Advanced Linear Continuous Control Systems
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Lecture - 37
Tracking Problem in State Feedback Design (Part-II)

Now, we start with Tracking Problem in State Feedback design part 2.

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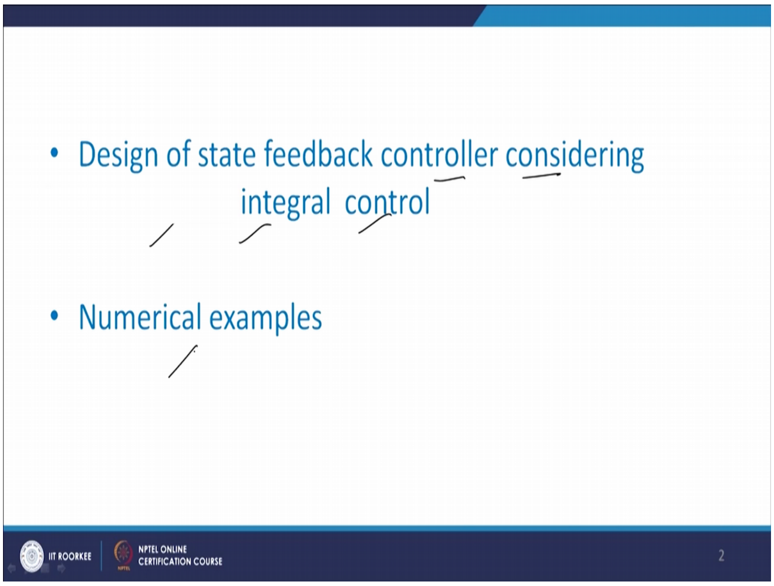

In this we will study design of state feedback controller considering integral control and numerical examples.

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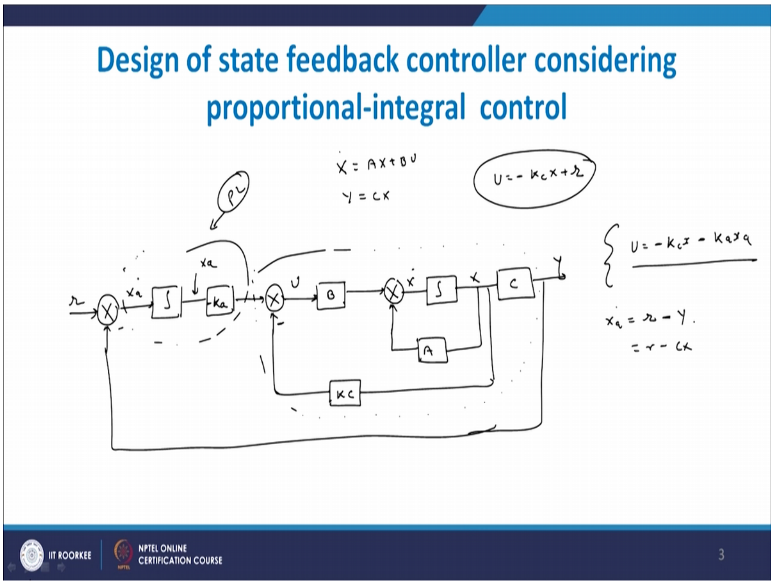

Now, about the design of state feedback controller considering integral control; In classical control aspects the different controls have been used, particularly proportion controller proportion integral controller proportion integral, derivative controller sometimes pd controller. The main role of derivative control is to reduce the overshoot; the main role of integral control is to reduce the steady state error. Now, we were seen that in our problem our problem of state feedback control design, there is issue of steady state error. So, now, in classical control there is proportion integral, which reduces the steady state error and in our advanced control there is a issue. So, can we combine the classical approach of pi with state feedback design?

So, it is possible that that if you use the concept of proportion integral in a state back feedback design, we can get gate the reduced steady state or we can say we cannot get any steady state error, that is the output track the refers input properly. So, here will see how you how we combined the proportion integral aspects in state feedback controller design.

As our plant is x dot equal to Ax plus Bu, y equal to Cx; Now the block diagram for this here is summing block, here is one integrator, now here output C this is c and Y and here is we have A matrix this Ax; now here is B matrix, now here is U and here control law we have control law U equal to minus k c x plus r. Therefore, here we are going for our control law U equal to k c x plus r. Now here, we have a controller kc and here is x so, U equal to minus; that means, this r will come into picture. So, this is our conventional approach that is u connect r here that is u equal to this is say minus k c plus r and here is x dot x x dot equals to Ax plus Bu y equals to Cx. So, by conventional approach we call will get this type of block diagram, but the these issue with the steady state error. Previously we were seen that adding a certain block here, that is we have seen that adding one block of ts and finally, we determined ts in order get the 0 steady state error or in order to track the reference input that we have seen.

Now, here some idea of integral concept we have to use therefore, here what we are doing here, we had add an another block k that is a minus k a, and here is we add one integrator and here is state say x a now here is x a dot and here is another summation block, now here is your r and now here is. So, now, here these particular block diagram, this particular portion is reusable to advance control state space, and this integrator x a this resemble to our integral controller issues.

This integrator means one bias that reduces the steady state error, and now you see the concept that how to reduce the steady state error by means of this one, but now here the control law u equals to minus k plus r this is this is earlier law now here because of this your control law will be modify. So, here the control law is a U equal to minus k c into x minus k a into x a. So, see that this control law u equal to minus k c into x this is x minus this is k a into x a now here 2 types of variable x and x a and will finded because of this arrangement definitely the number of state has been increased.

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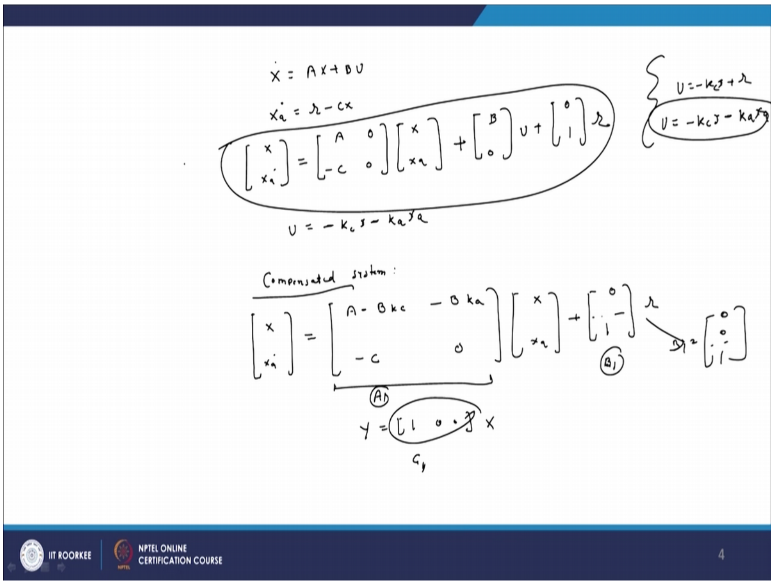

Now, we see now will see state equation as X dot equal to Ax plus Bu; this is the state equation for the original plant and because the new state has come. So, here the equation is x a dot equal to r minus c x you will find at here x a dot equal to r minus y that is here this x a dot equal to r minus here y here r y c into x. Therefore, here you got x and x a dot and if you combine this equation x dot x a dot here A 0 minus C 0 that is x here is x a plus B 0 into u plus 0 1 into r.

So, this is the state space model of a system, when the integral controller has been included in the state space model. Now here this is a complete system now here our control law also change because earlier our control law is U equal to minus k x plus r that is k c x plus r, but now here our control law has been changed to minus k c x minus k a into x a this is our control law. That means, your control law in this case is u equal to minus k c into x minus k a into x a.

So, this is control law u equal to minus k c x minus k a into x a this is the control law and now you have to use this equation here, and they solve it and finally, you will get the compensated system as. So, our compensated system is x dot x a dot that is, A minus B into k c minus B into k a minus C 0 x x a plus 0 1 into r and y equal to 1 0 0 x. Because of this new control law and this state space model, we will get the compensated system like this and now in order to simplify the result because our final aim is to reduce the steady state error or the your output must track the reference input error to be 0. Now in order to see this we will take this block as a A 1 now this block say as B 1 and here this block as a C 1.

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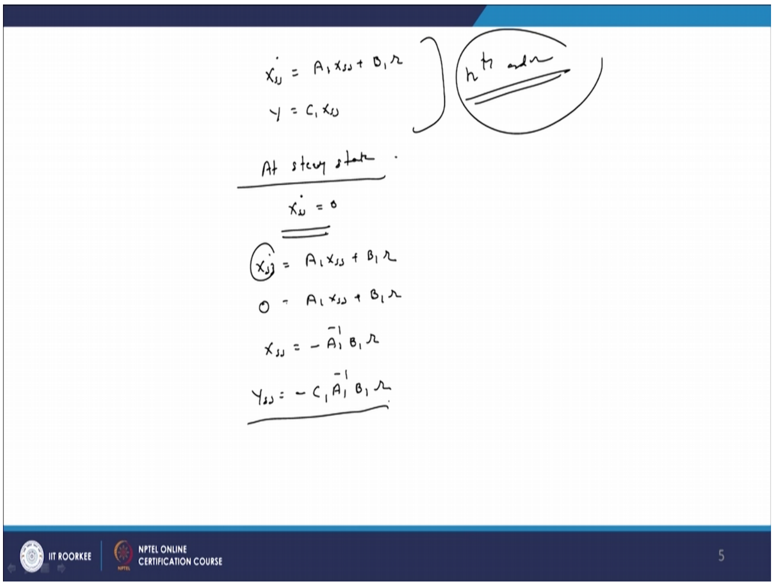

So, here the state space model is steady state equal to x dot equal to A 1 x s s plus B 1 into r y equal to C 1 into x s s; that means, the earlier which were taken. So, actually this B is a involvement of 0 0 say 1 this like this because as if this is increase the order we can get more elements, if you the second order plant is there (Refer Time: 11:19) 001. So, here output is you can say 1 0 0 as assume that the plant is of second order or if (Refer Time: 11:31) we can say that this is particularly only C 1 B 1 and A 1 only

So, here we come across x s dot equals to A 1 x s s B 1 into r, y equals to C 1into x s s. Now what at steady state at steady state at steady state we are x s dot equal to 0. So, x s dot equals to 0 therefore, this x s s dot equal to A 1 x s s plus B 1 into r though this is equal to 0. So, 0 A 1 x s s plus B 1 into r; that means, here we can writes x s s equal to minus B 1 into r and Y s s equal to minus C 1 A 1 inverse this B 1 into r now here will finded we have compensate system a b c.

Now, x x a now here 0 0 1 so, if as the if the order is 2 this for original plant is 2 then a 0 0 1 and output is correspond to 1 0 0. So, that is we can say is C 1 we are written. So, here x s s dot A 1 x s s B 1 r y equals C 1 x s s assuming that these or we can say consider these for n th order and therefore, x steady state x s s dot equals 0. So, x s s dot equals A 1 x s s plus B 1 into r 0 A 1 x s s plus B 1 into r. So, we get y s s equals to C 1 into B 1 into r.

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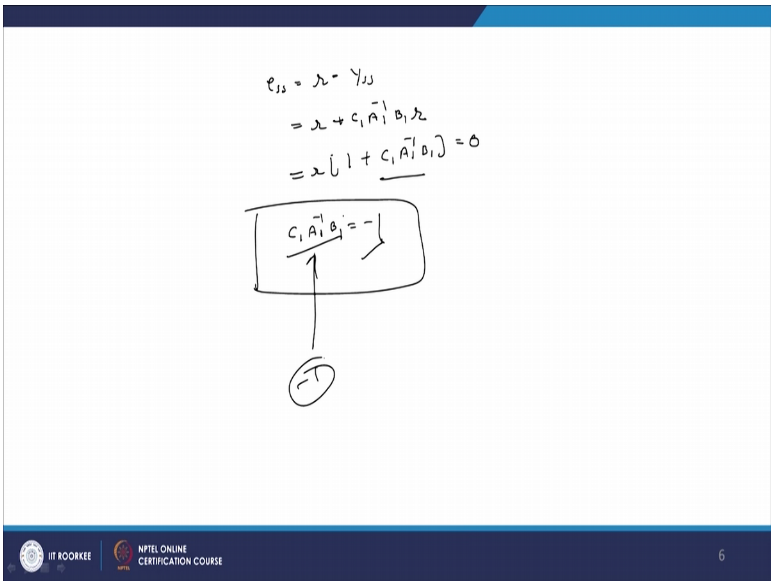

Now, about the steady state error; Steady state error e s s is given as r minus y s s. So, here we can write only this as r plus C 1, A 1 inverse, B 1 into r and here you take r is common you get 1 plus C 1 A 1 inverse B 1, B B 1 equal to 0. So, here steady state error is to be 0. So, when this is possible. So, this is possible when this is the conditions C 1 A 1 and this B B 1 equal to minus 1. So, this is the conditions.

So, when these conditions comes then we can say that steady state error equal 0 therefore, whenever problem is given to you in that case you have to determine the C 1 you have to determine the A 1, then take a inverse and B 1. If it comes minus 1 it means that; that means, the result of this is say minus 1 it means that we have 0 steady state error. That is output tracks the represent output correctly. Now we solve the example to see whether we get the 0 steady state error or not.

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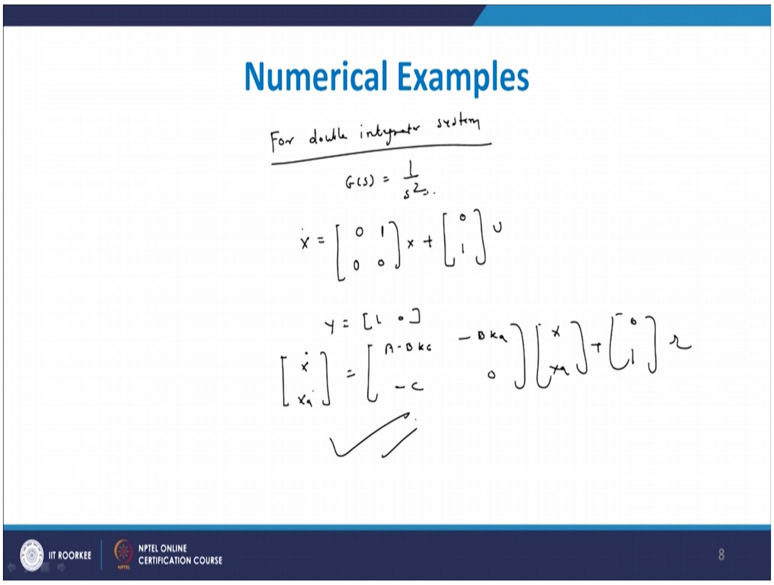

Now, about numerical examples then we see the double integrator problem for double integrator system. Double integrator means that there are 2 poles at the origin that is your G of s equal to 1 by s square. Now our main purpose is to design the controller for this plant, along with that is we had to reduce the steady state error. As G s equals to 1 by s square that is the order of the system is 2. And now here we are designing control using state space; that means, the maximum states in this case is to just looking up for the problem.

Now, there for first aim is to develop the state space model of this double integrator; for state space model we have written G s equal to 1 by s square the order of this system is 2. So, order is 2 it means that number of states are 2 and if you want to write the state space model of such type of equations, in that case we have to write like this x dot equal to 0 1 because now here we are writing down the equation in compelling form, the arrangement is 0 1 and we have to write the elements of the denominator now here the element of s square. So, when we write the compelled form through transfer function, they no need to take the elements of these we have to take the remaining elements here. So, the remaining elements in 1 by s square is 0 therefore, we can write down as 0 0 and here we write down as x plus 0 1 into u y equal to 1 0.

So, this is the trans state space model of a double integrator system. Now here we have to design a controller. So, the number of states r 2 and because of the integral controller worst state has been added; that means, if you see this one. So, this resembled to your C 1 matrix which you have taken, and here B resists for to 0 0 1 now here what we can do here write the given system as x dot, x a dot as A minus B into k c minus B into k into a, minus C equals to 0 x x a, now here 0 1 into r. So, now, here A B k c and this is k e element is there. So, if you write down the model of the system taking care of all the states; that means, here taking care of this x x A dot A 0 C 0 x x A B 0 0 1 that is this is the particular the moral of the system when we are added integral controller, now in this problem what we have to do?

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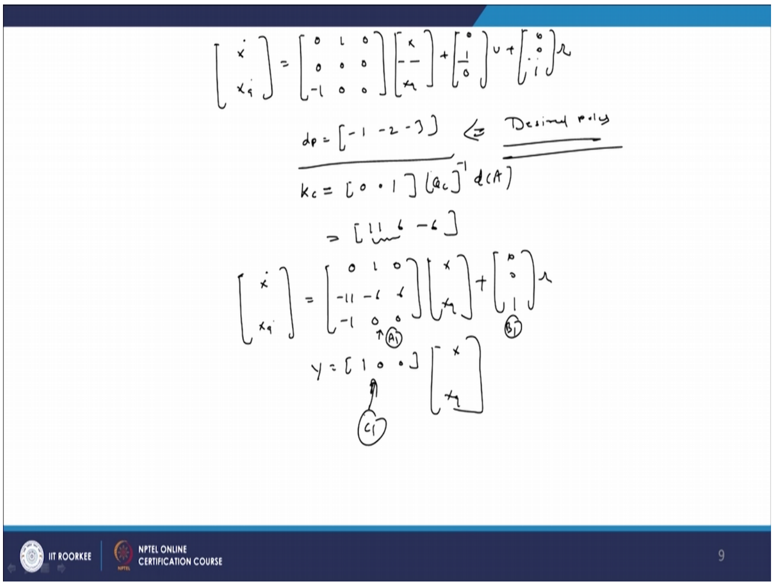

We have to write the elements of A elements of C x x a B 0 and we see the equations as the model is x dot x a dot 0 1 0 0 0 0 minus 1 0 0 here is x x a 0 1 0 u and here 0 0 0 1 into r. This is the state space model of the system when there is we can say no controller and we have got this results x a plot is the part of the integral controller, now our purpose is to design a controller suppose our pole plus pen we are debian or minus 1 minus 2 minus 3. So, this indicate your desired poles.

So, in the desired poles we have taken minus 1 minus 2 minus 3 all are real poles; that means, in the performers there will be no transiers involved. So, as we have seen the formula earlier the controller k c equal to 0 0 1 controllability inverse Q c inverse into 5 of A. So, if you use this formula you get the result as controller k c look at as any one 6 minus 6. The first part 11 6 is corresponding to the part of the original model and minus

6 is corresponded to another x a that means; every state there is a controller gainess. So, l a minus 6 for original state and minus 6 for we can say for x a x a variable.

Now, as we have seen this is compensated system. So, this compensated system for all these values we can write down as x dot, x a dot 0 1 0 minus 11 minus 6 6 minus 1 0 0; here is x x a 0 0 1 into r and for output one 0 0 0 x into x a. Now this is our compensated system and in the compensated system we take this as A 1 this as B 1 and this as a this is add C 1. So, A 1 B 1 and C 1 are there and we have seen the conditions. The condition for steady state error to be 0 it C 1 A 1 inverse into B 1 that must come as minus 1; So, here the condition for steady state error.

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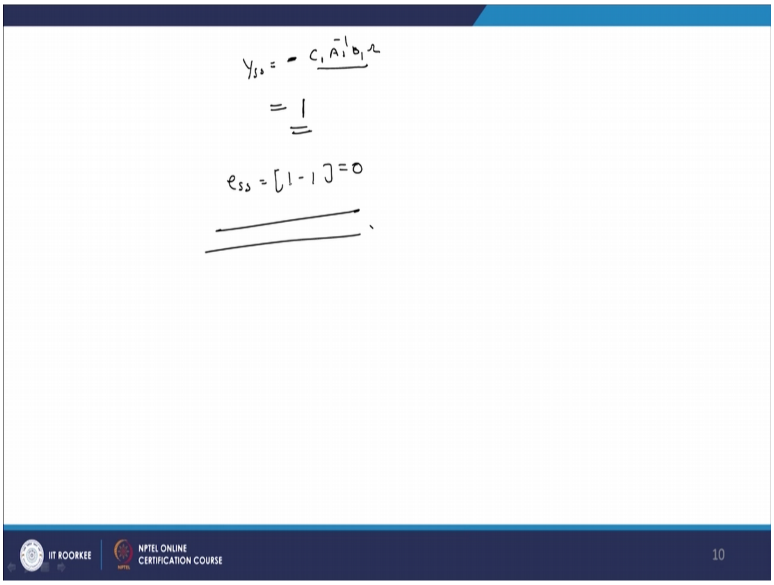

So, Y s s equal to minus C 1 A 1 inverse B 1 into r. So, you will finded this condition is satisfied here will come as minus 1 as have seen minus 1 C 1 A 1 into B 1 and here will get Y s s equal to 1 and the steady state error e s s equal to 1 minus 1 equals 0. So, replacing the all the values of C 1 e 1 B 1 you will get steady state error as 1 and e s s equal to 1 reference equal to one minus 1 equal to 0; that means, our the conditions of the steady state error to be 0 is satisfied.

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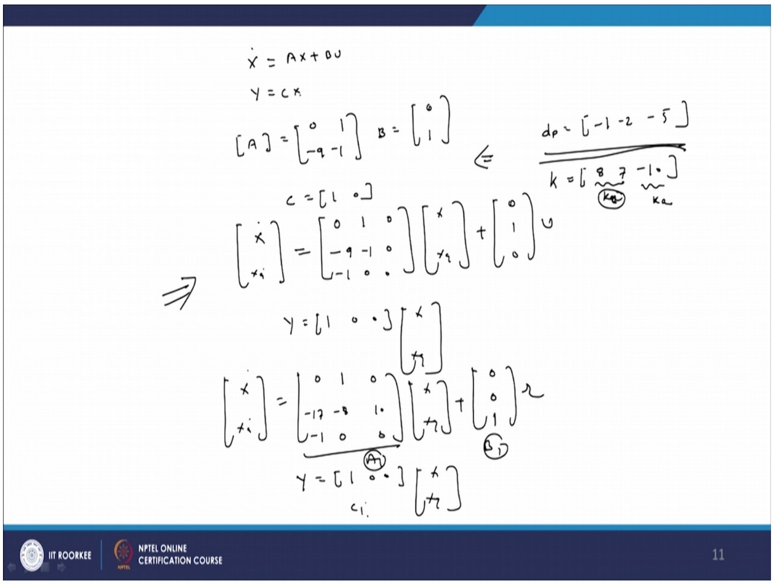

Now, we see the another example now we see the example, which you have taken last time particularly related to the compensating factor part that is x dot equal to A x plus B u y equal to C x. Now, here A matrix is say 0 1 minus 9 minus 1 and here B equal to 0 1 and here C is 1 0 here we have x x a dot. Now, here using this formula this one using this formula we have to determine the compensate system and for that purpose for this particular problem we take the case as a desired pole placement see dp equal to say minus 1, minus 2, minus 5 and we had to determine the controller. The controller k finally, we get this as as 8 7 minus 10. So, this 8 7 is corresponding to k and this corresponding to the additional state that is a kc and this part is considering the k, and compensate system will get as 0 1 0 minus 9 minus 1 0 minus 1 0 0 this x x a and here 0 1 0 into u.

And here Y equal to 1 0 0 here x into x a and this is the particular the model of the plant and here we get this final system in terms of compensate model as x x a dot that is equal to 0 1 0 minus 17 minus 18 10, here minus 1 0 0 to x to x a plus 0 0 1 into r and here Y equal to 1 0 0 into x x by x a. Now here is our A 1 here is B 1 and here is our C 1 and now if you solve this one we can get Y ss equal to minus C 1 that is A 1 inverse into B 1.

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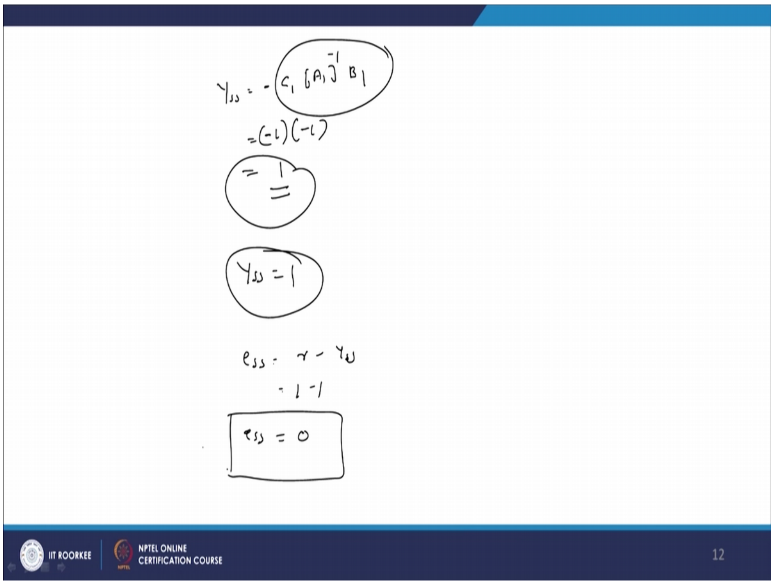

So, particularly C 1 A 1 inverse. So, if you use the values of A 1 B 1 and C 1 we have find that will get here minus and you get this minus 1 here will get minus 1 it is (Refer Time:26:58) as 1. So, steady state error or steady state output we comes to be 1; that means, you are the error the steady state error e ss equal to r minus Y ss that is 1 minus 1 that is steady state error equal to 0.

So, will find that if we use the concept of integral aspect in state space analysis, we have got the steady state error 0 that is the output track the represent properly, but here we find that the complexity in the calculation has been increased. Because in earlier times we have seen that we had to design the controller for the minimum state; that means, if you see the additional of (Refer Time: 27:50).

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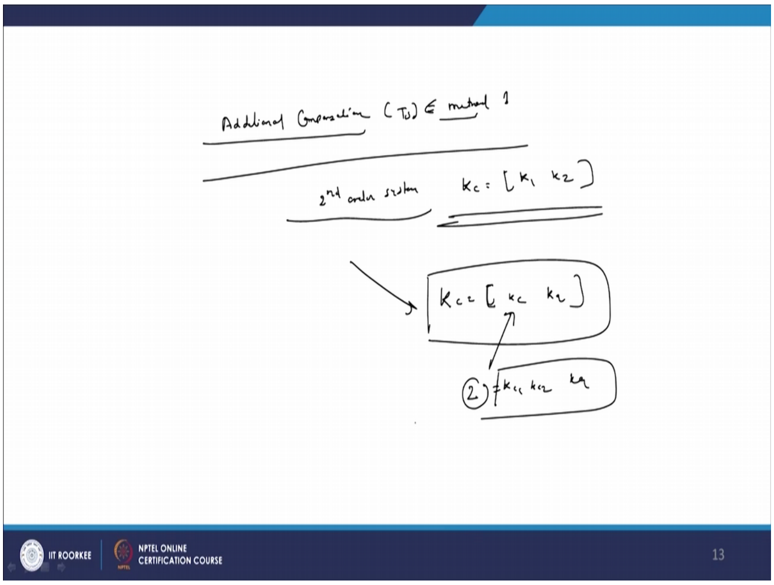

Additional compensation that is T s which we have taken, that is method one in this approach the controller has been designed for the original plant; that means, for the second order system, we have got controller k c as here k 1 k 2. And final system has to deal with the second order, but if we consider the integral aspects; that means, in that case we have to consider the state of that pole that is additional of the integrator and therefore, in that case here this k c is nothing, but this final controller k c equals to this k c divide by into k. So, this k c is correspond in to 2 part that is say your k c 1 and k c 2 and here is k a; that means, the complexity has been increased we have to taken the (Refer Time: 29:18) more state, but our error has B reduced.

So, as far as the stability issue is concerned as we know that as the order increases there can be issue of the stability therefore, the methodology which you have taken earlier that is method one is quits better from the stability point of view, because in that case there is need to taken care of minimum state. But where as in these case that is in case of the addition of integral, we had to consider more state therefore, the method one is beneficial in compare into other method.

So, now upon the state space issue we have designed we have develop the state space model, then we have designed a controller then we have seen the issues relating to the tracking. Now, this also a another issue about the states because the state feedback design is possible when we can measure the states, but there may be possibility that sometimes we cannot measure the states then how to handle this. Because if you cannot measure states this is never be possible because there is state measurable then control design is possible, but state cant measurable this type of thing is never possible. Therefore, there must be some arrangement by wish we can some estimate the states so; that means, the state observer. So, that part will see in the next part.

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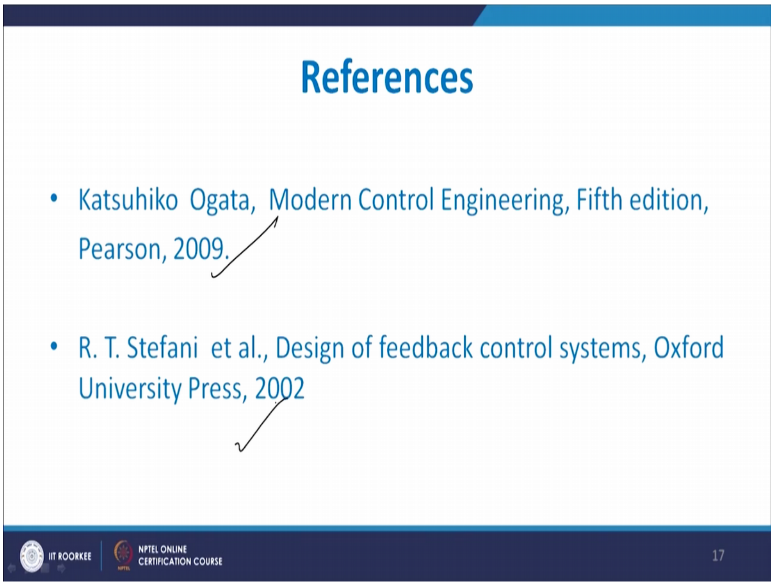

So, these are some references.

Thank you.