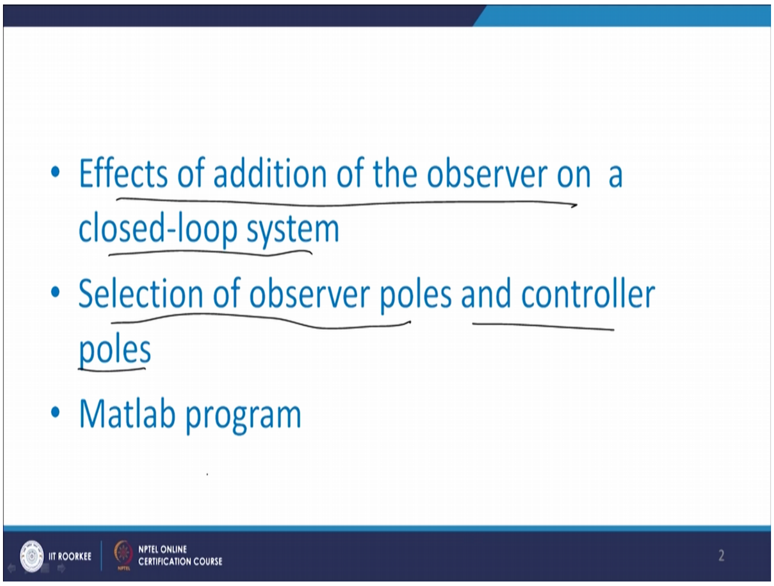
Advanced Linear Continuous Control Systems
Dr. Yogesh Vijay Hote
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture - 39
State Observer Design (Part-II)

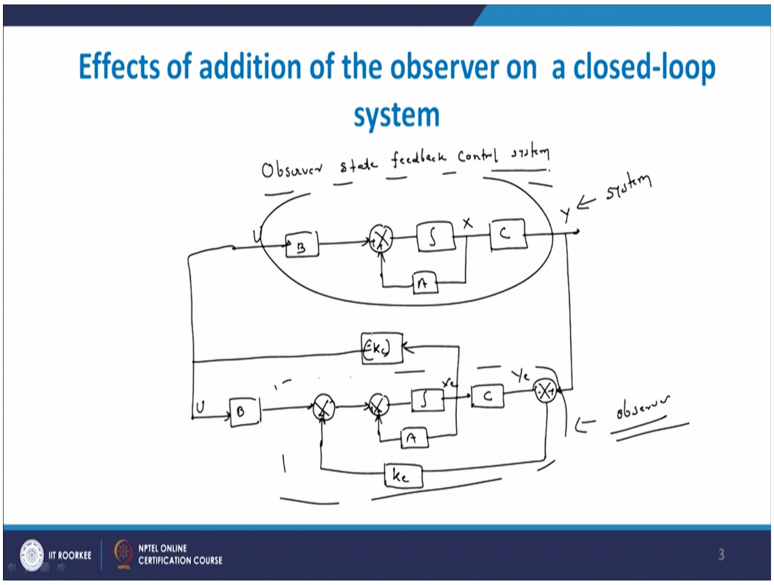
Today we start with State Observer Design Part-II.

(Refer Slide Time: 00:35)



In this we will study effects of addition of the observer on a closed loop system. Then selection of observer poles and controller poles and will see MATLAB programs.

(Refer Slide Time: 00:49)

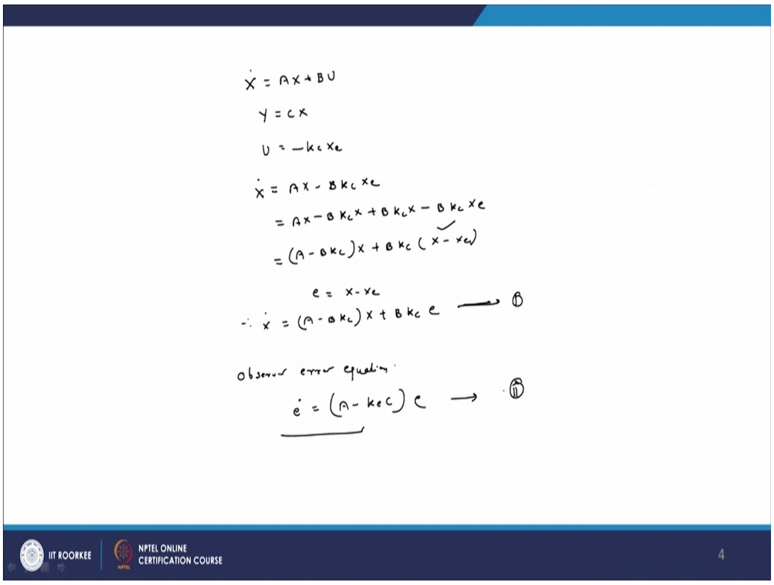


Now, about the effects of addition of observer on a closed loop system that means what is happened here that, we have a system and for the system we have a controller; along with this controller we have observer. So, we have observer controller so we have to see that that if you design a observer is it affected to the controller poles or vice versa, so that part we have to see.

So, first of all we draw the block diagram of a system with observer that is observer state feedback control system. Now here is the observer, here is A matrix, here observer control matrix. So, ke now here is the B matrix upon observer; now about the system this is C matrix, here is actual output Y here is A matrix again of a system. Now here is B matrix and now the U is common and here is state Xc and is controller is going for these state controller kc, and here is the output Y and this is input U and this is same U fine positive, here is positive C matrix. So, will find it the upper portion this portion you can see that this is your system this actual system and this portion is observer.

Now, the output of the observer system is Y and here is output Y and here we are reducing the error between Y. And there are 2 matrices or we can say 2 controller matrices k e and kc. This k e is for observer and kc we can say this kc is for our controller. Now we have to see the effect of this kc and ke are the kc and k e are related that is suppose if you change the k e can it effect the kc or vice versa.

(Refer Slide Time: 05:10)

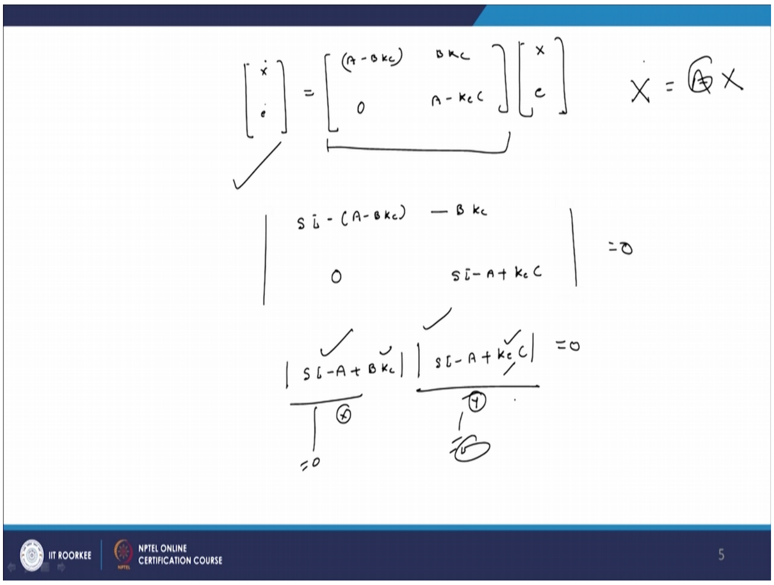


Now, we start deriving the result. So, here x dot equal to Ax plus Bu and Y equal to Cx, now here the control law your control law is now our control law is U equal to minus kc into x e. So, after this you to replace U equal to minus kc into x e in this equations, so you will get X dot equal to Ax minus Bkc into x e. Now here one factor we added Ax minus B kc into x plus B into kc into x minus B into kc into x e.

Now here a minus B kc into x plus B into kc common x minus x e; so you will find it x minus x e this factor this is nothing but error therefore, we write e equal to x minus x e. Therefore, will get x dot equal to A minus B kc into x plus B kc. So, this is the equations which we have obtained from the given observers state feedback block diagram, and base on the control law this results we have obtained.

Last time we have also determined the error equations, so the observer error equation that is equal to e dot equal to A minus k e C into e this is error equations. So now, we have dot got equation.

(Refer Slide Time: 07:38)



For x dot and we have got observer error equation e dot equation 2 now we combine 2 equations. So, we will get x dot e dot equal to A minus B kc Bk Bkc 0 as A minus k into C and here x and e. Now we have got a equation in terms of x dot and e dot. And now here is to determine the response or check the stability or to determine the Eigen values.

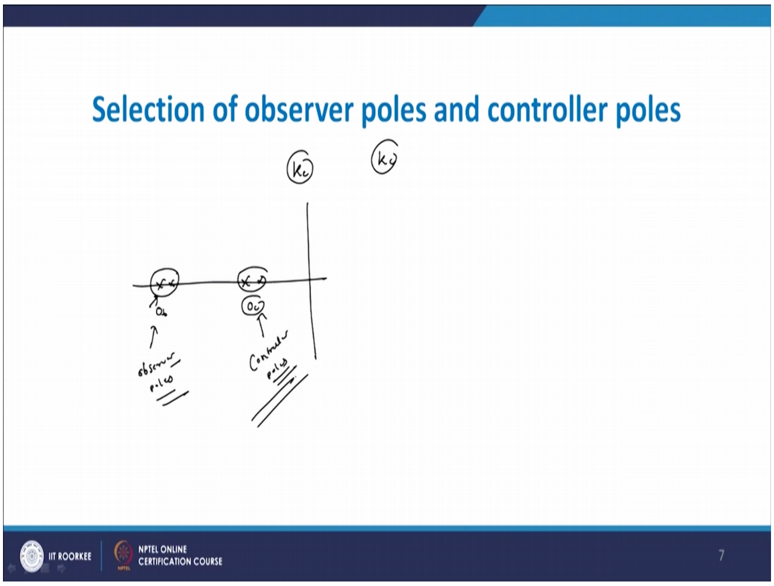
Now, we have to consider these equations, so this is similar to our equation let us say x dot equal to A x. Now here a is like this that means, here we have to determine the caustic equation for this system, the caustic equation for this system is given as SI minus A minus Bkc minus B into kc 0, SI minus A plus ke into C equal to 0. And now here in terms of determinant will get SI minus A plus B into kc, and here SI minus A plus ke into C.

So, after doing the calculations we found at the such type of equations we got and here will find at this particular first part let us say this first part let us say X and this is a Y part. So, x part consist of the Eigen values I with respect to kc that is a control kc and part Y which consist of a controller ke and as for as the Eigen values I of this is totally dependent, that mean this can be equal to 0 or this can be equals to 0. That means, the both these equations individually if you take they are independent.

Therefore will find at whatever changes you can make in this kc, it is not effecting the k e and similarly whatever changes you can make in the k e it will not affect our k c; that means, whatever poles we design that is whatever control controller poles we have fix and whatever with observer poles we fix, both are independent they are not dependent to each other.

Therefore, there is no any such issue while designing the controller that is the most important advantages of the advantage of this type of controller design. Now, about the next part selection of observer poles and controller poles; just now we have seen that the observer control matrix that is say k e and control matrix kc both are independent.

(Refer Slide Time: 11:18)



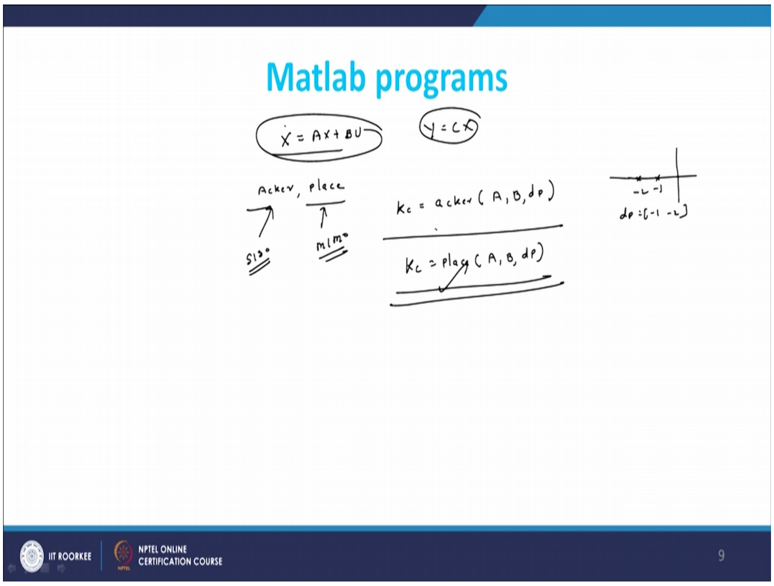
That means, you can place poles control poles anywhere in the expense similarly you can place the observer poles anywhere in this pen. But still is there need for any selection is there any selection issue related to it. Generally, it is say that you are observer pole should be 2 to 5 times far away from the control poles, that means it has been said that let us say I am taking 2 poles.

Now here Ob this is nothing but the observer poles, and now here these poles let us say Oc. So, Oc indicate you have a controller poles and here these are observer poles. That means, as we are selecting observer poles far away from the imaginary axis and in terms of magnitude they are greater than controller poles. So, these poles directly reached or easily reached to steady state in comparison to the control poles. And therefore overall system if you can consider it is dominated by the control poles.

So, generally we are following these type of things, but sometimes there can be issue that that is there can be a problem with a sensor that output can be noisy. Then, if the output is noisy it means that there is a issues with respect to the measurement to in that case. If you take observe pole very far away, so it may amplify and therefore our observation is not proper. So, in that case what we are doing we are keeping the observer pole near to the imaginary axis as compared to the control poles. Therefore, in that case your response is dominated by the observe poles only.

So, as far as added system is there you have to try is differentiate of value and see the which gives the better results and finally the that means which to be taken that is that is the tradeoff between the your speed of response as well as the effect of noise or disturbance, now will see some MATLAB programs with respect to this controller poles as well as the observer poles.

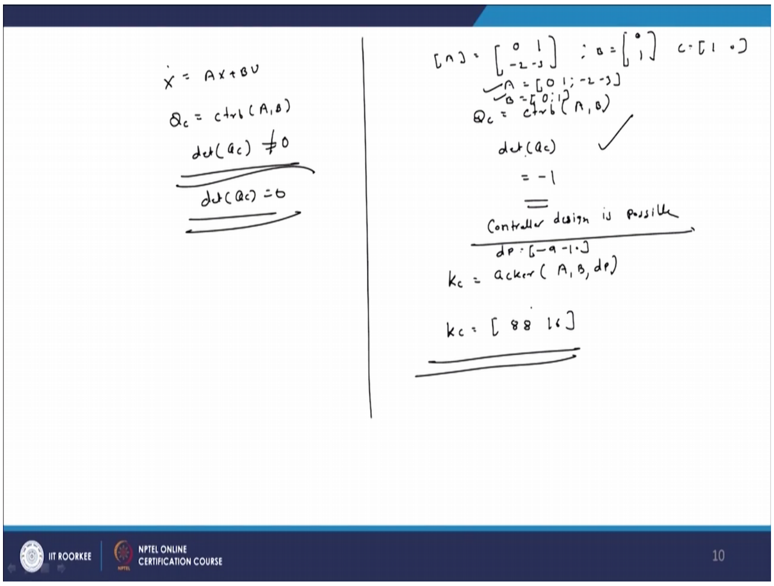
(Refer Slide Time: 14:04)



Now, our purpose main purpose is to design a controller using a MATLAB; so x dot equal to Ax plus Bu now this is planta and Y equal to Cx. Now when we are designing state feedback controller, so in that state feedback control design we are concerned with the controller that controller means because, of this controller we can place the pole anywhere in expense. So, in MATLAB there are 2 commands: one command that is called Acker and place with the both the command can be use for designing of the controller, but for SISO system single input single output we are going for Acker that is for SISO. And if the system of MIMO we had to go to the place command and the general format for this controller design is kc equal to Acker A poles sorry A matrix B matrix and d p and here d p is the desired pole; that means, at least location view all the poles.

For example, if you want the poles at this point say minus 1 minus 2, so we can say that you d p is minus 1 minus 2. And if use this command directly you will get the control matrix that is of single type or single command is sufficient to get the controller matrix. Similarly, you can get the controller by using place, so A comma B comma d p. So, you will get the controller using the place command, but for MIMO definitely we go for place and for SISO we had to go for Acker command. But when we are design the controller for any system, the first step is to check the controllability of the system.

(Refer Slide Time: 16:23)



Therefore for any plant Ax plus BU to write a command as controller matrix or control ability matrix as ctrb A comma B and we had to determine the determinant of Qc. If the determinant of Qc that means, this Qc gives you the matrix that is controllability matrix and then we have to take the determinant. So, command that use d t Qc so if it is non zero, then we can say that control design is possible and then you have to use the command Acker or place.

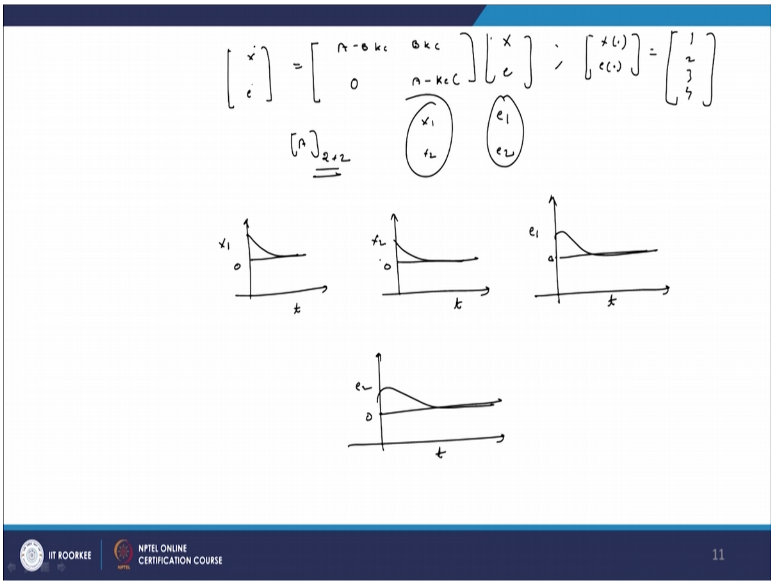
If this determinant of kc is equal to 0, it means that controller design is not possible and therefore we will stop here itself. Suppose if you take a systematic as 0 1 minus 2 minus

3 B matrix as 0 1 and C matrix as 1 0; now the now the first step we had to check the controllability of the system, so controllability Qc call to ctrb A comma B. So, first of all here we have define A that is 0 1 minus 2 minus 3 and here we have defined the B as 0 1 and the upper a and B you have to use command Qc equal to ctrb A comma B, we have to write in determinant of Qc and it will get as minus 1 and therefore we can say that controller design is possible.

Now, about the controller kc the command is Acker A comma B comma d p. Now here d p is the pole placement, so prior to this 1 prior to this 1 we have to use the command d p as minus 9 minus 10 this is the desired pole placement and therefore you will get the controller kc equal to 88 16.

So, in this way we have designed a controller for state space model and then the model is SISO system, so first of all what we have written we have check the controllability of the system and then system is controllable. And then, we have used the command kc equal to Acker A comma B comma d p and this is the controller which we have got.

(Refer Slide Time: 19:44)



Now, we have already seen 1 equation there is x dot e dot equal to A minus B kc and here B kc 0, A minus k into C and here x e and there is a condition that is x 0 and e 0 let us say a condition any value let us say 1 2 3 4. Now here our purpose is to determine the state response as well as the error response and what is our basic idea is that about the states state will start and it should quickly reach to the origin or 0. Similarly error is there error between the actual state and the estimated state that must be equal must be reach to 0 quickly.

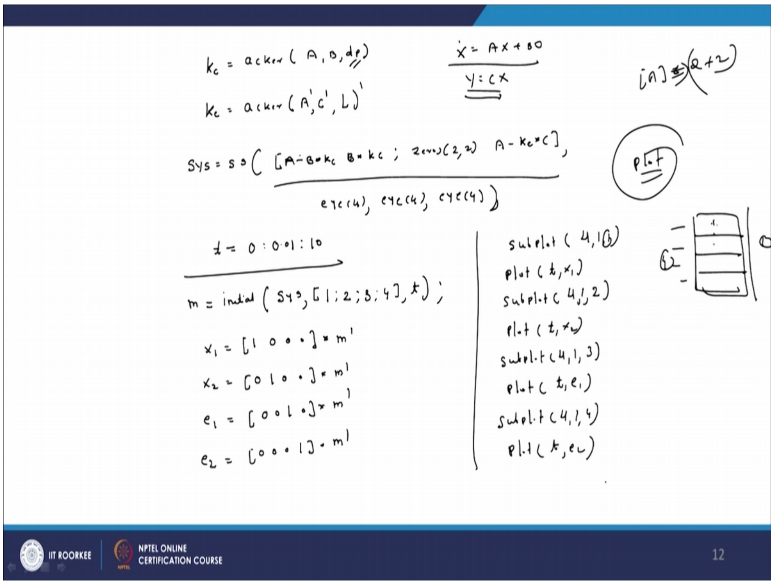
So, this is our aim. And therefore we have to determine them the response of the all states as well as the error response. Now we assume that this is our system this is involvement error equation as well as the system equation; whereas, the involvement of we have a controller as well as the observer and here we have to develop the MATLAB program.

Now, we assume that this A matrix is 2 cross 2 assume that this A is the matrix 2 cross 2 matrix. So, A is 2 cross 2 matrix it means that there are 2 states; that means, x 1 x 2 and here error is also e 1 and e 2; that means, here we have to determine the response of state x 1 with respect to t, state x 2 with respect to t and we have error e 1 with respect to t and here error e 2 with respect to t and what we want at all these errors all the state response should reach to 0 quickly.

That means, let us say this 0 that means whatever response start it will move like this similarly for here similarly for here and similarly for here also. That means, this is the origin this is the where requirement and for that purpose means whether we are getting this similar this type of response are not we have to do the MATLAB program.

So, when we had doing the MATLAB program for such state of things, here we require the 2 controllers 1 is the controller of the actual system and other controller because of the observer poles. And therefore the first part we had to write down the kc a command Acker A matrix B matrix and this is a d p.

(Refer Slide Time: 23:11)



So, d p means the actual pole placement of the controller A is the system matrix that is it is of x dot equal to Ax plus BU. So, A is the system matrix B input matrix and here d p is that design poles, now after these we want the observer controller. So, observer controller matrix that is k e we have to use the same command Acker acker, we have A transpose that is A dash C transpose means C dash. And let us say L transpose that is L complete system transpose, that gives you the observer controller matrix that is in that case C is Y equal to Cx. So, as further this equation about the observer and control equation we have got kc and we have got the k e.

Now, we have to represent the complete system this system into the state space form and therefore here the command we have to use sys equal to SS A minus B multiplied by star kc B star multiplication of kc; then 0 2 comma 2 and here a minus k e multiplied by C C. So, here these particular part is nothing, but this equation that is a minus B into kc B into kc 0 a minus k into C that is a complete matrix system. But there are B C d matrix they are represent by eye I 4 eye I 4 and this is eye I 4 and here the bracket completed and this complete bracket.

So, this particular part gives you the given system into the state space form, so I 4 it means that you are getting the identity matrix; that means, you are get the element in the in the diagonal form. So, here after we have to define a time assume that t equal to 0 0.01 say 10. Assume that we have taken the t 10 here. If I we have taken 2 by 2 because, we assume that here a is 2 cross 2 matrix A is belong to 2 cross 2 matrix and I 4 4 is variable is we have written because here x involve 2 2 state variable error the 2; therefore, we for complete matrix we have 4 cross 4, so in order get this we have written eye I 4 I 4 and I 4.

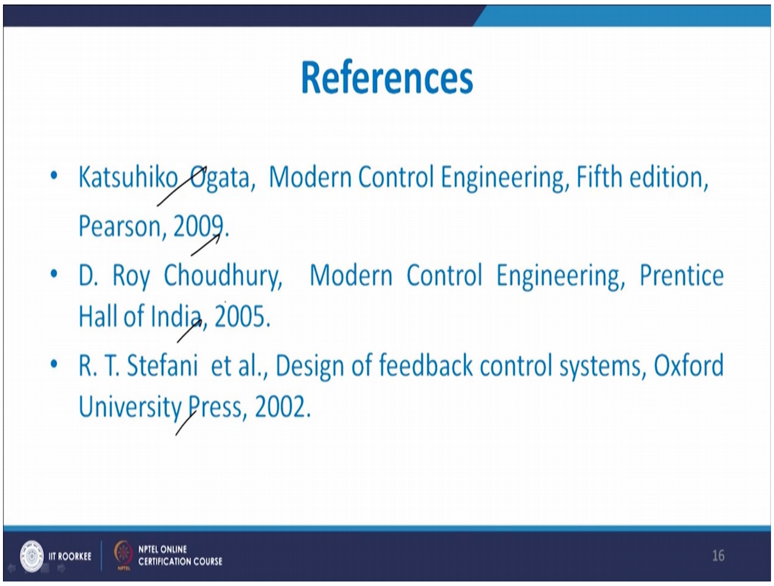
Now, here t has been defined now in order to get this response let us say m is equal to initial this is the initial command, now this sys now the initial condition. So, here initial condition we have taken 1 2 3 4, so we written 1 2 3 4 t bracket complete. So, this is the initial command where it shows the response of complete plant, but now our purpose is to get the response of individual system that individual state as well as individual error; therefore, write this x 1 is equal to 1 0 0 0 into m transpose m dash.

So, that gives you the response of state x 1 and if you write x 2 0 1 0 0 this gives response of state s 2 and then error e 1 equal to 0 0 1 0 thus m dash that is the response, because of the error e 1 then e 2 is the 0 0 0 1 to m dash this is the response of the error e 2, so all the 4 response s 1 s 2 e 1 e 2 we have shown. So, this is the result in terms of numerical we have got, but still we have not got any graphical picture; that means, we have to show all these result in a graph. That means, the command to be used to get result that is plot that is plot command is to be use.

So, here there are 4 responses, so these all is 4 responses we can draw on a single plot or we can take all these 4 plot on a single graph or single window we can show all these or individual windows can be taken. But now here we want the all these results in 1 window or 1 graph therefore, here we had to use the command sub plot. Therefore, here the command to be used sub plot 4 1 1, so sub plot 4 1 1 indicates that there are 4 rows; that means, these are 4 rows 1 2 3 4 1 this is a column. So, 4 rows 1 column and this 1 is the first positions. So, where will get the graph therefore, we write the command plot t comma x 1. Now we have got this response of this x 1 now about x 2, so here we have to write the command sub plot 4 rows 1 column and this position is second, so we write 4 1 2 plot t comma x 2.

Similarly, for error e 1 e 2 we can write down as sub plot 4 1 3 plot t comma e 1 then here we can write command sub plot 4 1 4 plot t comma e 2. So, using these concept you can get the response of all state as well as the error. So, all these step of programming is very much essential when we are doing the complete analysis of the system, it is because even though there is a error if we error is not coming to origin or these a error it take lot of time to reach the steady state. That means, that we do not require therefore this type of analysis using MATLAB is very much required.

(Refer Slide Time: 30:54)



Now, you see these are some references: there is Ogata, D, Roy Choudhury and Stefani.

Thank you.