Advanced Linear Continuous Control Systems
Dr. Yogesh Vijay Hote
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture - 33
Pole Placement by State Feedback (Part-I)

Now we start with Pole Placement by State Feedback.

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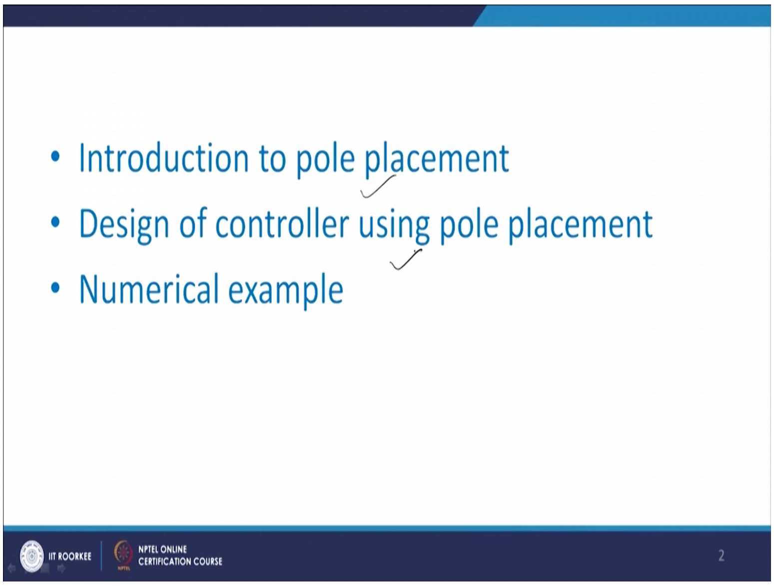

In this we will study introduction to pole placement, design of controller using pole placement and we will see one numerical example now.

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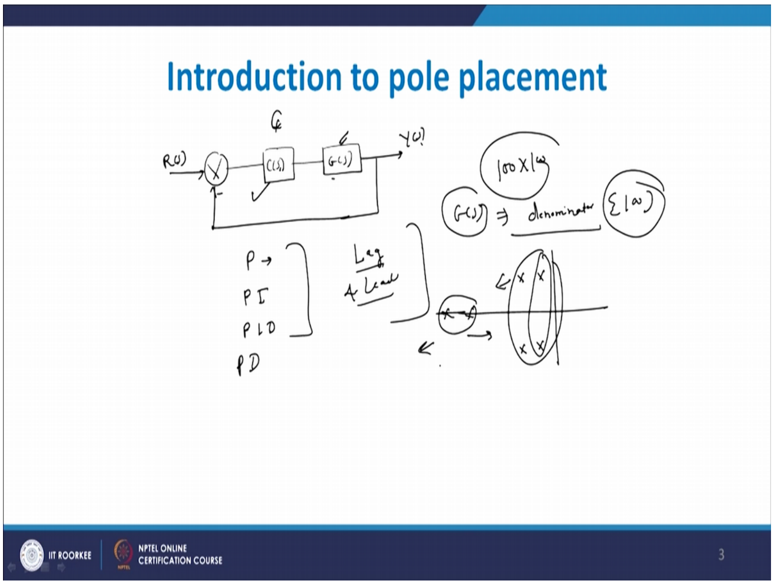

Introduction 2 pole placement basically what is mean your pole placement pole placement means that we have design a controller. It is similar to the controller which we are design in a classical approach. For example, in classical approach what we are seen? We have a plant it is like this.

Let us say this is controller C of s, then this is a plant G of s now this is feedback control system its negative say R s and Y s first of all why this need for controller? A controller is needed because this is a plant G s in open loop response is not satisfactory, then without controller means without taking this distance. We have taken a feedback control system; still performance is not satisfactory and because of that we have designed a this controller C of s. We know that the classical controllers are P controller there is proportional, then proportional integral then PID, P PI PID sometimes even a PD also other controllers are say lag and lead.

These are the classical controller. So, what is the problem with this controller is that as our purpose is to get a desired result whatever is our objective is that need to be achieved. So, we will find that if this G s all the plant of is a very high order, let us say 100 order plant 100 band rate or the G s G s the denominator order is say 100 denominator say order is a 100. So, in that case if you design a PID controller can we get the satisfactory results result means what? Let us say let us say there are this poles are here these are poles, and here is somewhere these are 2 poles.

Now, here are problem is that the poles which are nearer to imaginary axis that need to be shifted far away. So, if you what do shift this using the PID controller. So, what will happen these poles may shift this may shift, but what about these 2 poles? So, if may possible that this pole may more like this or may more like this, this no guarantees therefore, if you want a desired response. So, desired performance for higher order system this is not possible by means of a conventional classical controller.

So, here we can use a state feedback controller.

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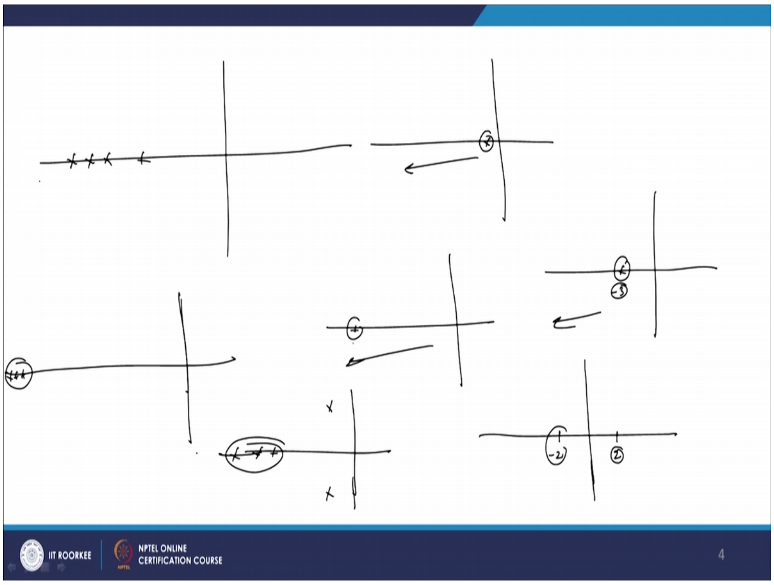

So, what is the advantage of this state feedback controller is that, you can place pole anywhere in the; you can place pole here, you can place here you can place here then you can place here someone we say no I we want to place the pole at the (Refer Time: 04:26) only here. So, this is possible because poles governed a performance. So, you have a desired object that you can place the pole anywhere it means that, you can get the desired response desired performance can be achievable.

But there are some issues with respect to this. Problem is that, if you move rgese poles let us say original pole is at here. So, the pole which is here it has say will last time constant, but if you moving here this pole this side. So, what is the time constant time constant is reduces and when we are doing a some measurement, because our approach is state feedback state we have to measured. So, when we are present pole far away where time constant is very very small, we required high quality sensors means acquire censored, then required actuator the large actuator that is the motors.

So, all these involve a more cost and again there should be again we needed to means we have to make it available, sometime it may not be available. Therefore, there is an issue about the pole placement. We got place the pole anywhere all though we can resign theoretically practically, there are some issues. Again the problem is this pole placement is that, that we have to measure the all the states. If some states cannot be measurable, then there are some issues we cannot design a controller. Although in future we will see design of the observer, but in actual practice if you want to design a controller we have to measure the all the states, and second as we have seen that as we move the poles far away from imaginary axis, then in that case we required a very high quality actuators so, there is some the problem.

Therefore, whenever we design a pole placement first of all we have to see the requirement. Suppose if you have a pole at here now let us say as minus 3. So, if you know if the pole at a minus 3, if it at proper positions. So, there is no need and to move this pole left side because our requirement is satisfactory; that means, you have to move the poles left side when there is a requirement.

If you want to reduce a some control energy let us say one pole on the right hand side there is 2 you can shift at say minus2. So, if you make it like this that is another system has pole has 2 if we move at minus 2 we required very less control energy. So, that is some say thumb rule we can also use. Other thing which can be used that, because the system sometimes required transient response. So, we can place the pole that is some poles are here. So, that the transients issues resolved and other poles we can place somewhat here on the real axis.

So, depend upon the problem, depend upon the availability, depend upon the cost associated and mainly the availability. So, we can design a controller, but the most important thing is that if we having the all facilities everything is available then, in that case this pole placement is better technique than the classical controller, but in most of the times we do not have the resources. So, in that case that this classical control is far from better. Because in a classical control it depend upon the input R s and output Y s, but whereas, if you (Refer Time: 08:05) takes place we have to measure the all the state which are involved in the system.

Now, we will see, how to design a controller using pole placement technique.

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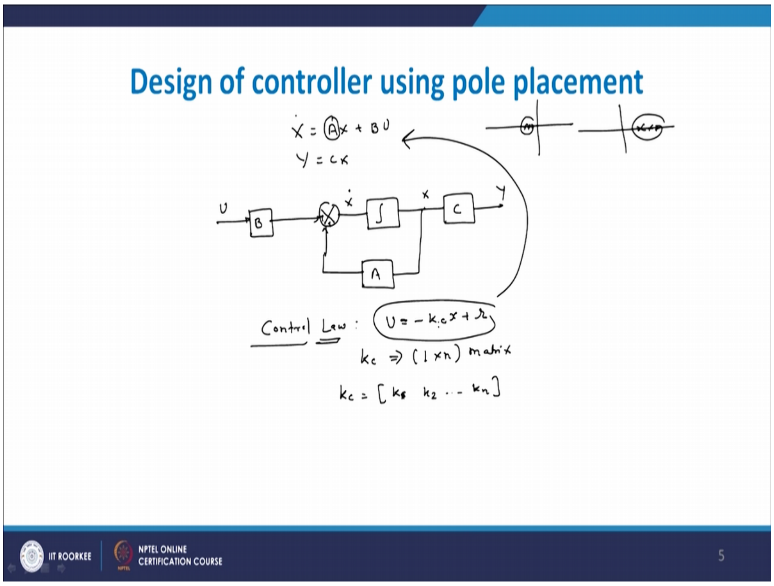

Now we are designing the controller using pole placement techniques so, how our system is basically X dot is equal to A x plus B this is our system Y equal to C x; and here we are designing controller because A is the system matrix, the system matrix A has pole maybe on the right hand side, it may have poles which are near to imaginary axis; that means, this is the conditions, A matrix may have eigenvalues which are nearer or it may possible that eigenvalue of the right hand side that system is unstable.

So, we have to stabilize this A or we have to improve the performers of the A; now if you draw the block diagram for this. So, you can relate now here is one integrator, now this x now here this is C this is output y now here is A matrix now here x dot this is x dot now here is B this is u. So, you will find that for this case your plant is this X dot that is equal to A x plus B u and y equals to C x.

So, this is the broader on this step. Now we have to design; a controller the controller we design; that means, you have to design a control law and this control law is through input; that means, u is place the important role how much input we have to provide? Therefore, you must be taken into considerations and this u must have relationship with the controller therefore, the control law which we can consider as the control law which we can considered as U equal to say minus k c x plus r this is a control law, where this k c is belong to 1 cross n matrix; that is this k c is nothing but ks k 1, k 2 up to say k n.

So, this k 1, k 2, k 1; that means, this one k 1 for one state, k 2 for other state k n for another state because this basically graded to the input and with the state that is why the U equal to minus k c x plus r. So this is the control law, now we have to use this control law in this equations f x dot equal to A x plus B u. Now if you replace u equal to this in this particular equations.

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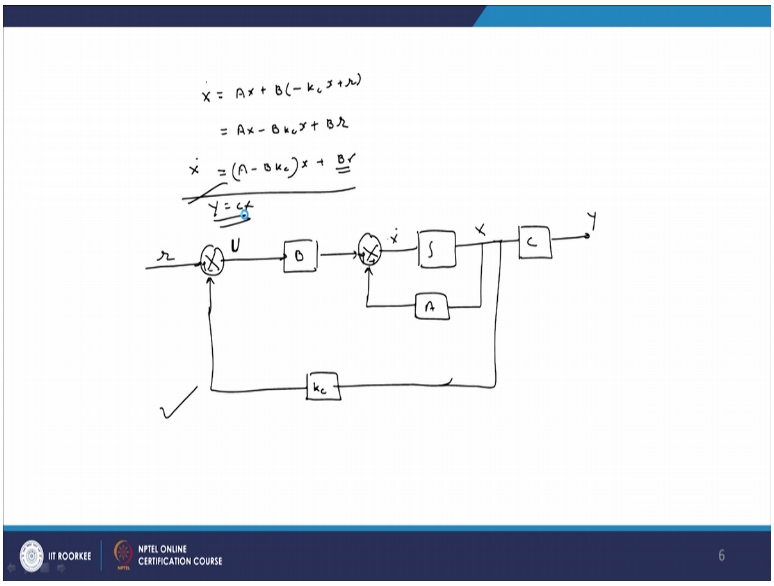

So, we will get we will get this X dot equal to A x plus this is B, u is minus k c x plus r now if you solve it. So, it will be the A x minus B into k c into x plus B into r and now if you solve it, it will be A minus say B to k c into x plus B r now this is x dot.

So, though this is the equation using the control law whereas, original equation is like this. So now, we have draw on the block diagram for this model now as our system has changed. So, you come into existence our controller is come into existence therefore, our block diagram is changed. So, now, we redraw the block diagram as. So, here let us say this is integrator C output y.

So, here x dot this is state x this is the integrator, now here this is A, this is B, here has like this is r, now it is positive and now here we have got x dot equal to A x plus B u and now what is this is u you will find that, this u equal to minus k c x; that means, here we have to use a control law as this k c is a controller. So, it is negative and it is coming to the state x. So, if you see is x dot equal to A x B u where B u, u equal to this equations has been computed and here Y equal to c x.

So, this is the block diagram of the system and now it become a close loop, because the state has been control here there is no access control in effect. Now we will see how to design a controller using state feedback control approach and whereas, or we can say the control law which we are taking here u equals to minus k c x so, here the plant which you will take.

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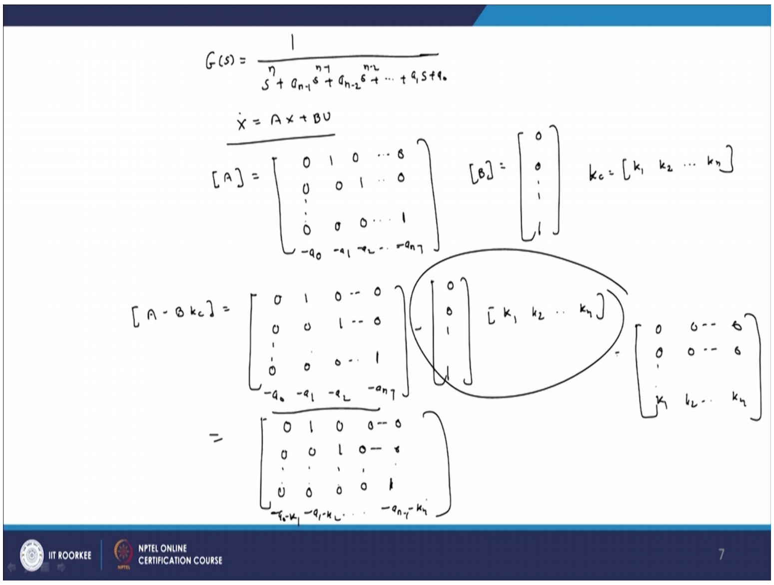

Suppose the plant G of s is taken has 1 divided by S raise to n plus a n minus 1, s raise to n minus 1 plus a n minus 2, s n minus 2 plus a 1 s plus a naught now, this is a G of s.

But now here we are designing a controller for a system which is represented in a states space form. Therefore, the state space model of the system it is in terms of x dot equal to A x plus B u and therefore, here we will take this A matrix as, this A matrix will write in a companion form that is 0 1 0 0 then here 0 0 1 0 and here this process repeated it is still 0 0 1. And last row you are write a naught minus a 1 minus a 2 up to minus a n minus 1 this is a matrix, and now here b matrices 0 0 say 1, and here this k c is nothing but k 1 k 2 up to say k n, this is controller k c a we have written b.

Now here we are designing a controller because the performer of this system is not satisfactory therefore, we are designing the controller. So, here as per the control law, u equals to minus k x plus r. So, we have got the system matrix as A minus B into k c. Original system matrix is A, A has some problem. So, the modified system matrix is A minus B into k c therefore, here we can write down this matrix as A minus B k c that is equal to 0 1 0 0 0 0 1 0 and here we are get 0 0 0 1 and last we write a 0, minus a 1, minus a 2 to minus a n minus 1 minus b is like this 0 0 say 1 and here it has been multiplied by k 1m k 2 say k n.

Now, this we have to multiply we have to subtract from this. So, after doing it we will get. So, we will get as 0 1 0 0 0 we have get 0 0 0 1 0 and lastly here 0 0 0 1 and in the last row we will get minus a 0 minus k 1, minus a 1, minus k 2 and here minus a n minus 1 minus k n. Because when this has been multiplied to this equation is nothing but 0 0 0 0 0 0 and last you will get k 1, k 2 up to k n therefore, when we subtract this from this you will get a 0 minus k 1, a 1 minus k 2 minus a n minus 1 minus k n.

Now, for this we have to determine the eigenvalues because the eigenvalues govern the performance.

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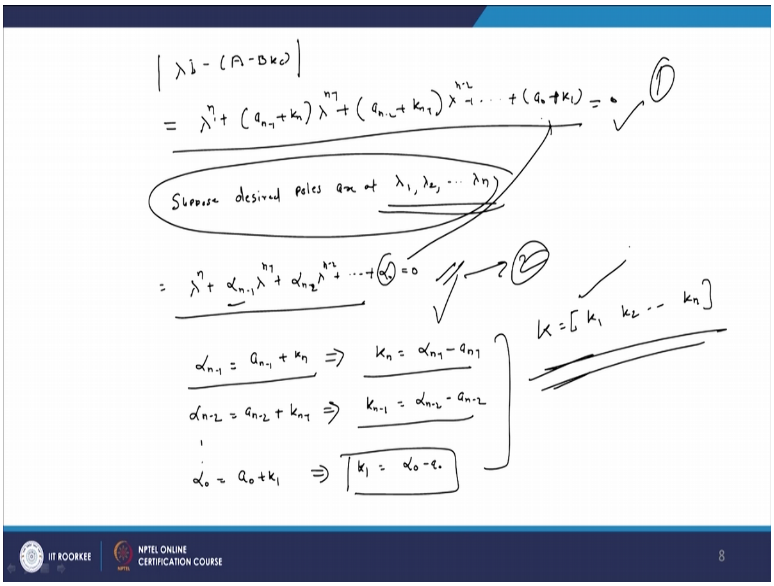

So, the eigenvalues of this system we can write down as lambda i minus A minus B into k c that is equal to. So, we will get lambda raise to n plus a of n minus 1 plus k n lambda n minus 1 plus a n minus 2 plus k n minus 1 lambda n minus 2 and this process repeated plus a 0 plus k 1.

So, if you see the equations. So, this is in companion form. So, what is the equation companion form; we can directly write down the equation like this. So, lambda raise to n a n minus 1 k n lambda n minus 1, a n minus 2 k n minus 1 lambda, n minus 2 a 0 plus k. Now here this is the equation this we want, but here we know the though the original system matrix say, but this k 1 k 2 k 1 we do not know. So, they need to calculate this they are how to get it. So, here in this case we have to decide where we want to place the poles.

So, we assume that our poles which are required to be place let us say it suppose the desired poles are at say lambda 1, lambda 2 say up to lambda n these are my desired poles. So, it can plus anyone in they explain. So, it has any values, but these are my poles. Now if you based on the desired poles we can also obtain the caustic equation. So, using these desired poles the caustic equation can be written as lambda raise to n plus alpha n minus 1, lambda n minus 1, plus alpha n minus 2 lambda n minus 2 plus alpha 0 equal to 0.

So, now this is the caustic equation this is our equations. So, these 2 equations are there and here we have find that this is my desired one; and this and this they have the same form lambda raise to n lambda raise to n. These are the non quantities and here a n minus 1 a n minus 2 a 0 these are non quantities, but which is are unknown is k 1 k 2 up to k n and this is nothing but the controller. So, if we compare these equation with respect to this equation this 2 equations here, this less the equation over 1. And let us say this is equation number 2 if we compare this 2 equations we can easily get the balance of k 1, k 2 up to k n now we compare this equation.

So, if you compare this equation let us say alpha n minus 1 that is equal to a n minus 1 plus k n. So, if you solve it will get this k n equal to alpha n minus 1, minus a n minus 1 this we get then about this alpha n minus 2 that is equal to a n minus 2, plus k n minus 1. So, after solving it will get k n minus 1 alpha n minus 2 minus a n minus 2 and now we come to this equation alpha 0 and this. So, if we compare this part. So, we will get alpha 0 that is equal to a 0 plus k 1 and again if you solve it, we will get k 1 equal to. So, here k 1 we will get similarly get k 2, k n minus 1 and k n.

So, in this way we how got the controller as k 1 k 2 up to k n. So, this controller k we will stabilize the system as per our requirement. As per the simulation is considered you can desired the pole anywhere whatever we required that desired pole space model is possible, desired controller is possible, desired performance is possible. So, this is we can see the beauty of the advanced control approach or a state feedback approach where this is not possible in case of the classical approach. But although, I will told that all the theoretically is ok, but practically there are some issues. As I told you that we required high quality actuator even a sensors because we have to sense.

So, if the if you far away the control input is more and that times if may got some problems that is we required very high quality actuators. So, now, this is the finally, controller and our main purpose is to design this controller k and we have already design it, but it has been design through a mathematical approach that is for nth order. Now we will see how to get the volume of the controller from a numerical example. So, we will solved a numerical example.

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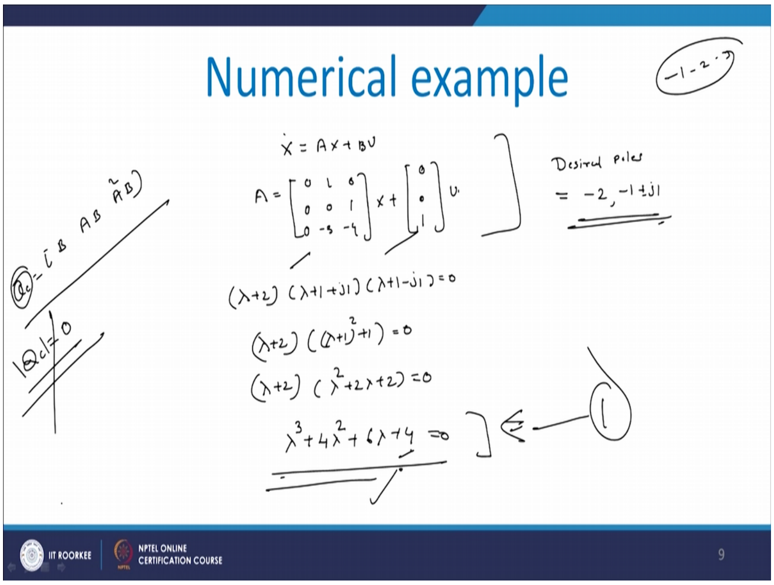

So, your problem is X dot equal to A x plus B u, now here A matrix is 0 1 0 0 0 1 0 minus 3 minus 4 x plus B is 0 0 1 into u.

And our desired poles that the desired poles are at minus 2 minus 1 plus minus j 1. These are our desired poles you will find that the desired poles in what we were taken the desired pole we have taken in terms of minus 1 to j 1 that is basically for the satisfied by the transient issues and minus 2 if the real poles. You can take desired poles as minus 1 minus 2 as well as minus 3 also.

But in that case your system is completely odd and then there are no transients, sometimes for faster response we have to make this type of arrangement also now these are my desired poles, you can take other poles also. And now we want this poles so, what will be the value of the controller? So, for desired poles we solve the equation as lambda plus 2, lambda plus 1 plus j 1 lambda plus 1 j 1 equal to 0 then here lambda plus 2 lambda one whole square plus 1 equal to 0.

Now, if you solve it lambda plus 2 lambda square plus 2, lambda plus 2 equal to 0 and after solving it we will get lambda cube 4 lambda square plus 6 lambda plus 4 equal to 0. Now this is my desired caustic equation, this is we have got that is this is similar to our equation lambda raise to n plus alpha 0. So, this equation we have got here. Now this is desired one, but when we are using the control law that is u equals to minus k x in the system. So, we have got the equation in terms of a minus b into k c.

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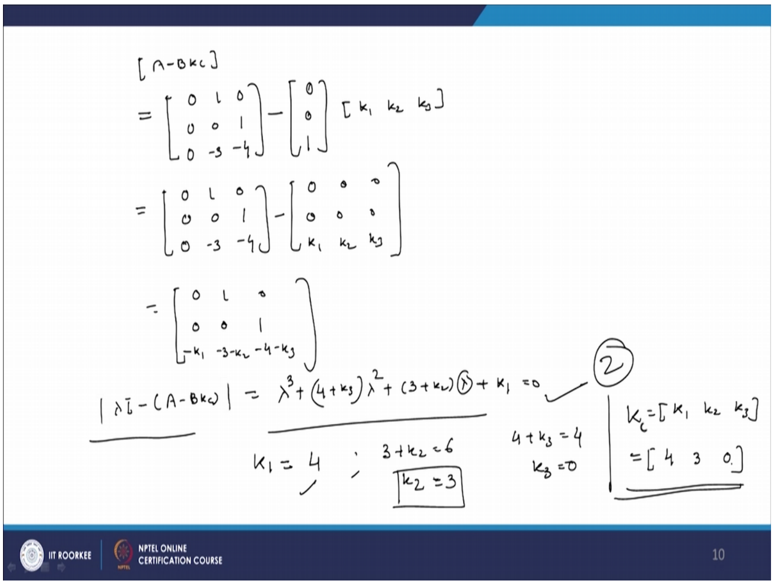

So, we determined this equation as a A minus B to k c that is equal to 0 1 0 0 0 1 0, minus 3, minus 4, 0 0 1 k 1 k 2 k 3. Now, we have to solve it. So, here we will get 0 1 0 0 0 1 0 minus 3 minus 4 and here after solving it we will get 0 0 0, 0 0 0 and we will get k 1 k 2 k 3 now we subtract this separation from this. So, we will get 0 1 0 0 0 1 because all the zeros. So, after subtracting we have same 0 1 0 0 0 1, here we will get 0 minus k 1, that is minus k 1 then here minus 3 minus k 2 and here minus 4 minus k 3.

Now, here A minus B into k c equation we have got, now we have to determine its caustic equation in terms of lambda; so lambda i minus A minus B into k c. So, it gives lambda cube plus 4 plus k 3 into lambda square plus 3 plus k 2 into lambda plus k 1 equal to 0. So, this equation in a companion form reverse way you see say minus k 1 minus 3 minus k 2 minus 4 plus k 3. So, this equation we have got.

So, this is the equation we have got from a from the original plant and in that case we will find that k 1 k 2 and k 3 are there. So, now, this equation we obtain and now we see the this equation, this is the desired equations now if we compares let us say this equation number 1 and this equation number 2, now if we compare these 2 equations. So, what will get? We will get this k 1 equal to. So, we will just see what is k 1 here; so k 1 equal to 4. So, we will get k 1 equal to 4 then about another term this lambda term and here is a lambda term. So, you have 6.

So, this 3 plus k 2 this equal to 6, so here k 2 equal to 3, so k 2 equal to 3 and about this last one: so here 4 plus 4 into lambda square, so here 4 plus k 3 into lambda square. So, if you compare its. So, 4 plus k 3 equal to 4 So, we will get k 3 equal to 0 therefore, your controller k it is k 1 k 2 and k 3. So, that gives you 4 this k 1, the k 2 is 3, and k 3 is 0 now this is the controller k.

So, this controller k we stabilize the system in such a manner that we are getting the desired performance. If you change the pole press band you will get the different values of the k. Here we will find that this given system matrix x dot equal to a x the view, that is in terms of a companion form that is basically a transfer function form and when the transfer function is there directly or when the system is given in the companion form, we can directly write the caustic equations there are no such issues.

But when the system is in the in any of the general form, then this type of things it is not easily possible we cannot direct write down with the correct equations. Therefore, we required some alternative method by which we can design a control room, that is our next expertise that how to design a controller when the given system matrix say it is in general form. And other most important point is that whenever we design a controller we have to check the controllability observability of the system. Here we are directly design a controller, but actually when we designed a controller for any system we have to check the controllability and observability of the system; that means, particularly this case we will find that we have to basically important the controllability that is where you determine B AB A square into B.

So, all this operation is based on the rank of this matrix. If the rank of this matrix or a determinant of matrix let us say 0 or whether rank is false, then this process we cannot use this process we cannot design a controller. So, controller design when this Qcs non zero and the as I told you that this is the problem or these analysis we have done only when, you having a G s that is in a transfer function form. But now our main task is to how to design a controller when a given system matrix in a general form.

So, that part we will see in the next class.

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So, now you see the some references.

Thank you.