative of input view we have considered to be 0. But now, today we have to considered the derivative of the input. That means, we have to consider the elements in this numerator.

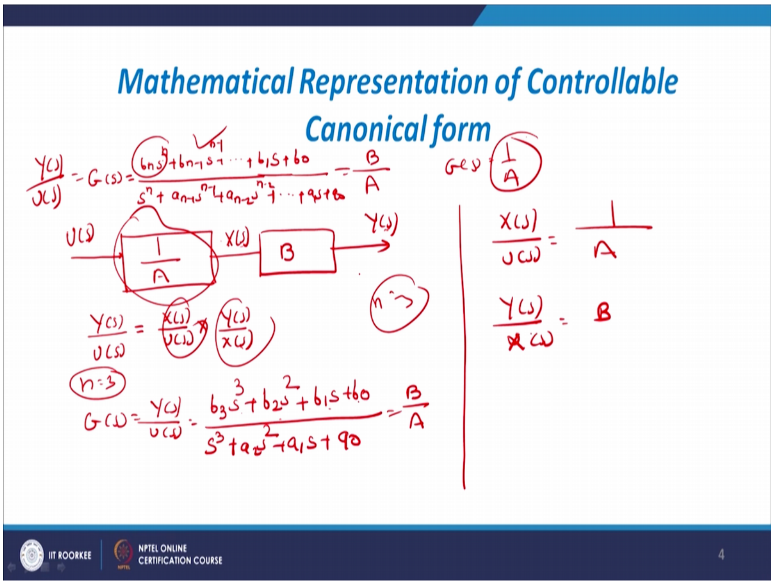
Now, question has come to our mind that if we add 0 or derivative of input is present can it affect our performance of a plant. For the purpose what we will do I will take one transfer function say G of S is a standard transfer functions omega n square omega n square plus S square plus twice zeta omega n S plus omega n square. This is the standard second order transfer functions, you are studied this in a classical control. In this case omega n is the (Refer Time: 02:34) frequency, zeta is a damping factor.

Normally, when we design any controller or when we check its response of a any system we are considering a different types of zetas. Suppose, if you consider zeta equals to 1 your 60 means critical damp. So, you will get response of this nature for example, if I will taking I will take here omega n equals 2.

So, what will get this G of S that is equals to omega is 4 S square and here zeta is 1. So, what you get 2 into 1 is 4 S plus 4 now, we will try to determine the response. So, you will find that for this system also you will get response like this you will get a critically damped response; there is no over shoot. But if you at 0 0 means let us say if added 4 S plus 1 here now, transfer function is becomes 4 S plus 4 by S square plus 4 S plus 4. So, in this case if you added 0 and normally when we compare we are comparing with this 1 this with this zeta is also 1.

So, what happen? We will not get this type of critically damped response, there may be certain over shoot or there will be change in the study state error. Therefore, 0 plus very important role in our control applications therefore, today what we have to do we have to derive the state space model of a plant when there are 0’s as poles both are involved. So, now, we write standard transfer function which we are written in the last time as universal view as a denominator.

(Refer Slide Time: 04:29)



So, we write as G of S equal to b n S n plus b of n minus 1 S raise to n minus 1 plus plus b 1 S plus b naught S raise to n plus a of n minus 1 S raise to n minus 1 plus a n minus 2 S raise to n minus 2, process repeated plus a 1 S plus a naught. So, in this case we will find that b 0 b 1 up to b 1 they are the coefficient of numerator and a 0 a 1 a of n minus 2 a n minus 1, they are the coefficient of the denominator. So, both numerator and denominators are involved.

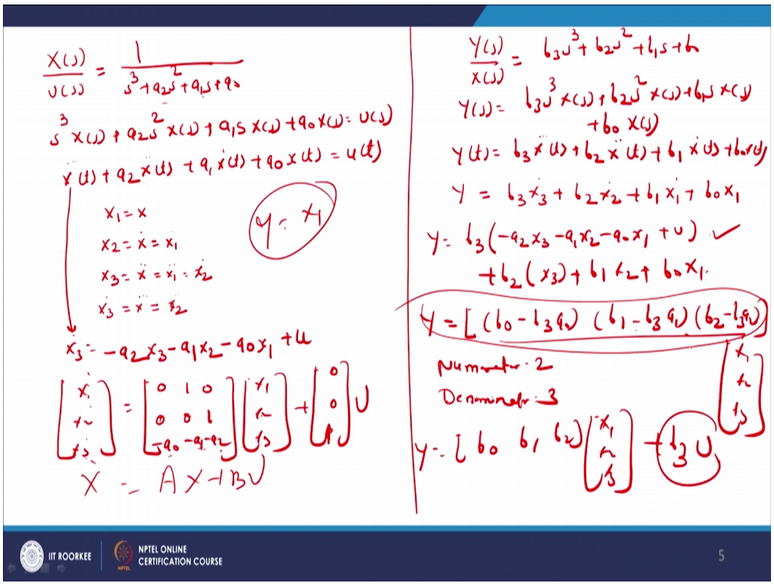
So, last time what you have done? Last time we have taken reduce model that is what you are done last time (Refer Time: 05:32) so, this is B and this is A. So, we not taken last time G S equals to 1 by A and we are determine model of this that is controllable canonical form of model, which we are determined for 1 by A 1 upon A. Now, here both B and A are involved. Now, how to do it?

So, what happened in this case we will use the concept which has being earlier; that means, whatever thing we have to do last time that will take into account and additionally we will considered the effect of this 0's or the numerator we can say. Therefore, we will write these transfer function as let us say here this is 1 by A, this is B and this is now, here U of S Y of S because your transfer function is actually Y S by U S. So, what we have written here we have written 1 by A B so, here result is Y S by U S B by A. So, what will do here this part will carried out by conventional approach that is last time which you are seen and here this thing we are added. So, here something root take here.

So, variable we have to assume therefore, what we will do we will assume the variable as X of S so, here we have assume variable X of S. So, now, this complete transfer function we can write down as Y S by U S equal to X S by U S multiplied by Y S by X S. So, if you see this will cancel it become Y S by U S. So, now, we will treat X S by Y S independently and Y S by X independently and we will determined the model that is controllable canonical form a model when 0's are present. But if you see take nth order sometimes difficult to understand. Therefore, for understanding purpose what I am doing here, I am taking only n equals to 3 n equals to order of the plant I will take as n equals to 3. Therefore, this transfer function we can write down as G of S equal to Y S by U S.

Now, here n is 3 so, we write b n 3 so, b sorry b 3 S cube see n is 2. So, 2 minus 1 n minus 1 that is 3 minus 1 b that is b 2 b 2 S square b 1 S plus b 0, this is numerator and your denominator is n is 3. So, S cube plus a n n is 3 3 minus 1 2; so a 2 S square plus a 1 S plus a naught. This is your transfer functions and now here we write this is B and this is A. And this can be final we write as from this logic X S by U S equal to 1 upon A and Y S by U S sorry Y S by X S equal to B. So, X S X of S by U of S is 1 by A and Y S by X S equal to B.

(Refer Slide Time: 09:37)



Now, we move further so, we will take the transfer function X S X of S by U of S equal to 1 upon S cube plus a 2 S square plus a 1 S plus a naught. Now, what we have to do here? Now, what will do we solve this equations will get S cube plus X of S plus a 2 S square into X of S plus a 1 S into X of S plus a 0 into X of S into U of S. So, here S cube X of S a 2 S square X of S a 1 X of S a 0 X of S and U of S. Now, what we will do? We will apply inverse Laplace transfer assuming initial conditions to be 0.

So, we will get what you get here we will get x triple dot t plus a 2 x double dot t plus a 1 x dot t plus a 0 into x x of t into u of t, you see here. Now, we use the approach of getting model which we was seen last time that is you have take the output as the first variable and remaining variable are the derivative of the output. Last time you seen that we have output y so, we have taken x 1 equal to y, but here we have not taken the variable as the x. So, last time what we have done we have taken variable as y equals to x 1. Now, here you have to take the variable as x 1 is equals to x that is the first variable and now what about the other variables they are the derivatives.

So, now, we write these x 2 equal to x dot and this x dot equals to x 1 dot. Now, what the other variable? So, as the order is 3 so, we got 3 more variables variable that is we write this x 3 x 3 equals to x double dot that is equals to x 1 double dot and that is equals to x 2 dot. Now, here we want x triple dot therefore, what we have doing you are taking x 3 dot now, here x 3 dot equal to x triple dot equal to x 2 double dot. Now, what we will do? We have to write this equation is such a manner that all this variable we have to adjust.

Now, here x triple dot here so, here x triple dot we write as x 3 dot you see that this one is write we had x 3 dot like this and you get as what we will get here, x 3 triple dot equal to minus a 2 and what is the x double dot x double dot is x 3. So, a 2 x 3 minus a 1 that is x dot x dot equals to x 2 a 1 x 2 minus a 0 and this is x t that is we can get x equals to x x 1 plus u. Sometimes you can try t here I have removed it so, that get simplified the results.

Now finally, we want its state space model. So, how you write the state space model? So, here we will take the variables as x 1 dot x 2 dot x 3 dot that is equal to while x 1 dot x 1 dot equals to x 2 that is 0 1 0. So, we write is as x 1 x 2 x 3, but x 2 dot x 2 dot equals to x 3. So, we write 0 0 1 and what about these x 3 dot so, x 3 dot we write as you just see here minus a 0 minus a 1 minus a 2 plus 0 0 0 1 equals to U. So, I have we have derived this one, but without derivation also we can write down the results.

What we have done? What we do in third row? We write a 0 a 1 minus a 2 and 0 0 1 only simple. Now, that we have done for a state model that is the state model is X dot equals to AX plus BU. Now, we want to write output equation. So, last time when the numerator is 1. So, what we have done? We have taken output Y equals to X 1. What we have done last time? Y equals to X 1 only, but here this is not like this because if you see here Y S by X S. So, Y S by X S there is even transfer function. So, the transfer function is b 3 S cube b 2 a square b 1 S b naught. Therefore, what we do we write I will take this side so, Y S by X of S that is equal to b 3 S cube plus b 2 S square plus b 1 S plus b naught. So, now, here what we do? We have to solve it.

So, we will get Y S equals to b 3 S cube into x of S b 2 S square into x of S b 1 S into x of S plus b 0 into x of S. Now, what we do here? We can apply again inverse Laplace transform assuming initial conditions to be 0. So, what we will write? We get Y of t b 3 S cube x of S that is x triple dot t plus b 2 x double dot t plus b 1 x dot t plus b 0 into x of t. So, we have already determined (Refer Time: 16:23) a x triple dot t x double dot t x dot t here now, we replace these values here. Now, we solve this equation we will get Y equal to b 3 here x 3 dot plus b 2 x 2 dot plus b 1 x 1 dot plus b 0 x 1, you can write t also there is no problem, but as we have to complete it as early as possible. Therefore, I am not removing this t terms ok.

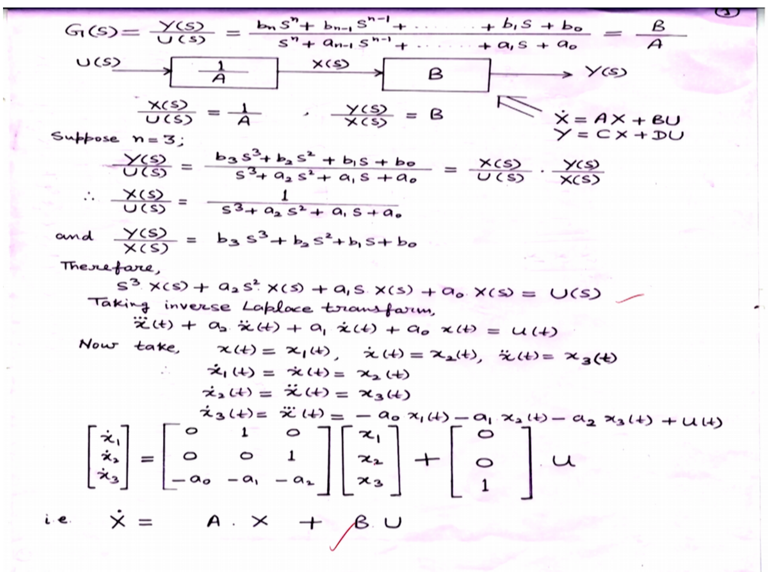
Now, what we do? We write here Y equals to b 3. Now, what is the x 3 dot, x 3 equals to a 2 x 3 a 1 x 2 a 0 x 1 plus u so we write down this equation that is minus a 2 x 3 minus a 1 x 2 minus a 0 x 1 plus u plus; this b 2 we can write down as what is x 2 dot your x 2 dot equals to x 3 plus b 1. What is x 1 dot, x 1 dot equals to x 2 plus b 0 b 0 into x 1. Now, we solve this we solve this equations and after solving what will get will get Y equals to will get as b naught minus b 3 into a naught b 1 minus b 3 into a 1. And after what will get b 2 minus b 3 into a 2 is bracket completed and is equals to x 1 x 2 x 3 plus b 3 into u.

So, after solving this we called this result term. So, we will find that in this output both the elements of numerator and denominator are involved. So, as we know b 0 b 1 b 2 these are the terms of the numerator and a 0 a 1 a 2 they are the terms of the denominator. So, both is are involved whereas, the earlier only Y equals to 1 0 0 is being involved, but assume that that is that your transfer function is strictly proper. Because, if you see earlier you we have taken S raise to n S raise to n that is this transfer function is a proper transfer function, but if the degree of numerator is lesser than degree of denominator that transfer function is called strictly proper transfer function.

And for control applications we always preferred strictly proper transfer function. If a transfer function is strictly proper transfer function, what will take the degree of numerator we take degree of numerator equals to 2 and degree of denominator equals to 3, this is numerator and denominator. So, what will happen in this case this b n so, b n here n is 3. So, b 3 is 0; that means, Y equals to b 0 b 1 b 2 x 1 x 2 x 3. So, we will find that when the transfer function involves no numerator sorry, we can say that when the transfer function is a strictly proper transfer functions in that case output it depends on only the numerator terms, whereas when transfer function is not strictly proper or we can say only proper transfer function you will find that the involvement of the denominator elements.

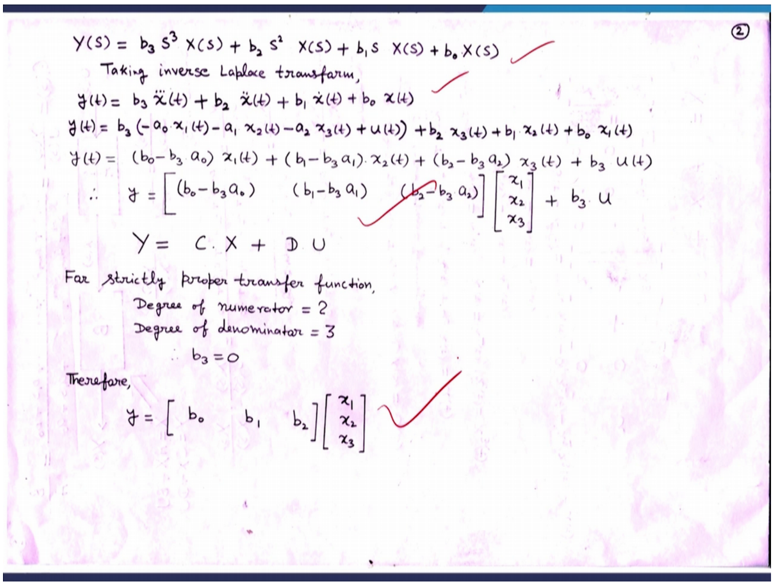
So, that is the deference between the strictly proper transfer function and a proper transfer function. Therefore, as far as the performers is conserved. Therefore, in that case we cannot sometimes we cannot get desired performers, if your given system is proper transfer function. We always prefer is strictly proper transfer function.

(Refer Slide Time: 20:51)



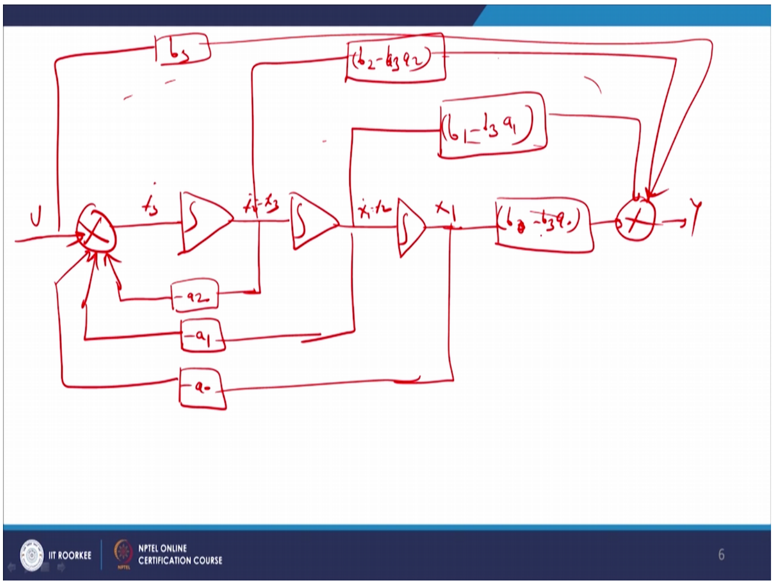
Now, we take one example and we will try to get the result ok, try out to this one what are you are studied again see it. So, here will find this a transfer function this at a 1 by A B, this is Y by U S you have taken then here X S by U S then Y S by X S you have taken. And then this is your solved then, we have got state space model X dot equals to A X plus B U.

(Refer Slide Time: 21:18)



Then we take this Y S, then we have got y of t and this y and this is a final model. So, these things will be definitely helpful to you. Prior to this one, what we will do? We will develop a model or a block diagram of the controllable canonical form that is when both numerator and denominator has been involved. Now, we will find that in this case order is 3. So, how many integrators are used? There are 3 integrators are used so, we will draw 3 integrators.

(Refer Slide Time: 21:54)

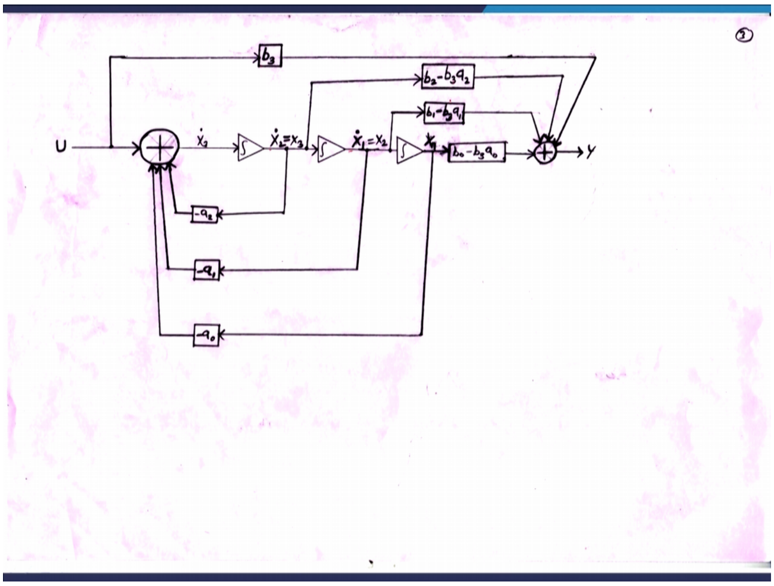


So, we have this is integrator 1, this is integrator 2 and this is integrator 3. So, we write variable as x 1, this is one integrator, this is another integrator and this is another integrator. So, here x 1 is one variable. So, how we got x 1 that is the integration x 1 dot because of the we got x 1 so, we write this as x 2 then here write x 2 dot equal to x 3 and there we write this as x 3 dot. Now, here another summation block and here this is. So, last time what we have seen you can see so, here the elements of a 2 a 1 a 0 has been involved that is particularly is concern with this x 3 dot equals to a 0 a 1 a 2 x 3 1 that is x 3 dot equals to a 0 x 1 a 1 x 2 and a 2 x 3.

So, this we have to show here. So, how will show it? So, here we write here this 1 as minus a 2 correct like this then we will write like this here minus a 2 minus a 1 and here we write minus a naught this is x 1 and this is U. So, this is the block diagram of only denominator, but here the numerator is involved. So, we have to write the output Y, this we have to write that is b 0 b 3 a 0 into x 1 b 1 b 3 a 1 into x 2 b 2 b 3 into x 3 here in this Simulink diagram. So, I will write down here the first part here x 1.

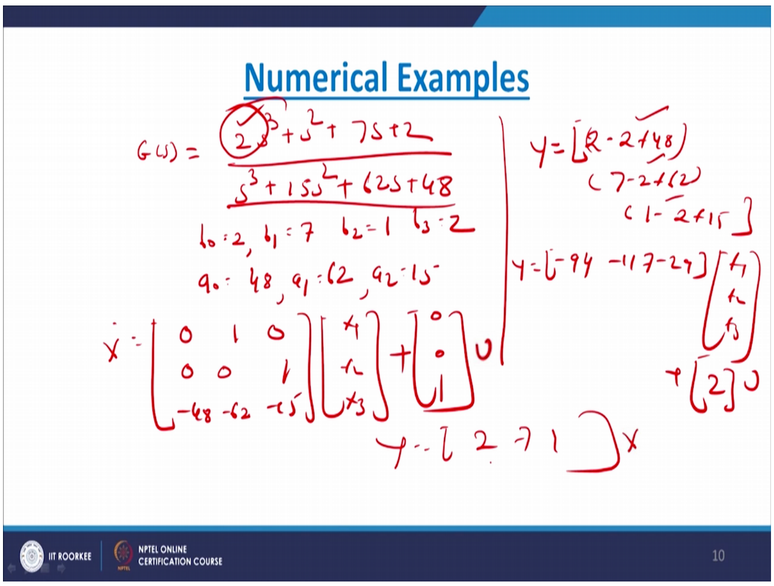
So, we write down here b 3 b 0 minus b 3 into a naught this is Y. Now, here another variable will come that is form x 1 dot equals to x 2 b 1 minus b 3 into a 1, we can connect here and another is x 2 dot b 2 minus b 3 into a 2 connect here and lastly we have from element U we have got b 3. So, we can connect here b 3 it will come and it will (Refer Time: 24:51). So, you will find that here that equations of Y plus here b 3 U so, all these portion these portion has been shown here. This is the portion this one, this portion this portion and this is U portion. So, this is Simulink diagram or block diagram and we can show it again see the again for clearly.

(Refer Slide Time: 25:17)



So, we will see (Refer Time: 25:17) here Y elements of X 1 X 1 dot X 2 dot X 3 dot. So, all the elements are shown a 2 a 1 a 0 U b 3 and all this has been shown. Now, we will take numerical example.

(Refer Slide Time: 25:31)

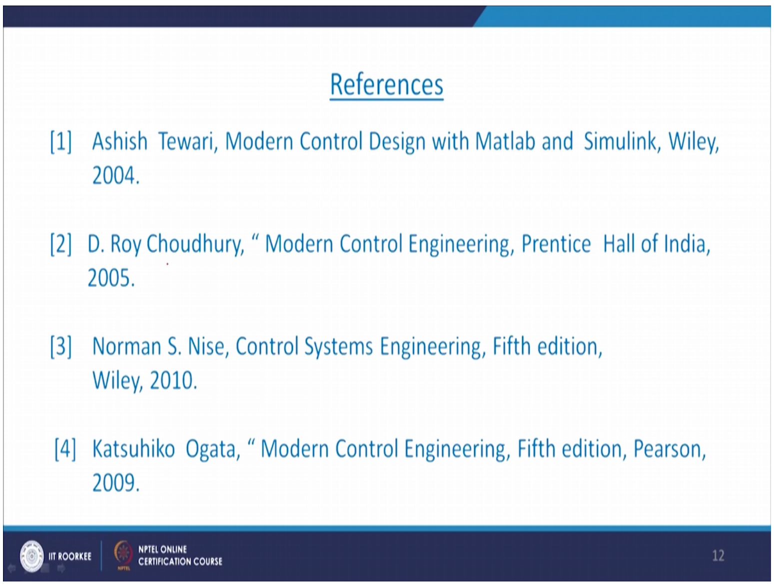


So, here we will take one example as G of S equal to 2 S cube plus S square plus 7 S plus 2 divided by S cube plus 15 S square plus 62 S plus 48 this is example. So, earlier what transfer function we have taken like this a 0 a 1 a 2 b 0 b 1 b 2 b 3 therefore, we write the elements like this. So, we have b 0 equal to 2 b 1 equal to 7 b 2 equals to 1 and b 3 equals to 2 and here we write a 0 equals to 48 a 1 equals to 62 a 2 equals to 15.

Now, we want to write down the state space model. So, how will write the state space model for this X dot equals to the write 0 1 0 and about this we write 0 0 1 and about the third row write minus 48 minus 62 minus 15 into x 1 x 2 plus x 3 plus we have to write here the other elements that is. Now, we first of all we will take care of the Y only. Now, in this case Y, What is Y? Y equal to write down, what is the Y formula for Y this here b 3 b 0 and b 3 into a 0 b 1 b 3 into a 1 b 2 b 3 into a 2. So, this we can write down here so, Y equals to 2 minus 2 into 48 second elements 7 minus 2 into 62 and third element we write at 1 minus 2 into 15; so first second third.

So, what we write Y will get as minus 94 minus 117 minus 29 and here this is x 1 x 2 x 3 plus b 0 is 2 this one 2. So, 2 into U and here we write 0 0 0 1. So this, we can say this state space model of a system in controllable canonical form and it is called extended because; here we have taken into taken care of the 0's. And suppose, if there is strictly proper transfer function and this part is not there you have simply write the output as 2 7 1 only.

(Refer Slide Time: 28:29)



Now, these are some references Ashish Tewari, then D. Roy Choudhury, Nise, Ogata.

Thank you.