Advanced Linear Continuous Control Systems
Dr. Yogesh Vijay Hote
Department of Electrical Engineering
Indian Institute of Technology Roorkee

Lecture - 38
State Observer Design (Part –I)

Now, today we start with State Observer Design.

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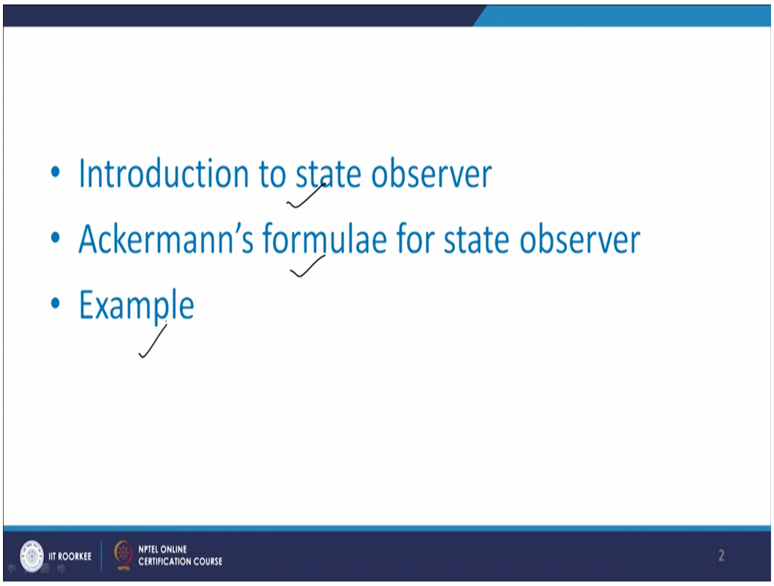

In this we will study introduction to state observer, Ackermann's formula for state observer and we will see one numerical example. Now, about the introduction to state observer why there is need for observer; first of all to see the state space block diagram of a system.

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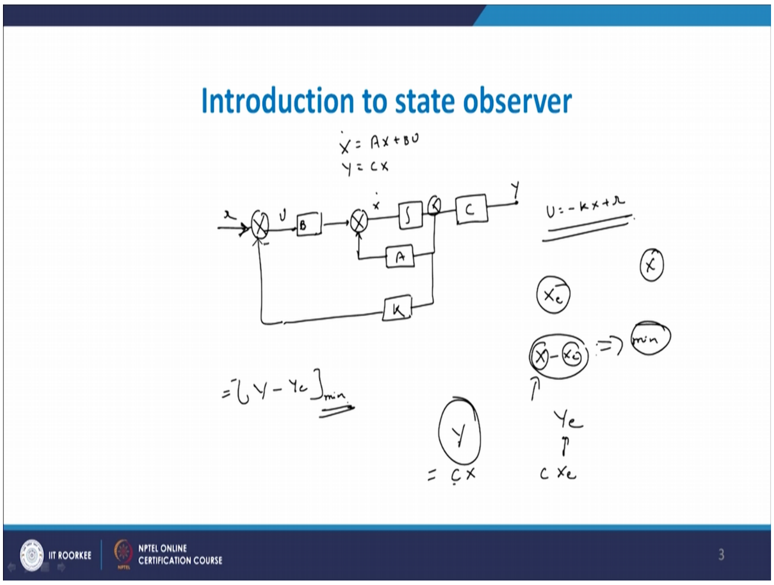

Let us say X dot equal to A x plus B u this is your system; now here say an integrator, now here is our C matrix and here is output y here is X dot, now here is B and input U and here is matrix A. So, here X dot equal to A x plus B u and y equal to CX. Now, when we talking about the controller design, so controller design basically is state space is with respect to state, now here this is a state x here.

Now, this state has to control, so when we when will we control the state when it is possible to measured it may possible that the state s cannot be measurable; that means, let us say here kx is there and now we are designing controller through this x. So, this is arrangement of the controller that means, here this U equal to kx U equal to minus kx plus r this is the control law. But here it is not possible to measure the state x then how the controller will design? So, that is the problem in a state space analysis because all states need to be measured.

So, in that case there is need to estimate the states, that means we have to assume or somehow we should know that states. For example, assuming some problem we occurred to us and we are going to doctor as doctor, ask some information and base on this medicine is given and if you not cured then doctor will say you go to higher center. But assume that it is not possible to go higher center; that means, whatever the internal thing in our body let us see urine, blood everything in will be check. So, that can be check by a some assumptions, so sometimes what doctor will take doctor will take your hand and check the nerves like this and see the condition of the patients. So, that checking of this checking of nerves here it means that we doctor is estimating the states.

So, here also there is need to estimate the state, so then how to estimate the state? So, the estimation of the state it definitely it depends upon the elements of AB and we have input U. So, assume that we have estimate the state X e this is the estimated state and X is the original state, so X e is than estimated state and X is the original state what we want that whatever state will estimate it should be nearer to the original state; that means, what we want the difference between X minus X e it should be minimum. But how will you know that the different between X and X e minimum, because this X this cannot be estimated that is why we are determine X e.

Now, the here the option is that we are knowing the output, so here we are knowing the output of a system although states we do not know, but we know output. Similarly we know the output of an estimated state or we can we know the output of this estimated state with respect to X e. Let us say output is Y and what is the Y? Y equals to C into X and this Y e equal to C into X e, that means here C and C is common and Y is with respect to X and this Y e is also with respect to X e. It means that it is better to compare output Y as well as the Y e, if the error between actual output and the estimated output if it is minimum; it means that whatever state we are estimated that is proper and properly we can design the controller.

It means that here the difference between Y and Y e it must be minimum. Now this is our problem now; now how will you make all these arrangement. Now in order to understand everything we need one device which can do the estimation of this or which can do the correction between Y and Y e, so that is called observer.

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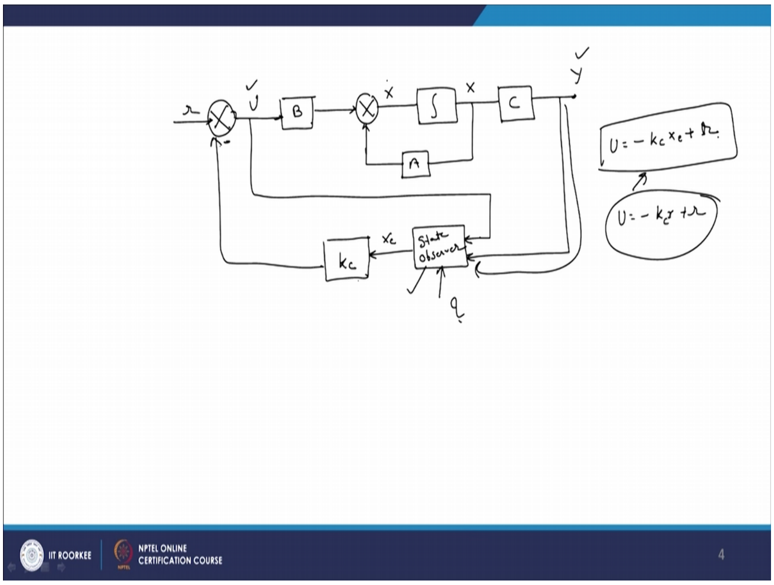

Now, we will draw the diagram for this observer say a original plant is like this is C and here is output Y here is X dot here is X, now here is matrix A, now here is B, now here is U and now here is the reference r. So, this is the block diagram base on the state space model, now here this state X has to be estimated; but at X we cannot know, but what information we have? We have input U this input is we know and as well as output y is knowing. That means, whatever estimator or a device which estimate the state it depends upon this input U and output Y, therefore what we are doing here we are taking an observer that is called state observer, which estimate the states and now it depends upon this is the output and here is the input, which is coming and for this.

So, base on the information of input U and output Y now here this is the state X e and here is our controller kc and system is completed. So, now here will find that that there is a controller kc which is a controller which is useful for stabilizations along with these there is called observer that is called state observer, which is which estimates the states of the system. Now, this produces X e and it depends upon input U and output y, now what is present inside this state observer.

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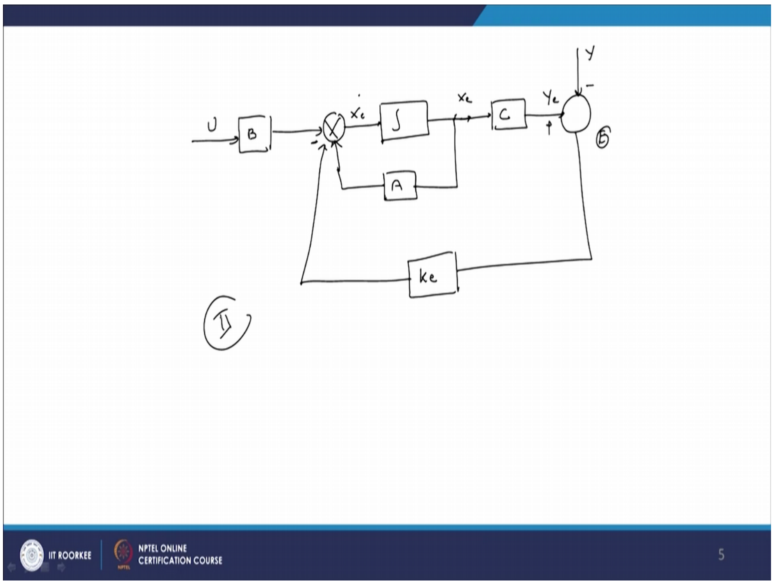

Now, will draw the block diagram of this state observer, now here is estimated state X e dot and now here is X e; now here B input U here is A matrix, now here is C matrix and as state X e is the estimated state therefore, here output is instead of Y it is Y e and here Y e is the output. And next the output Y which is coming from the actual system you just see here. Here this output is coming here this output is coming and it is shown here this output Y e is the estimated output.

And because of that because of Y e and Y there is an error and this error is to be minimize and therefore what we are doing here we are adding another control matrix that is called k e and here negative. So, here this output actual output and Y e there is a error and is error has to drive through k e and what we want that error which is coming here is should be minimized and this minimization is possible because of these k e and this particular state X e.

So, once we have got this X e we have to apply to the actual system; that means, this X e which was seen here this X e is nothing but this X e and now here it will goes to controller to the reference. That means, this is closed loop control system and we are law is satisfied, that means the our control law is here kc to X e plus U plus r earlier your law is minus U. But here because of the modification in the actual system the our control law has been changed to U equal to minus kc into X e plus r.

Now, this is the block diagram involved the system with state observer, now state observer we had drawn independently. Now, what we will do? We combined the both figure that is this block diagram and this block diagram and let us see the what in what way you will get the final structure.

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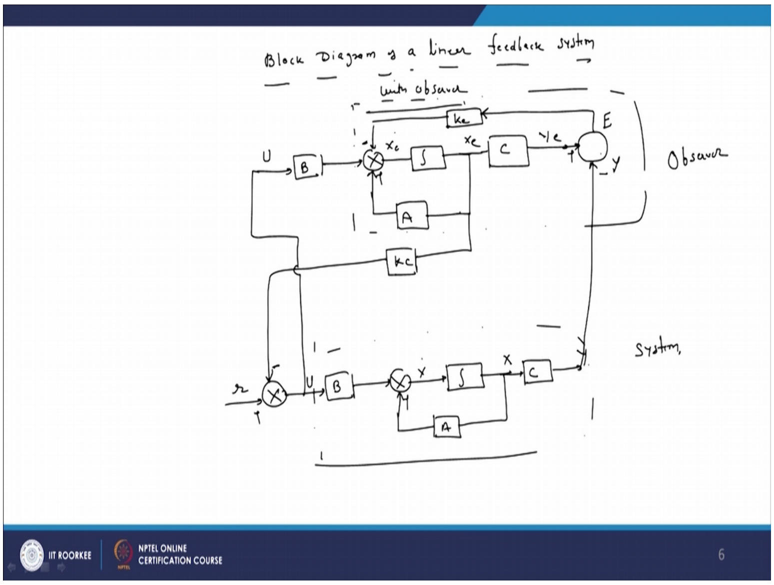

Now, we are drawing the block diagram of a linear feedback system with observer. So, here there are two parts the first part is related to aqua system and second part is related to observer. Now, first of all we draw a system now system in the system this is states called as X dot.

So, here integrator X dot here is X here is matrix A, now here is C and here is output y now here is your B matrix and here is of reference r. Now will draw the block diagram of an observer in observer the states are referring by X e. So, here X e dot now here X e this is integrator now here is C matrix here is Y e. Now here is the observer gain matrix or we can say observer type controller k e, now here output Y e now here is this is B matrix and this is U and here is a matrix A, now here is X e.

So, here this upper portion indicates the observer and the bottom portion indicates the system. Now, the what is the role of observer? Observer is basically estimates the state by reducing the error between the estimated state and the actual output; that is the output of the system as well as the estimated output. So, here estimated output is Y e, now here is the actual output, now here we have to move upper side now here negative it is positive and because of positive negative this is y these a error e and it error has to minimize and as I told you that it is minimized through k e and that is complete this is minus k.

So, now the error has been minimized, but for actual system the state X is not accessible now here these state X e we have to move through controller kc come here and this input U is same is coming like this. Now this upper portion is basically an observer and this portion is our actual system; the input U is common here this is U and these X e through kc, this kc is the controller that is the controller of actual system and this k e is the is the controller for the observer, what is the main purpose of this k e is to reduce the error between y e and y.

So, now this is the complete diagram of a linear feedback control system with observer, now we have seen the block diagram and we have seen that the error is to be minimized. But how will you prove mathematically that this k e matrix as we have seen this k e matrix minimize the error, therefore base on this block diagram we have to write down the equations.

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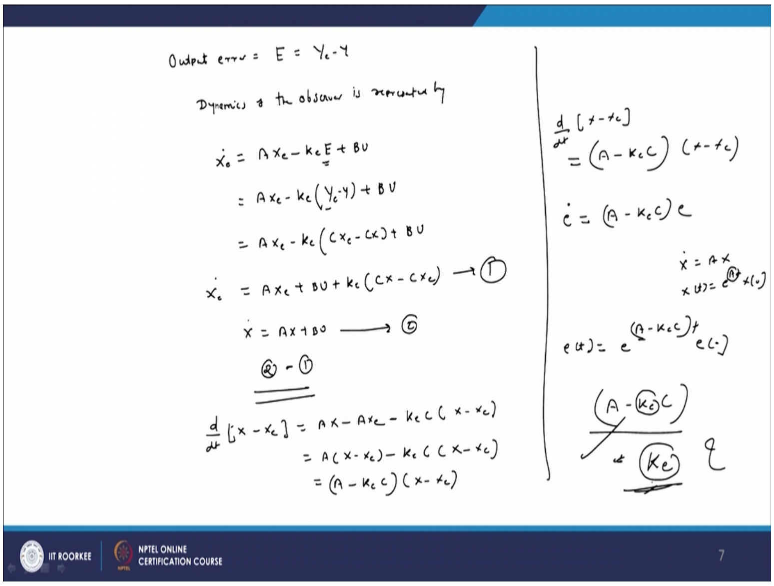

Output error that is e that is equal to Y e minus Y and the dynamics of the observer dynamics of the observer is represented by X e dot equal to A x e minus k e into e plus B into U. So, if you see this X e dot equal to a into X e minus k e into e plus B into U, now this A x e minus k e now this error e is represented by y minus y plus B into U now here

A x e minus k e this y is nothing but C into X e this y equal to C x plus B into U and this can be written as X e dot equal to A x e plus B into U plus k e this negative has been changed into positive. Therefore, this negative becomes positive; that means, Cx minus C into X e now let us say this is equation number 1, so we have got X e dot.

Now, the original state space equation is X dot equal to A x plus BU, now let us say this equation number 2 now we subtract 2 from 1. So, 2 minus 1 so we subtract equation 2 from 1 that is differentiation of this X minus X e equal to this A x and this minus A x e. Now, here B U B U will cancelled and here will get minus k e into C X minus X e, that means here a X minus X e minus k e into C X minus X e. Now here A minus k e into C X minus X e, that means equations differentiation of X minus X e that is equal to a minus k e C into X minus X e.

So, now will find that this X minus X e, that means the error between the state so that means, this X minus X e we can write down as error dot e dot and here a minus k e into C into e. If you write equation as X dot equal to A x. So, what we are writing down X of t equals to e raise to X of t into X of 0.

So, here what we have seen that in order to reach the steady states very quickly what we want the eigenvalues of a should be very far away from the image x; that means, it depends upon the eigenvalues of a this similarly here also. If you write the error equation let us see in terms of t e raise to t equals to e raise to a minus k e C into t into e of 0. That means, the minimization of error that is the minimization of error it depends upon the eigenvalues of a into k c; that means, it depend upon a minus k e into C

As early the as early the error reduces to 0 it is better for our estimation. So, this is the possible when the eigenvalues of A minus k e C it is far away from the image axis and; that means, the whether is far away from the image axis or near to it depends upon these k e matrix. Therefore, this important of k e matrix, k e matrix is very important because it governs the error equation that is why here will seen that we not taken the k matrix k e from that is the error between Y e and Y when it comes to here the (Refer Time: 23:30) important role this is because of this is only here will find that.

Because of this eigenvalues depend upon the subtraction of a minus kc as eigenvalues is far away from the imaginary axis error reduce to 0 and this depend upon this k e matrix.

The study of as we have seen the observer design, now we see the various points related to state observer the device that performs.

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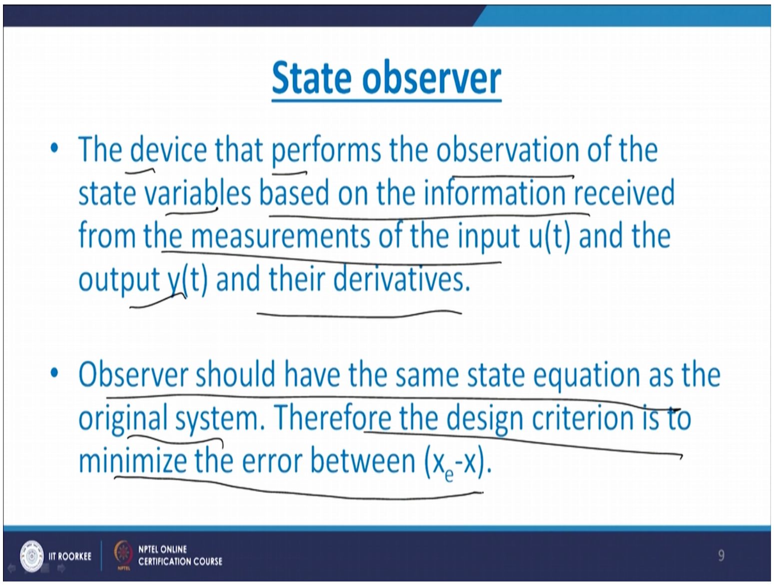

The observation of the state variables base on the information received from the measurement of input and output and their derivatives then second point observer should have the same state equation as the original system therefore, design criteria is to minimize the error between X e and x. So, we already seen that here see here that we require the error between X and X e should be minimum.

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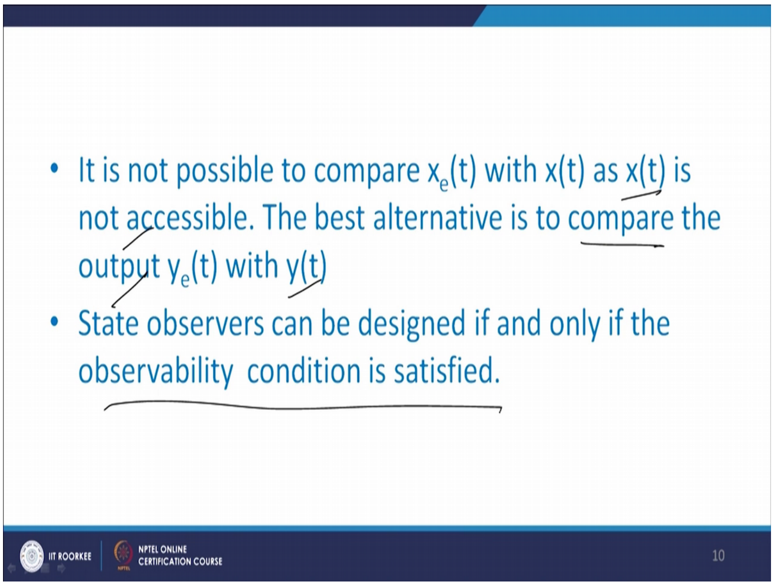

So, it is not possible to compare x e of t with x of t as x of t is not accessible, the best alternative is to compare the output y e t with y t and state observer can be designed only if the observatory condition is satisfied. So, whatever we have discussed, so this has been reported in this 4 points. So, in actual system it may possible that there are few states, let us say there are 10 states out of these 10 states 5 are known 5 are unknown, but what we prefer that 5 are known 5 are unknown it is better to estimate all state. So, if you are estimating all states even though some are measurable still we are estimating that is called full order observer.

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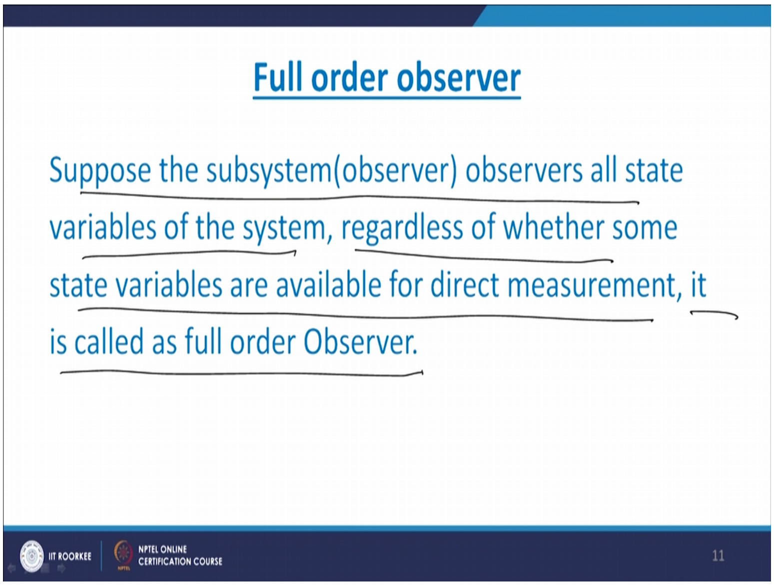

So, suppose the subsystem observes all state variable of the system regardless of whether some state variables are available for direct measurement it is called as full order observer.

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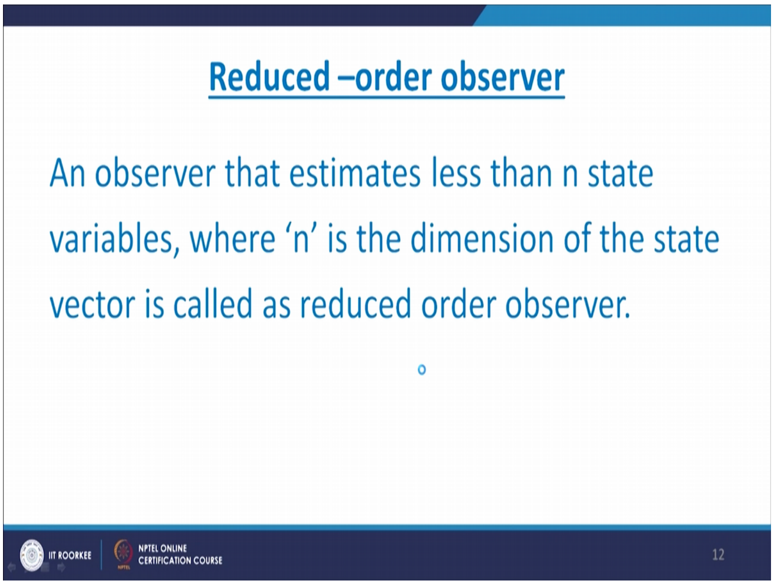

And similarly the reduced order observer reduced observer means there are some states which are known which are unknown. So, what we are doing the states which are which can be possible to measure we are measuring it and which are only unknown that only estimating that is called reduced order observer.

So, an observer that estimates less than n state variable where n is the dimension of the state vector is called reduce order observer. Now will see that how to determine the observer matrix k e. So, this is possible by Ackermann formula earlier we have used the Ackermann formula for updating the controller matrix that is the state feedback control matrix. So, this Ackermann formula is also useful for determining the observer matrix, so let us see how will you get it.

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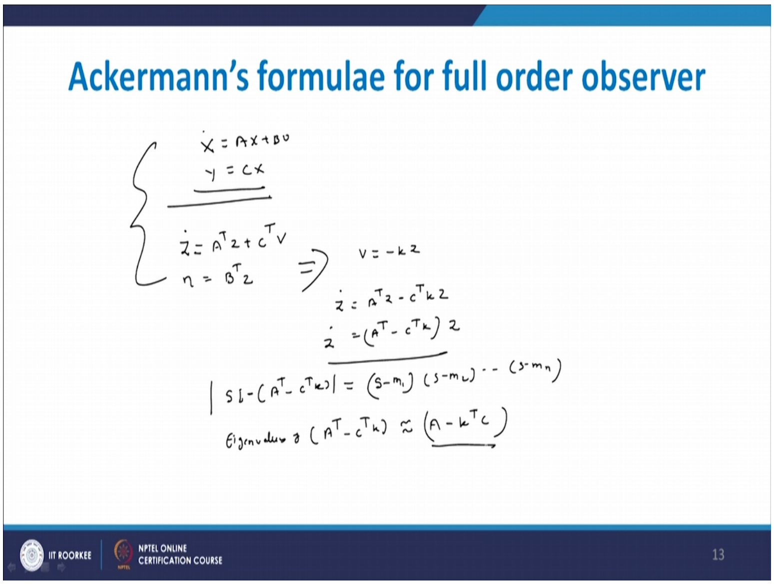

Now, consider our original plant as X dot equal to A x plus B u and y equals to Cx this is the plant. So, this is the plant without changing the properties we have seen that this plant can be converted into another form that is dual form of this, 1 we can represent by states Z dot equal to A transpose Z plus C transpose v and say another variable we have taken n B transpose Z; that means, what we are doing we are changing the position of B and C. So, B is earlier here it has been in the state it has been transpose output equation as B transpose similarly the C in the output it has been transferred to state equation as C transpose, so both have the same properties

Now, suppose if we are dealing with this, so what is the control law the control law in this case let us see v is the output equal to minus k into z. So, you can get z dot equal to A transpose Z minus C transpose k into Z, now here equation is A transpose minus C transpose k into Z dot equal to like this.

So, now we have got the we have got equation as Z dot equal to A transpose C transpose k into Z. Now the in order to get the roots of the system or in order to find out the value of the controller k, we had to assign some eigenvalues; that means, here this equation we have to represent in terms of casting equation. So, here this can be written as SI minus A transpose minus C transpose k that is equal to say S minus m1 S minus m 2 say S minus m n.

So, this is m 1 m 2 up to m n that are the assume that there are the observer poles that are like this and our purpose is to determine the values of this k matrix. Now if you find at eigenvalues of a transpose minus C transpose k it is same as. So, if you take that transpose of this particular matrix, so what will get; will get a minus or transpose of this is nothing, but k transpose into C.

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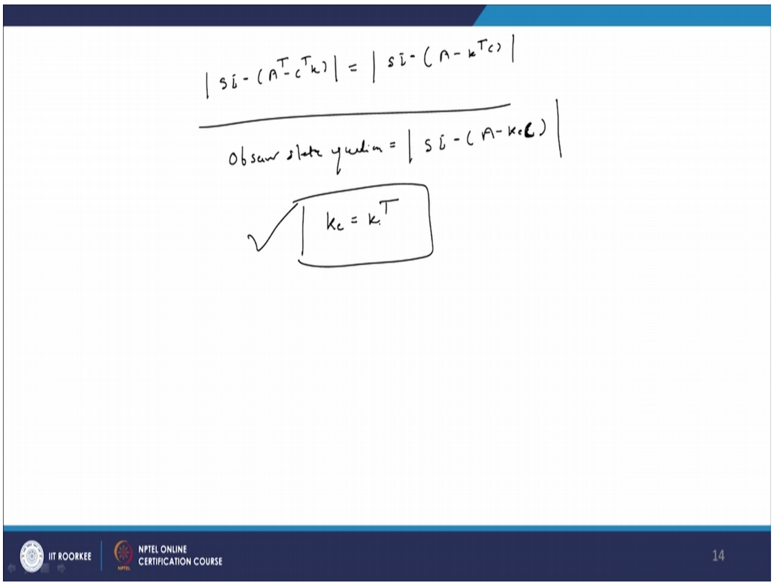

It means that SI Minus A transpose minus C transpose k that is equal to SI minus A minus k transpose into C and now our observer state equation is observer state equation is SI minus A minus k e into C or observer state equation equals to SI minus A minus k into C and this equation we have got it means that here we have got this k e equal to k transpose.

So, here what we have done we have taken the a system we have taken the dual of the system and after taking the dual of the system, what we have what we have got we have got A minus k t into C this equation we have got and our main purpose is to get the value of k. And therefore we have decided the observer poles here these are some poles which has to be placed on the left hand side of the experiment to get the value of k. And there afterward what we have done this A transpose C transpose k can be done in terms of A minus k transpose C from here and our actual observe equation is SI minus k into C. So, if you compare this 1 we got k e into k transpose.

So, it means that we have to determine the k by actual post space technique and verse we get the k we have to make it k transpose and we will get the k e matrix. So, as we have seen will get the k e matrix now in a pole placement techniques, we have seen the different metal methods for calculating the controller matrix or we have design a controller by using the pole placement approach.

So, here also we found at the pole placement concept that is concept of pole placement has been has been used in determination of the observe matrix. So, therefore whatever methods which was studied for pole placement they are applicable in observer state equation also; that means, we can get the observer state observer matrix or observer base controller using the pole placement techniques, so the formula which we have taken k e into k transpose. So, here we see only one method as similarly you can use other methods, so here we see the Ackermann approach.

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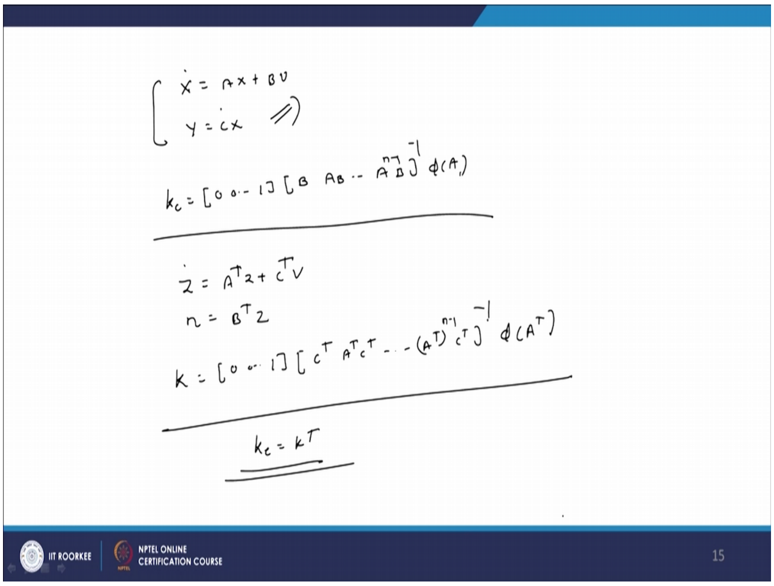

So, according to Ackermann approach X dot equal to A x plus B u y equal to Cx. So, we have seen last time the controller design in such cases using the Ackermann formula, the Ackermann formula says the controller kc is equal to 0001 and here B AB A raise to n minus 1 B inverse. So, this is the controller for this state space model and now here we taking the dual system, now for the dual system Z dot equal to A transpose z C transpose v, n equal to B transpose z now here if you compare this and from this will get k equal to 0001.

So, the B it change to C transpose here will get C transpose and a change to A transpose will get C transpose A transpose C transpose and here will get A transpose n minus 1 C transpose inverse into phi of e transpose this is the controller. Now our k e is nothing but k transpose our k e nothing but the k transpose because, the actual observer controller is k e now we have to make it transpose. So, when we make the transpose of this 1, so all this elements we shall like this we have to write in the reverse order.

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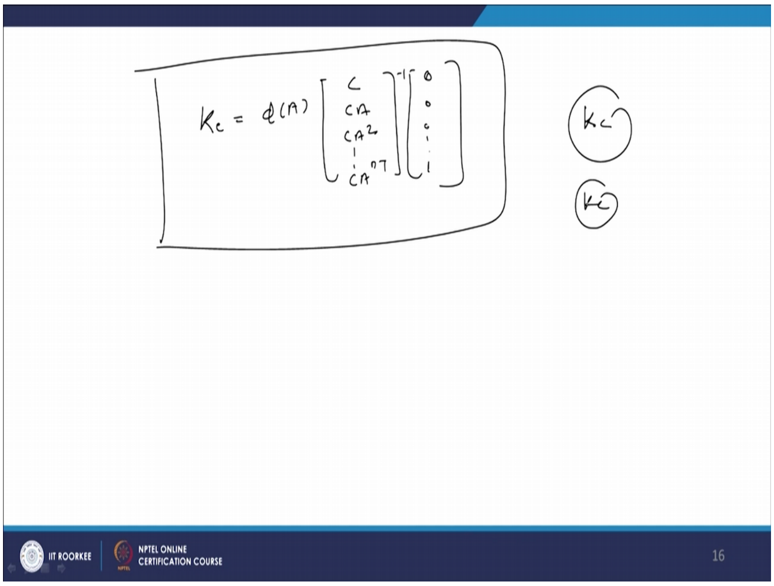

So, when you write the elements in the reverse order the controller k e will get as phi of A C CA CA square up to CA raise to n minus 1 here 0001is. So, this is the Ackermann formula for getting the observer matrix or this k e is mainly responsible for reducing the error between the output and the estimated output.

So, this is the kc formula for getting the controller pole placement controller and this is observe controller. So, when they are when it is not possible to measure the all states the in that case there are two types of controller, we have to design one is the controller because, of the pole placement and second controller because of the observer design. So, kc and k e are the important things, now we solve one example and will try to determine the observer base controller now.

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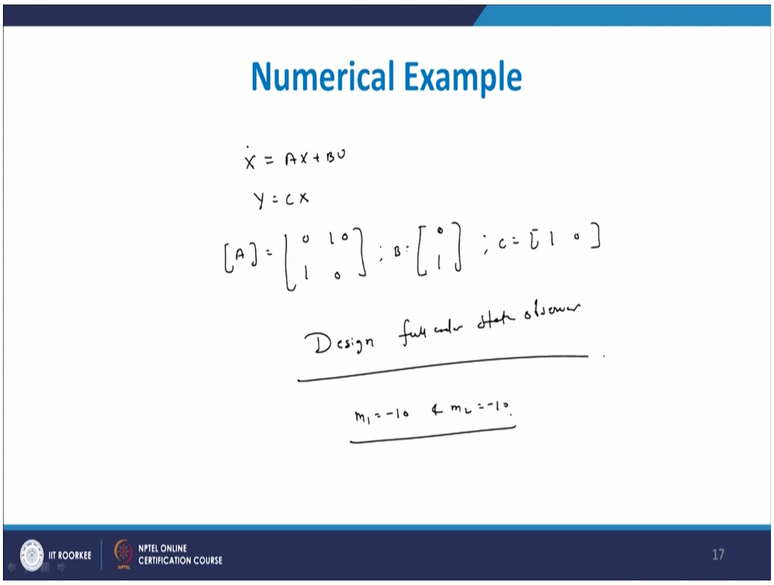

Consider the system as X dot equal to A x plus B u y equal to Cx, now here a matrix as 0 10 1 0 B equal to 0 1 and C equal to 1 0 and our problem is we have to design full order state observer. So, when we are designing the full order state observer we had to decide where we want to place the observer pole; that means, similar to control poles we have to place the observer pole let us say for this problem, the observer poles let us say m 1 equal to minus 10 and m 2 equal to minus 10.

So, when we determine the observer or full order observer first step is to check the observative of the system, so in this case first step is to check the observatory.

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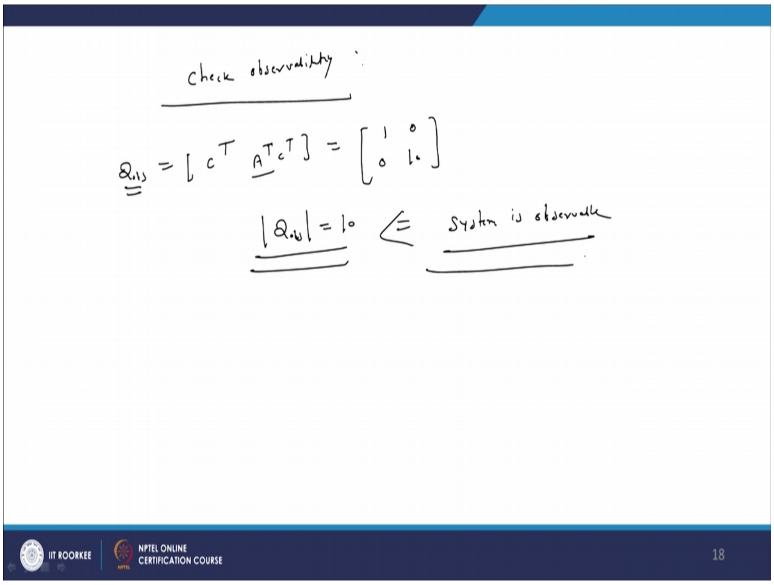

So, check observalibity for checking observatory we have C transpose A transpose C transpose. So, here C transpose is 1 0 and A transpose C transpose if you do the multiplication will get 0 10. So, it is observatory matrix, so determinant of observatory matrix is 10. So, your system is we can say observable. Now, system is observable and to get the full order observer we directly use the formula for getting the observer gain matrix.

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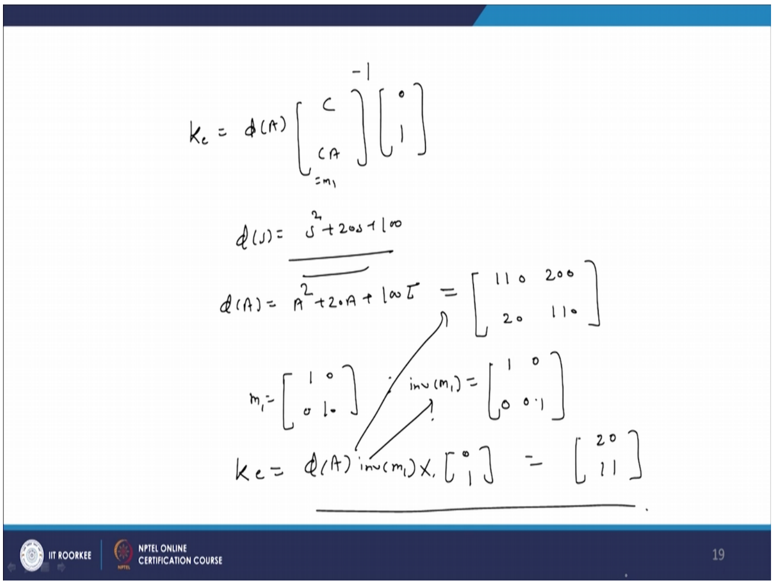

So, the Ackermann formula that is this k e equal to phi of A and here C CA inverse to 0 1; now desired poles desired S square plus 20 S plus 100 because, we have pole at m 1 minus 10 m 2 minus 10. Therefore, we have got this S square plus 20 S plus 100. Now your phi of a will get as A square plus 20 A plus 100 into I. So, here if you solve it you will get 110 200 20 into 110.

Now, here m 1 let us say this C CA let us say m 1. So, this m 1 it as 1 0 0 10 and here if you want to determine this m that is inverse of m m 1 will get as 1 0 0 0.1. And now this controller matrix k e is nothing but this phi of A which we determined here then C CA that is inverse of m 1. And this is that means here inverse of m m 1 multiplied by this 0 1 and if you do the calculations. So, what will get you will get the controller matrix k e as 20 11.

So, because of this controller matrix k e or we can say because, of this full order observer we are in position to estimate the state of the system, but now there are other things which is also important to be discussed. Now in this problem we have design the observer design the observer full order observer, but in the system we have to also design the controller. Now here we have placed the pole minus 10 and minus 10 is it the right position to place the pole or observe pole can we place very nearer or can we far away.

Second case is that about the controller poles is there any relationship exists between the observer poles and the controller poles or is there any some conditions that we have to place observer poles far away from the control poles. So, this also important point second thing that are the observer poles and control poles are dependent on each other. So, this point is to be discussed so we will try to discuss this point in the next part.

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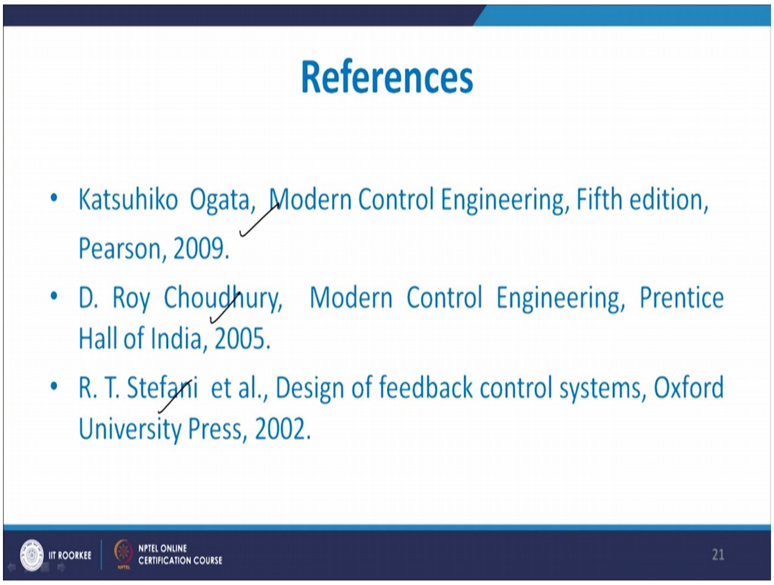

Now, these are some references Ogata, D, Roy Choudhury, R.T. Stefani.

Thank you.