Advanced Linear Continuous Control Systems
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Lecture - 36
Tracking Problem in State Feedback Design (Part-I)

Now, you start with Tracking Problem in State Feedback Design.

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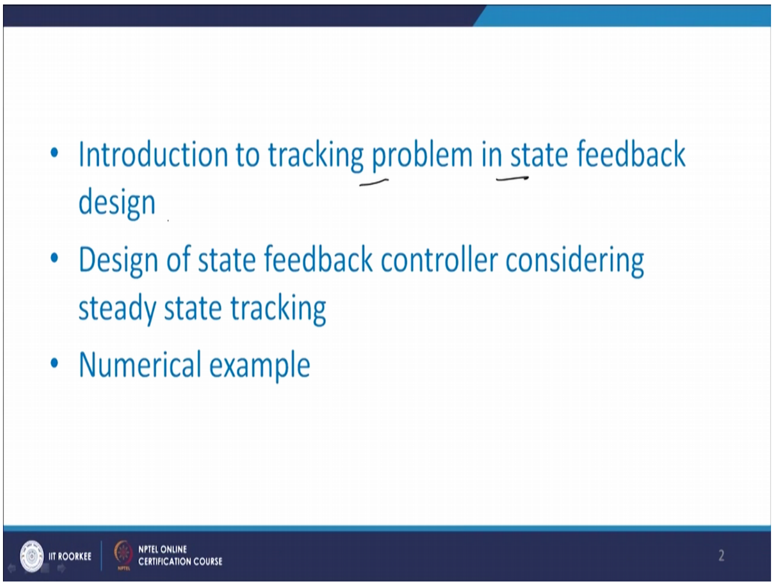

In this we will study introduction to tracking problem in state feedback design, then design of state feedback controller considering steady state tracking, and numerical example; now about the introduction to tracking problem in state feedback design.

If you see the any system for that system there is need to design a controller. So, what is the basic purpose of controller is to clear the transient issues as well as the steady state issues.

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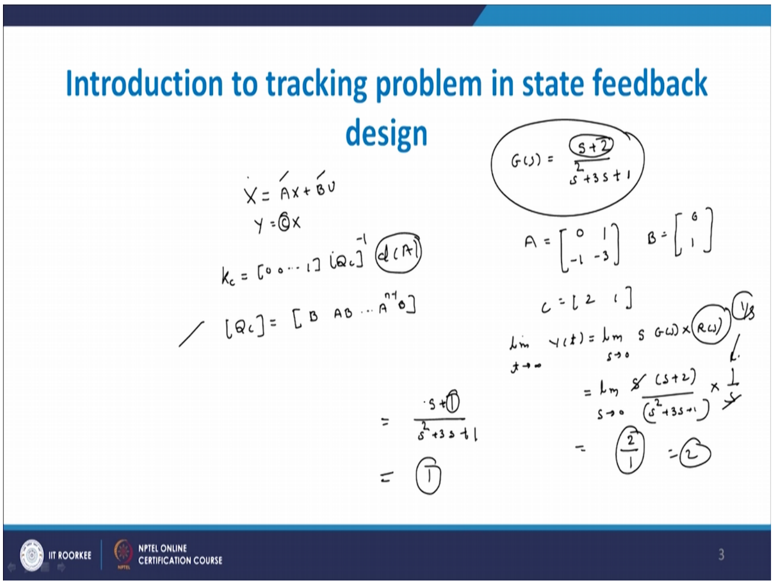

Now, we have seen state feedback control design. And we have seen that for state feedback design there is for X dot equal to AX plus BU Y equal to say C X, the controller k controller k c is determined as 0 0 0 1, a controllability inverse phi of A. And here this controllability matrix Q C is repaired by B AB A raise to n minus 1 b. So, we will find that the controller in a state feedback design is depends upon B matrix, A matrix, and phi of A is the desired characteristic equations.

Now you will find that, there is no role of C matrix; C is the output matrix. So, whatever the things in output it is not play important role in a controller design. It means that, there can be some issues. Now here this issues is respect to steady state error is because if you write any transfer function G of s equal to s plus 2 s square plus 3 s plus 1. So, this is transfer function. Now the state space model of the system is given as 0 1 minus 1 minus 3 and B matrix a 0 1 and C matrix is a 2 1.

So here we will find that A B C is there, and if you find out the steady state output by using final value theorem it is given as limit t tend to infinity of Y of t equal to limit s tend to 0 of S G of s into say R of s. Now here if you see limit s tend to 0 s G s is s plus 2 divide by s square plus 3 s plus 1 and R s is for step input, this R s is 1 by s. So, here s and s will cancelled and we will find that replacing s 0 there is 2 by 1.

Now here we will find that the steady state output for this plant is depend upon this matrix that is we will get 2 by 1 that is means 2. Suppose instead of this s plus 2 let us say there is say s plus 1 s square plus 3 s plus 1, to in that case your steady state is 1 this is 1. That means, the steady state is depends upon the C matrix and here we are not taken care of the C matrix. Now again see here.

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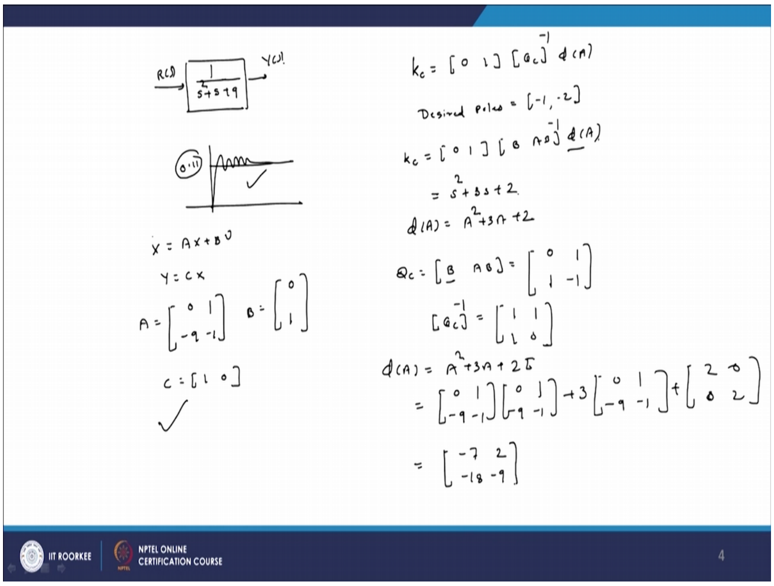

Now, we take one transfer function as 1 by s square plus s plus 9. This is the transfer function. Now here we apply input R of s and output Y of s. We want to take the step response of the system. And the step response of the system for this plant is given as like this, there is some oscillatory behavior and there is some steady state value.

Now, the steady state of this is 0.11. But suppose if you want a steady state of 1; that means, that here we have to design a controller. Now what we do? We develop the state space model of this plant, the state space model of the plant is given as X dot equal to AX plus BU Y equal to C X and here is A matrix is 0 1 minus 9 minus 1 B matrix as 0 1 and C matrix as a 1 0.

Now, for this system 1 upon s square plus s plus 9, we have determined its response see here this is response and then we have also develop the state space model of this plant. Now here we will find that there is a oscillatory behavior. Now our purpose is to design the controller for this system. So, controller means we have to design the k c.

So, here the controller k c is given as 0 1, then controllability matrix inverse into phi of A; phi of A considering with the desired poles. So, here our desired poles are say minus 1 minus 2, and now here k c is given as 0 1, and here B AB inverse and here this phi of A. So, how to get the phi of A? So, this phi of A is depend upon this characteristic equation for this; so s square plus 3 s plus 2. And now corresponding this phi of A equal to A square plus 3 A plus 2.

So, here the controllability matrix Q C is B AB and it is given as 0 1 as B is 0 1, and this AB we will get as 1 minus 1, and if you make the Q C inverse of the same we will get 1 1 1 0, and this phi of A equal to as A square plus 3 A plus 2 I. And here, if you replace the A as 0 1 minus 9 minus 1 multiplied by 0 1 minus 9 minus 1 plus 3 0 1 minus 9 minus 1 plus 2 0 0 2. So, after solving we will get minus 7 2 minus 18 minus 9.

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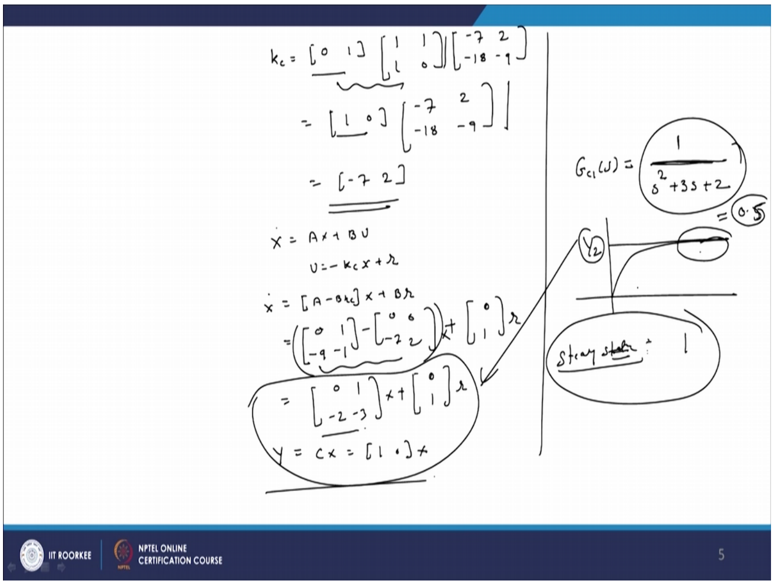

And therefore, the final controller k c is given as 0 1 Q C inverse is 1 1 1 0, and here minus 7 2 minus 18 minus 9. Now if you multiply this by this, so after this we will get 1 0. And here we will get minus 7 2 minus 18 minus 9 and here multiplication by this we will get minus 7 2.

Now for this particular plant that is X dot equals to AX plus BU Y equals to C X AB C we know, there we have desired poles minus 1 and minus 2 and by means of Ackermann's formula; this is Ackermann's formula we have got the controller k c.

Now, when a controller will come into picture, what will be the, it is closed loop system or a closed loop transfer function, and we have to see the response of that is closed loop transfer function. So, here as X dot equal to AX plus BU and as U equal to minus k c X plus r and here we can get A minus into X plus B into r. And if you solve it will get 0 1 minus 9 minus 1, 0 0 minus 7 2 plus B this is 0 1 into r. And after swallowing this, we will get 0 1 minus 2 minus 3 this is a, this is x, x plus 0 1 into r. And output Y is C x, so it is 1 0 into X.

So, we will find that by considering in the controller in a system we have got the state space model in this in this form X dot equal to A minus B k X plus B into r and Y equal to C X. Now we will find that this particular system it is in a companion form; 0 1 minus 2 minus 3 and here 0 1 C 1 0. So, this from this from these equations we can directly write down a transfer function as G C l of s that is closed loop transfer function. So the numerator, the numerator it depends upon say c so here is 1 and the denominator it depends upon this. So, here we can write down as s square plus 3 s plus 2. Now this is the closed loop transfer function.

And if you determined the response of the system; so response as we have press the poles here minus 1 minus 2; that means, system is of over time nature, but about the steady state. So, here steady state for this plant for state input is 0.5. So, it is 1 by 2 there is 0.5 and response is like this. So, 1 by 2 like this, but our reference is 1, but output will got 1 by 2; that means, there is a steady state error.

And therefore, by means of state feedback design we cannot say that we have got the desired result, because the desired results depends upon a transient issues as well as steady state. Transient issues we have solve because of minus 1 minus 2 their pole place, but there is a issue with the transients; that is the issue with respect to steady state because steady state is 1 by 2 and 0.5, but our requirement is that steady state steady state that is equal to say 1. This is the our requirement. Now how to solve this problem?

Now our main purpose is to solve this problem, now we will see how to solve this problem.

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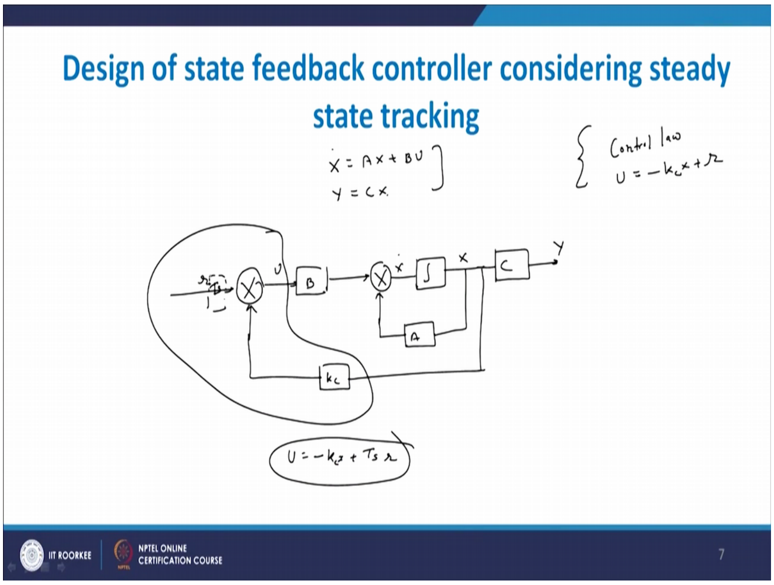

So, here we are trying, we are taking the plant as same X dot equal to AX plus BU and Y equal to C X. Now, we develop the block diagram for this plant. So, we have a summing block, and here is one integrator, then we have output say C, and here is Y the here is state X because of integrator it is X dot. Now this X dot equal to AX, and here is this is B, and here is U. So, here X dot equal to AX plus BU and Y equals to C X. Now our control law, our control law is that is U that is equal to minus k c X plus r. Now here this is U, so this U here this is a control law, now here is a reference input r and here is a controller k c X plus r.

So, base this is a control law, this is state space equations, and now we have got this diagram. Now here is the problem about the output Y, whenever we get the output there is a issue with the steady state. That means, all know we are controlling the states, we are getting the in good transient phenomena or transients have been improved, but there is a problem with the steady state. Now how to make the arrangement; now the arrangement which will going to make like this.

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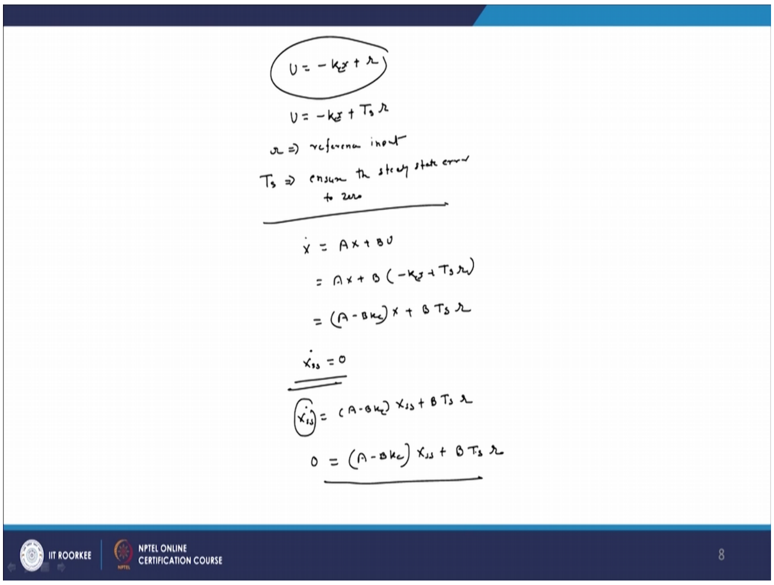

Now we will design a control law in a such a manner that suppose if you take U equal to minus k X plus Y k X plus r this is the original as per this logic. Suppose, here if you write the control law as U equal to minus k X plus this small T s a constant value multiplied by r; where r is the r belong to the reference input and T s which is to be computed in order to reduce the steady state error; that is the T s is computed to ensure the steady state error. That means, a T s we have to design such (Refer Time: 18:00) that the error to be 0, therefore now here control law has been changed is like this that is here r you can add 1 block of T s. And now here we will find at here this is your control law has been changed to U equals to minus k c X plus this T s into r.

So, this is a T s is a constant which helps in reducing the steady state error. Now we will see mathematically how this T s helps in reducing the steady state error. So now, here X dot equal to AX plus BU, now here AX plus BU write as k X plus this T s into r. Now here equation has changed to: A minus B k X plus B into T s into r. Now this equation we have determined. Now what is the steady state? Steady state is nothing but the equilibrium point. So, it is a constant. So, the derivative of any constant become 0; therefore, what we have now here steady state X ss dot that is equal to 0. That is a condition for steady state.

Now here we write the equation as X ss dot A minus B k X ss plus is controller is control is k c B into T s into r. Now your controller is k c, A minus B k c k ss plus B into T s into

r. Now our main performance is that at steady state this is equal to 0, so this equation can be written as A minus B into k c X ss plus B into T s into r. Now we have to solve this equation and see the result.

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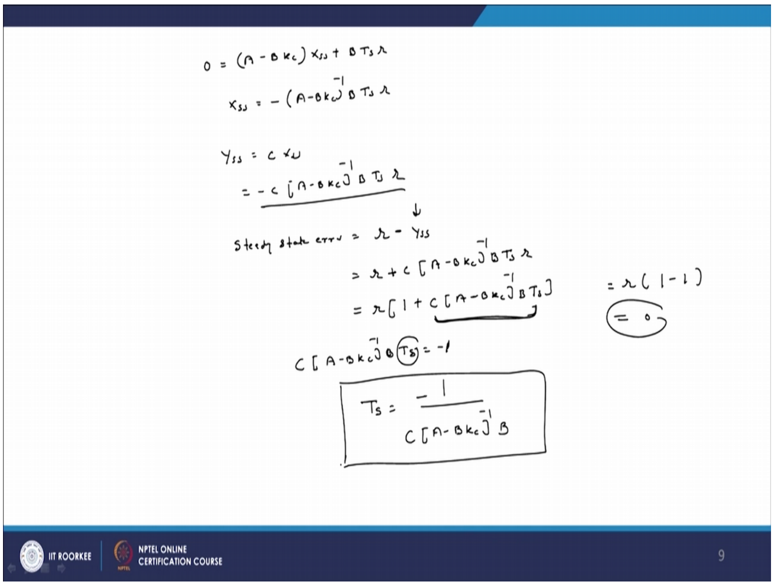

So, we will we can write 0 minus A minus B into k c X ss plus B into T s into r. Now here X ss steady state we can write down as A minus B into k c inverse into B into T s into r.

Now, here output Y ss equal to C X ss; so we can write down as minus C A minus B k c inverse B into T s into r. Now here is the steady state output. Now in the steady state output we have C A minus B k c inverse B into T s into r. If you see if there is no T s the equation is in this equation there may be no T s. Now we have got Y ss. Now about the steady state error, the steady state error the steady state error is given as a reference r minus steady state output that is Y ss. So now, we replace the value of this here we can write down as r plus C A minus B k c inverse B into T s into r. Now take r common, 1 plus C A minus B k c inverse B into T s.

Now, what is, what we want. What we want that the steady state error must be equal to 0. So, when this is possible. So, this is possible if this, if this equal to minus 1; so if this is equal to minus 1 this equation is there that is called steady state error and what we want that; we want that this particular error to be 0, so this is possible with this particular equations which we have written that should be minus 1. That means, your equation should be r plus 1 minus 1. If it is there, that means, your error is 0 that means, this equal to minus 1. So, we can write down as C A minus B k c inverse B into T s equal to minus 1 and now here we written minus 1.

But here the C a AB and here all this parameters which are known and that cannot change; the change we can do here in the T s. That means, we have to design T s in a such a manner that that makes steady state error is 0; that means, your result is minus 1 divide by C A minus B k c inverse into B. So, this is the compensating factor for steady state error that is a T s. So, this T s we have to arrange here. So, if we make the arrangement of T s r we will find that error will be 0.

Now what you do in order to see whether the result or the mathematics we derived is matches with the matches with the desired result or not. For that purpose we will take an example which you have taken earlier and we see the result.

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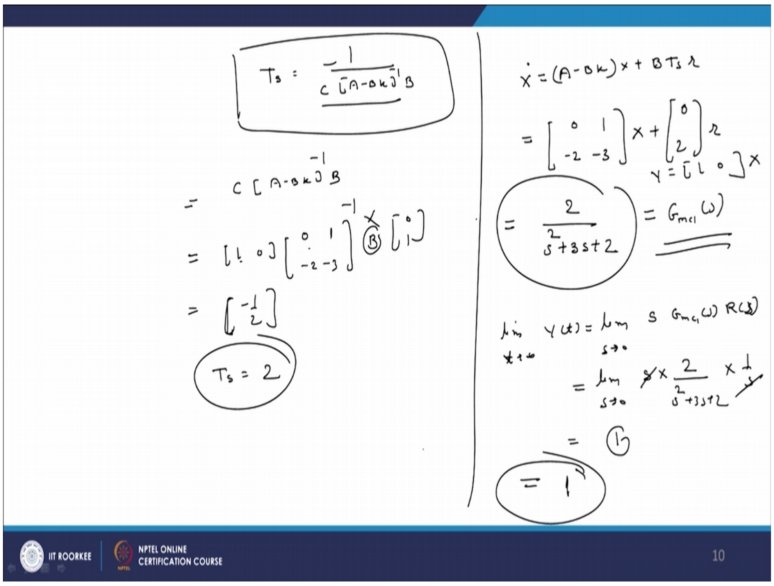

So, this T s equal to minus 1 divided by C A minus B k inverse into B; now this is the T s. Now we calculate this factors that is C A minus B k inverse into B. Now we will see A minus B k earlier. So, A minus B k is 0 1 minus 2 minus 3. So, here as same problem C is 1 0, and here we write down as 0 1 minus 2 minus 3 this is inverse into B, and here it is equal to if you solve it because here B is here 0 1. So, if you multiply this, this and this we will come minus 1 by 2. And here, if we put it here in this T s we will get this T s as 2.

So, now for the problem which you are taken here we are got additional parameter T s therefore, now my our equation is X dot equal to A minus B k into X plus B T s into r. Now here our A minus B k is 0 1 minus 2 minus 3 X plus. So, B 0 1 and because of T s is 2. So, we can write down as 0 into 2 into r. And therefore here, if you solve this that is X dot equal to A minus B k X 0 1 minus 2 3 r plus 0 2 into r and now if you solve it we will find that the equation for this is 2 upon A square plus 3 s plus 2; that is the modified transfer functions that is this is called as G modified C l of s. And here this is evolve this problem Y equal to 1 0.

And therefore, the transfer function for this type of model equals 2 divided by s square plus 3 s plus 2. Now if want to determine the steady state error. So, steady state error means limit t tend to infinity of Y of t equal to limit s tend to 0 of s into G m C l of s into step input R s. And here, we will find that limit X tend to 0 of s divided by 2 upon s square plus 3 s plus 2 into R s is 1 by s, so this is cancelled. And now here replacing s 0 we will get 1.

The steady state 1 we have got; the steady state output is 1 error is 0 and therefore, the whatever is the required results that is satisfied.

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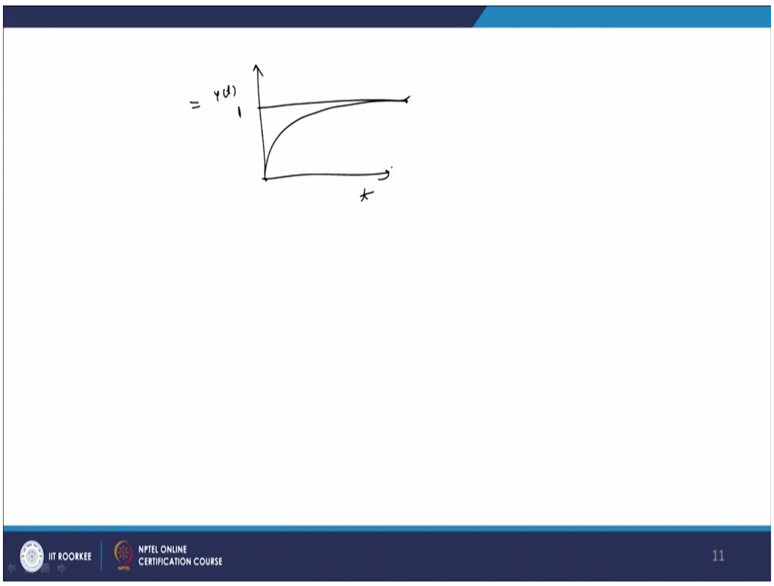

That means, in this case the response of the system let us say t say Y of t we will get steady state of 1; this is 1. So, we will find that because of their additional parameter T s; T s in this plant we have got the steady state error 0 and that means output has reached properly to steady state; that means, the output track the reference input. So, this was not possible because of the conventional state feedback.

So because of this, this is possible. So, we can use this type of ripples for different problems. Now here I have solved 1 example you can solved as many example as possible. And let us see whether this methodology has satisfy the steady state error or not.

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Now, references for this. This is Ogata, then this is Stefani et al.

Thank you.