Advanced Linear Continuous Control Systems
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Lecture - 32
Observability in State Space (Part-II)

We start with Observability in State Space: there is a Part-II. In this we study detectability.

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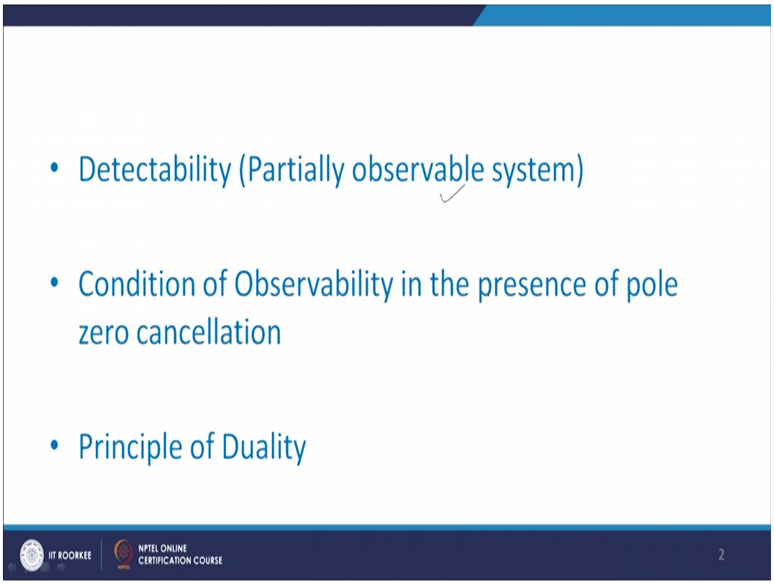

That is partial observable system; then condition of observability in the presence of pole 0 cancellations, then principle of duality. Now start with the detectability that is partially observable system. In earlier time we have seen a general example of understanding of controllability observability. Again we have seen stabilizability which is the call a partially controllable system here also we have to see a partially observable system what is mean by partially observable system?

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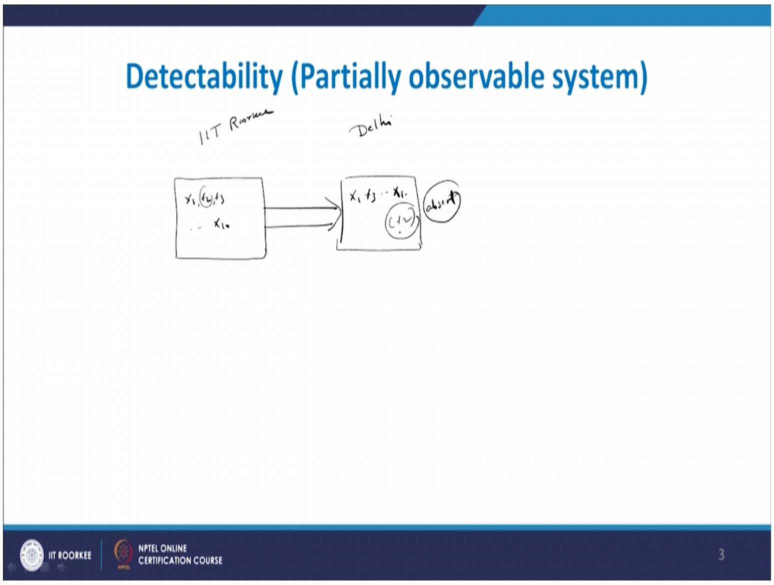

Let us say there are 10 students, they are going from let us say from IIT Roorkee there are 10 students. So, each student we represent by state. So, x 1, x 2, x 3 let us say x 10 these are 10 students and they are going from for industrial treef; let us say they are going for to say Delhi, and Delhi these have industry and they have to go. So, all the state which are here they have to wish to Delhi. So, it is found at out of this all the states x 1, x 3 up to x 10. So, these nine state have been observed whereas, this x 2 state it is not found here. So, this x 2 is absent.

So, here we found at if any of the state is missing system is unobservable, but here we found at the state x 2 which is absent or the student which is has to wish 2 industry he is not found, but what we are found at he somewhere at a safer locations that is he is safe this no as such issue. So, you get instructor who is been travelled with the student he is found that that particular student, he has to he has to reach to the company or industry, but he has not reached, but somewhere he is safe and these no as such worry. So, what happen the teacher what will do he will take nice student to industry and living only that that particular state.

So, in that case we called as it is called a partially observability system; that means, all 10 has to reach, but some problem you cannot reach, but still you can say that system is an observable, but here it is called as partially observable system.

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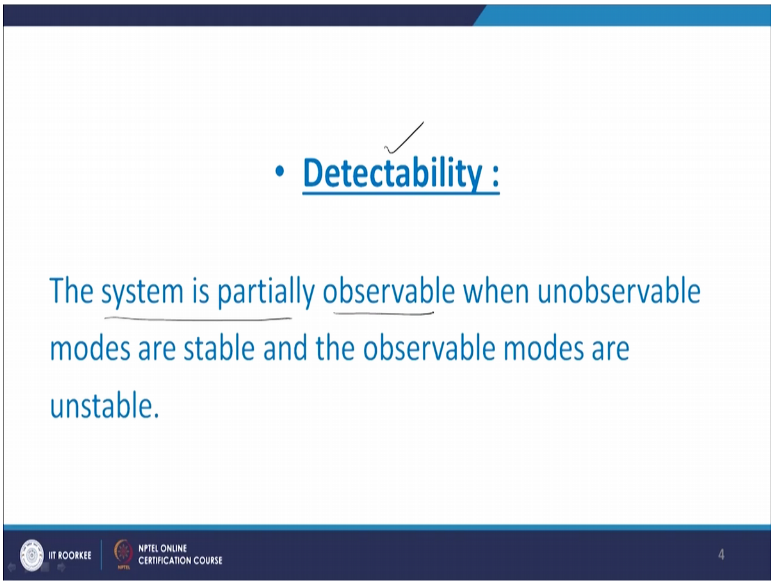

So, this partially observable system it is call as detectability. So, how you define this? The system is partially observable when unobservable modes are stable and observable modes are unstable now we will see two example.

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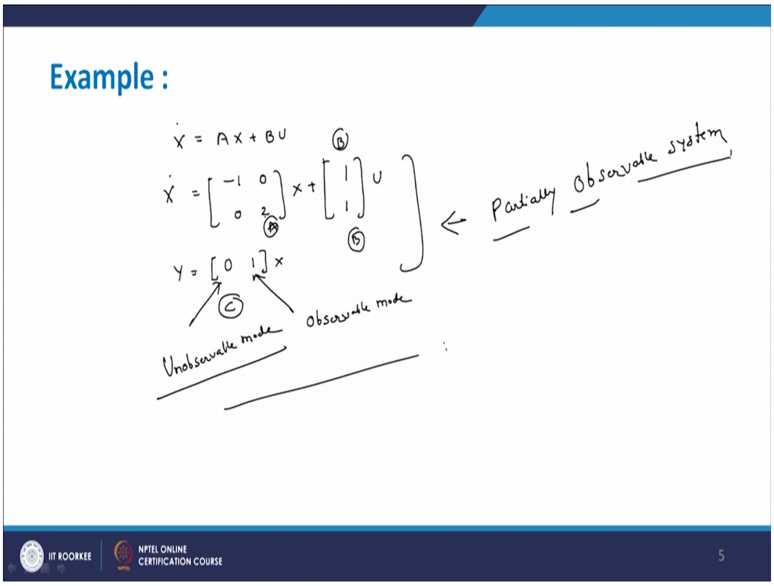

Suppose write x dot that is A x plus B into u, x dot equal to A x plus B u now here this a matrix is represented by minus 1 0 0 2 x plus 1 1 u that is here b this b matrix; so x dot equal to minus 1 0 0 minus 2 x plus 1 1 into u. So, here this is a matrix in a matrix minus

1 and 2 are the diagonal elements and 11 this is A B matrix. So, will find at these particular system is controllable because here elements are 1 1.

Now, we think about the observability, nowhere output of this system we can write down as 0 1 into x. Now this is a c matrix we have seen that controllability depends upon A and B whereas, observability depends upon A this A matrix and this is C matrix. Now we will find that corresponding to this minus 1 this is a stable mode, but this part issue unobservable whereas, this 2 is unstable, but it show that one as a observable mode. So, when this is the case. So, this is called as a partially observable system. So, this 0 is show here it is unobservable mode whereas, this one indicate observable mode therefore, in a real practice we can consider this type of system also.

Now, the condition of observability in the presence of pole 0 cancellations; when we are considered the system then in terms of x dot equal to A x plus B y equals to C x plus D u and here we are using either Coleman test or Gilbert test to check the observability of the system. Let us say if the system is not observable, then if you take that particular system convert into as a function can it possible that there is pole 0 cancellations? Now we will see this effect.

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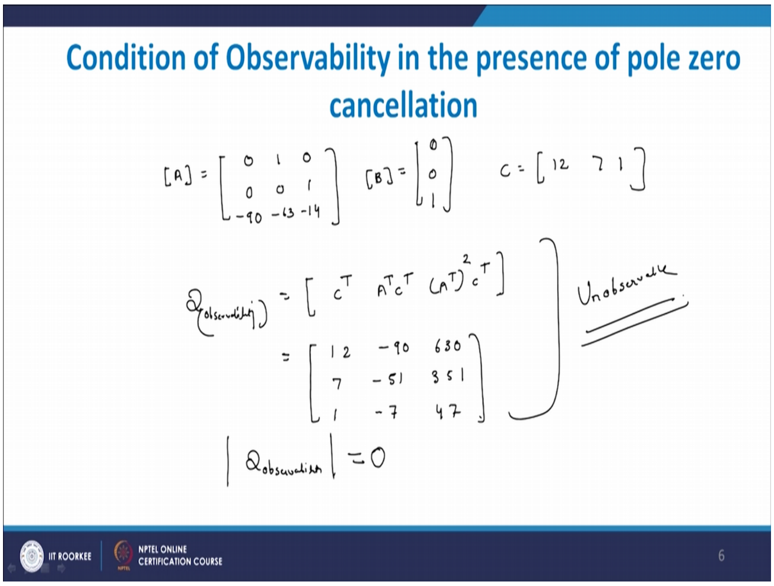

Suppose if you write A matrix as this A is equal to 0 1 0, 0 0 1 and here minus 90 minus 63, minus 14, this is A matrix now here we write a B matrix as 0 0 1 and C matrix as 12 7 1. So, steps involve A B C. So, we have to an a b and c now we have to check the observability of the system. So, how do you check the observability? So, for observability this Q observability that is C transpose A transpose, C transpose, A transpose square C transpose. So, we have to an A C transpose A transpose C transpose A transpose square into C transpose. So, it is this 2 means n minus 1. So, is a order is 3. So, we are written 2; so n minus 1 3 minus 1 that is equal to 2. So, C transpose A transpose C transpose A transpose square C transpose.

So, this is basically a Coleman test now what is C? C is 12 7 1. So, we have to take C transpose then A is this one. So, we have to take A transpose and multiplied by C transpose and similar this one. So, after a doing multiplication here, here will get the observability matrix as 12 7 1, 12 7 1. Now, if you multiply A transpose and C transpose we will get minus 90 minus 51 minus 7 here we will get 630 351 and this 47. So, this observability matrix you determined. Now if you take the determinant of this. So, be determinant of this observability matrix. So, we will find at if you take determinant of this it comes 0. So, this particular system it is unobservable; now this system is unobservable.

Now, what we can do we have a b c, now we convert this given state space model into transfer function.

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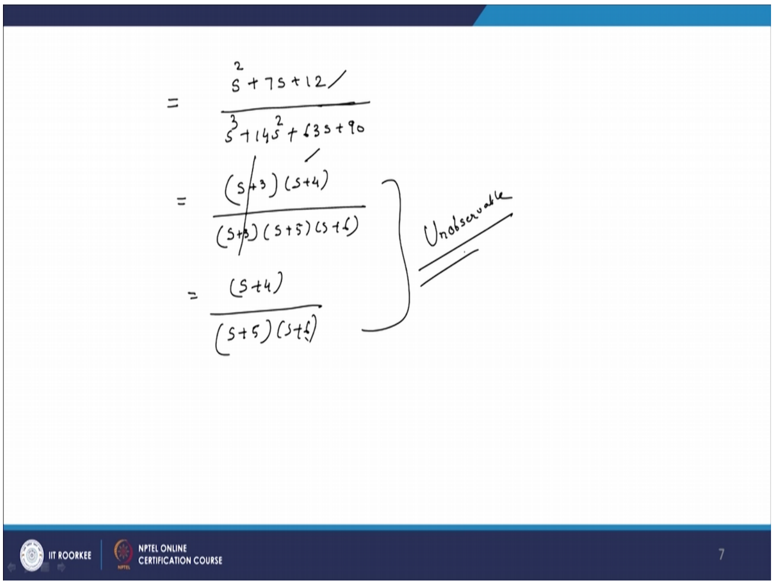

So, we will find that the transfer function of the system is repeated by s square plus 7 s plus 12 divided by s cube plus 14 s square plus 63 s plus 90. If you just see here this is a matrix. So, this is in a compendium form or controllable canonical form because 0 1 0 0 0 1 minus 90 minus 60 minus 43 and 0 0 1. So, this can be easily written in this form, we already seen this how to represent it. So, this transformation we have got, but just looking after this transformation we do not know whether these, a pole 0 cancellation or not.

So, in order to know whether this pole 0 cancellations we have to solve this equation and this equation that is we had to calculate the roots of numerator as well as the roots of the denominator; so the roots of the numerator here s plus 3, s plus 4. So, if you solve it s q s square plus 7 s plus 12 now about this denominator. So, denominator is nothing but the roots of the denominators are s plus 3 s plus 5 s plus 6. So, we will find that s plus 3 and s plus 3 is common and here we can cancel it and remaining is s plus 4 divide by s plus 5 and s plus 6. So, we will find that this s plus 3 have been cancelled.

So, here 16 is unobservable. So, this observability of the system has been tested by even transfer function, this can be tested by using this Caleman form. So, any of them we can use, but here we will find that that if this is the pole 0 cancellation system is itself is unobservable. And here this s plus 3 and s plus 3 these particular state is not available for the measurement, because in a state space design or in a actual practice, we have to design a controller. So, the control design is only possible when states are available.

So, here we will find that for this particular plan, the state is not available therefore; the controller design cannot be possible. So, there how to design a controller in all these issues, we will see later on, but for time being we will find that these particular state is not observe is not exist it is cancelled and therefore, our system we can say as call as unobservable system.

Now, about the principle of duality: so what is mean by principle of duality? So, it is say that if the system s 1 is completely state controllable, we have seen what is mean by controllability.

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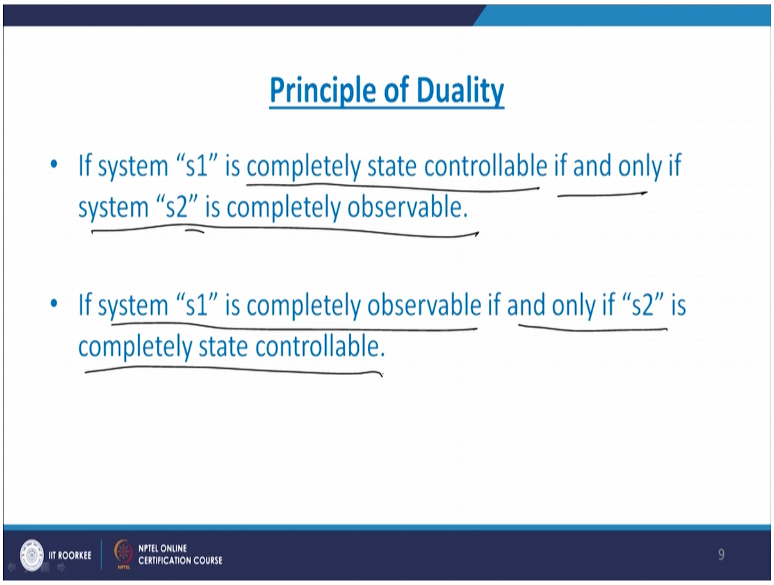

So, if it is completely state controllable if and only if the system s 2 is completely observable that is if we take one system s 1 and take the dual of each it found that they are related that is s 1 is completely state controllable, s 2 is completely observable again the reverse case, if system s 1 is completely observable if and only if this s 2 is completely state controllable. That means, initially there are controllability then observability, then here observability and controllability they are relate to each other.

Now, we will see how they are related; we have seen the different types of form companion form canonical form in this different form different companion forms, we have seen the controllable canonical form then observable canonical form. Now we will see this form and how they are related in case of controllability and as well as in case of observability. So, here first of all write one system s 1 s 1 is a one system as we are written here s 1.

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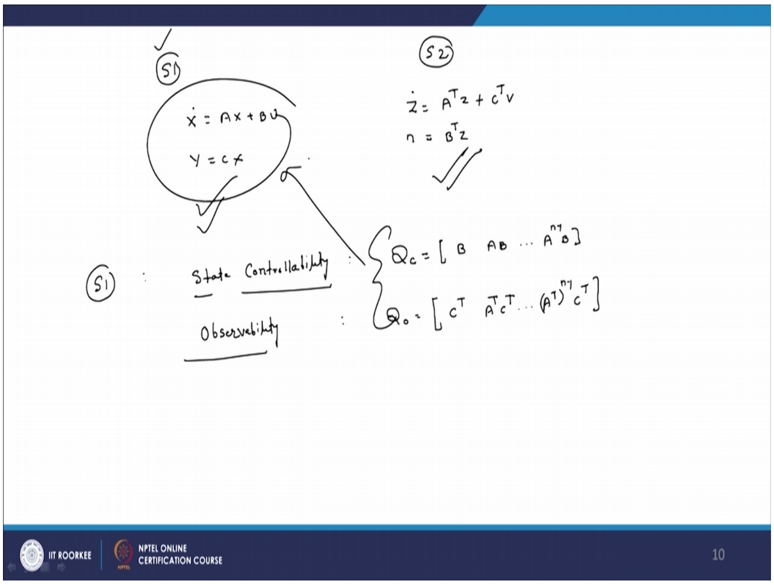

So, this way I have written s 1 here; for s 1 i have we have to take a one model suppose this model is represented by x dot equal to A x plus B u and y equal to C x this is system s 1 this s A is system matrix B is input C is the output matrix.

And now we assume that this particular system it is in controllable canonical form; and now the same system we can write in terms of observable canonical form. So, we will write observable canonical form, we take a different states let us say we have a system s 2 like this. And this s 2 written as say Z dot to A transpose z plus here the B in this case is nothing but the transpose of c that is C transpose

So, vector we can take instead of x we can take v and output let us say n can be as B transpose into z, that we see here this B instead of B we have got C and instead of C, we have got b now there system s 1 and system s 2. So, here we can take a different puts different outputs, but now assume that the that the inputs are for s 1 inputs for s 2, but the thing is that this a and A transpose c is coming as C transpose and b is coming as B transpose.

Now, what will do we will check the controllability and observability for this system also the controllability and observability of these system s 2 s 1 and s 2. Now for this system s 1 system s 1 the state controllability state controllability that is say Q c. So, here controllability by Coleman test we can write down as B, AB and this process repeated A raise to n minus 1 B that is state controllability for s 1. Then observability for this system s 1; so observability that is Q 0 Q naught Q naught equal to. So, observability means we have to write C transpose, A transpose, C transpose and this process repeated we can write down as A transpose, n minus 1 C transpose. So now, this is the controllability observability equation for this particular system, this system this part these are related.

Now, coming to system 2; now for system s 2.

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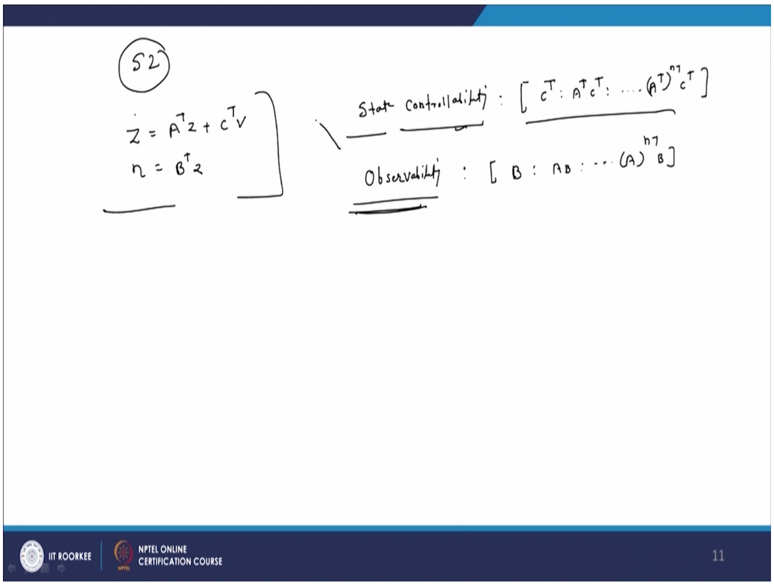

Now, for system S 2 now we write the equation as same equation Z dot to A transpose z plus C transpose v and end B transpose z. Now here for this system we have to determine the controllability and observability. So, for this system the state controllability for this system the state controllability can be written as. So, here now here earlier if you see if checking the controllability we have taken B A B A raise to n minus B. So, here the equation it is in terms of A transpose and C transpose. So, here state controllability is starting with C transpose. So, C transpose is nothing but the B of the original system. So, here state controllability we can write down as C transpose, A transpose, C transpose here A transpose C transpose.

And now about observability now observability now observability depend upon the c matrix, but now here C matrix is nothing but the B transpose. Therefore, we can write observability as B A B here to A raise to n minus 1 B. Now we have got a state controllability equation like this and observability equation like this, this is for s 2 plot this plot. So, if you compare the state controllability for the system s 1. So, we will find that the state controllability equation for s 1 B AB A raise to n minus 1 B.

So, for this plant this same equation will come here in terms of observability B A BA raise to n minus 1 B, that is controllability in s 1 is nothing but the observability in s 2. And similarly if you find that observability here this C transpose A transpose C transpose A transpose n minus 1 C transpose, it is similar to your state controllability. That is a observability here it become the state controllability. Therefore, here we can say that the controllability and observabilities are duel of each other. All these controllability observability are very much important concepts they are useful in applications various applications.

Now, here we are looking after the control design you will find that this for control design it is a first step, without controllability observability we cannot design a controller. So, along with this controllability observability has also very wide applications, it is useful in a (Refer Time: 21:44) stability analysis, it is useful in a various model order ratio and techniques. Even we will find that this type of concept is also useful in a image processing application also, but as our main purpose is to control designs. So, we will see further that how this controllability observability is helpful in a designing of the controller.

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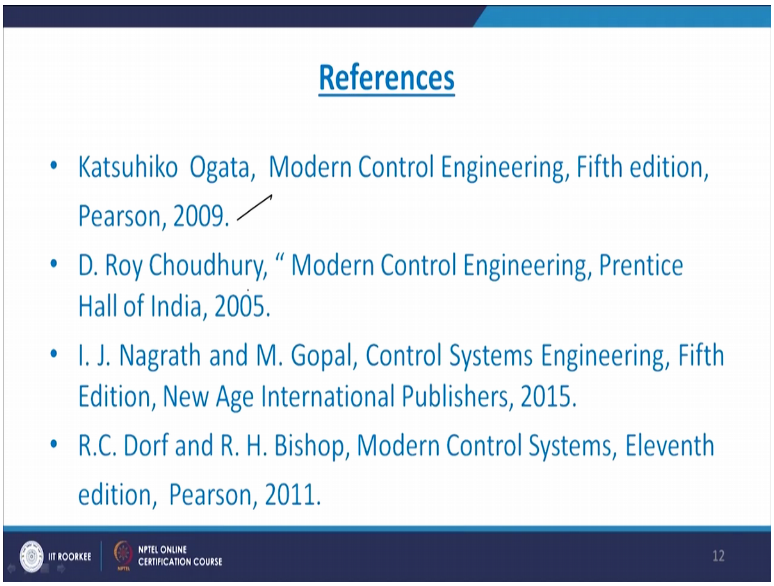

So, these are some references.

Thank you.