Advanced Linear Continuous Control Systems
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Lecture - 30
Controllability in State Space (Part-II)

Now, we start with Controllability in State Space that is Part II. Last time, we have seen state controllability that we have seen that the states which are at initial time that needed to be transferred. And all state if you transferred then that is called state controllability. Then this there is another word or another terminology that is called output controllability.

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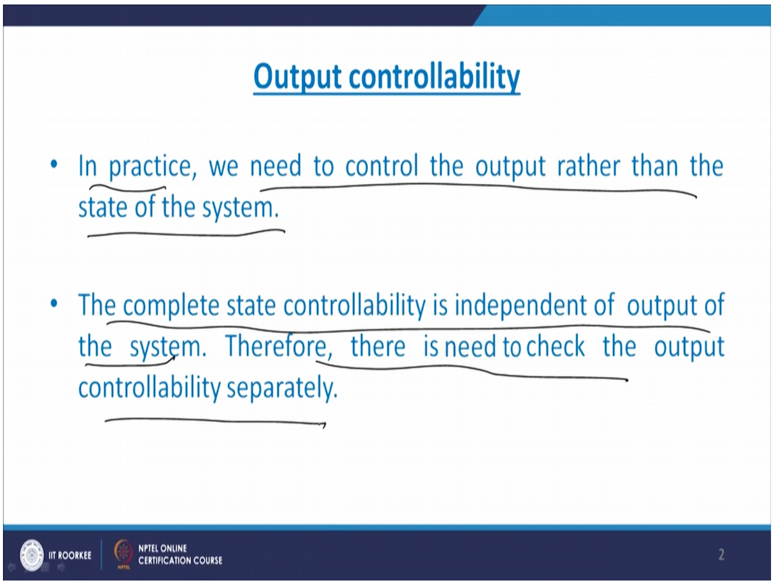

Now what is the meaning of this output controllability? In practice, we needed to control the output rather than the state of the system. If you considered a classical system, we have output and input; finally, output has to control. So, therefore, along with the state controllability, we have to control the output of the system.

The complete state controllability is independent of output of the system. Therefore, there is need to check the output controllability separately.

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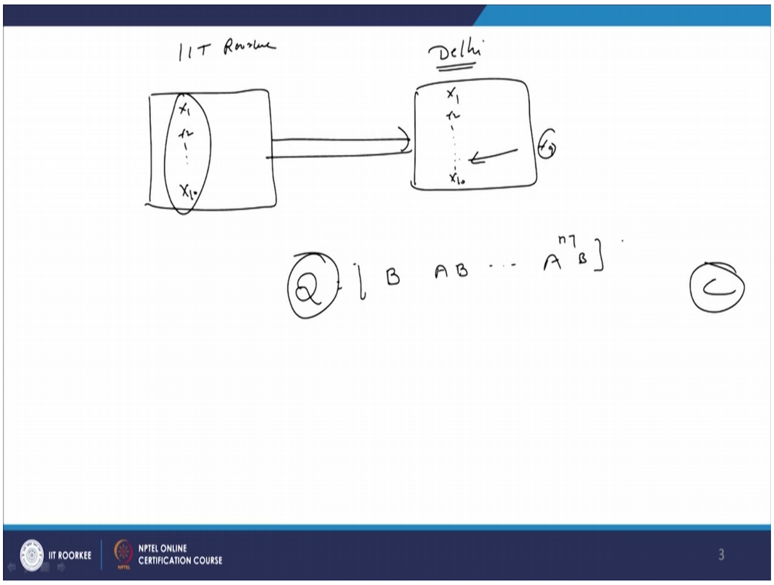

Now, what is a logic behind it? Last time I have I have given you one example about the students who are going from one place to another. Last time you have seen that an industrial trip is going from some place just like IIT, Roorkee. And they are going to some a Delhi. And there are 10 students are going assuming that 10 students and now we are we consider that the 10 students are the states of the system. And all the states has been started at some time and they have reached to Delhi, let us say X 1 X 2 X 10.

So, this state have been reached and we have seen that if any of the states has been missed, that is any of the student has been miss from this place, let us say this is X 9. This has been miss, then we have seen that this that particular system is not controllable. Now, but, but assume that the all states are here, they have will reach here. All students at IIT Roorkee started from here, they have reach to Delhi. Then why there is need for output controllability is because the students have come here to Delhi for some industrial trip.

Assume that all student have reached, but let us say the company has denied to enter because the output in this system is to reach to the industry. But certain reason, it is not possible; that means, that even though your state has been reach here, but you are not get giving the output. Therefore, there is need to have output controllability also. That output also be control: so but how to test it in a mathematical aspects?

Last time we have seen, If you want to change the controllability of the system, we have written the controllability matrix as Q equal to B AB just like that a raise to n minus 1 B and we are checking the determinant or rank of this matrix. If rank is full, the system is controllable. And in this case in case of controllability, we have seen there is no C matrix has been involved. But this is the output controllability. It means that something; that means C matrix which is important and output that must be involved because that is concerned with the output. Therefore, the test which is required to change the controllability is represent as shown here.

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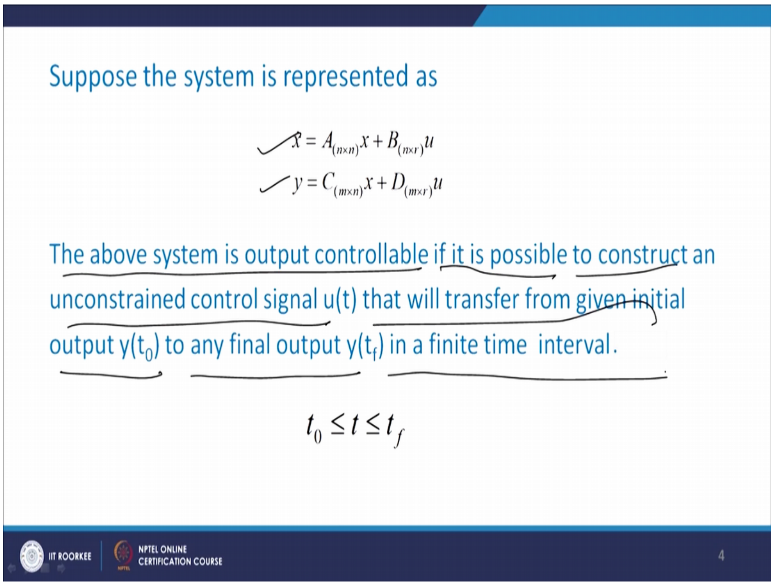

Let us say x dot and y this is our system, state space system and we are talking about the output controllability. So, what I said? The above system is output controllable, if it is possible to construct an unconstrained control signal u of t that will transfer from given initial output y of t 0 to any final output y of t f in a finite time interval.

Here we are talking about the state at initial time, then is to final time; but output controllability is concern with the output at initial time, what is the final time? As we have seen that here we are output, we are concerned with tourist industry or some practical problem occurred output in that possible. So, you have the state has to reach, but we are not getting the output. That means that output is not controllable. Therefore, output controllability is also very important tool whenever we are studying the control in all respect; in terms of stability, more living, in terms of controller design.

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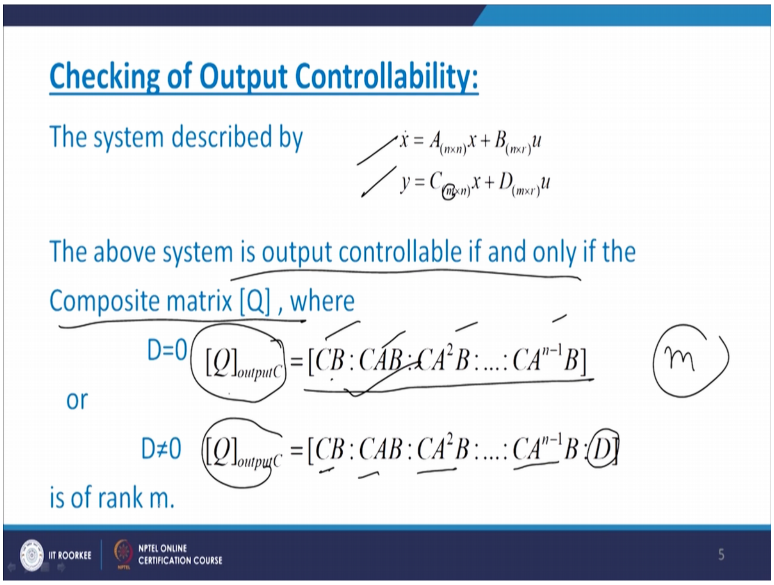

Now, how to check the output controllability? Now this is a plant. The above system is output controllable if and only if the composite matrix Q. Now this is Q is output controller matrix and we have to write in this forms C B C B C A n minus 1 B; this.

So, we have to determine the rank of this matrix. The rank of this matrix must be m. So, m is order of C is m cross m so, this particular m. So, order of this is must be m. Similarly if D is not equal 0; that means, when these existing; that means, our output controllable matrix, we have to write in terms of C B C B C A square B C A N minus 1 B and this D parts is will come whereas even D 0. This is D is not present and for all this case your rank must be m.

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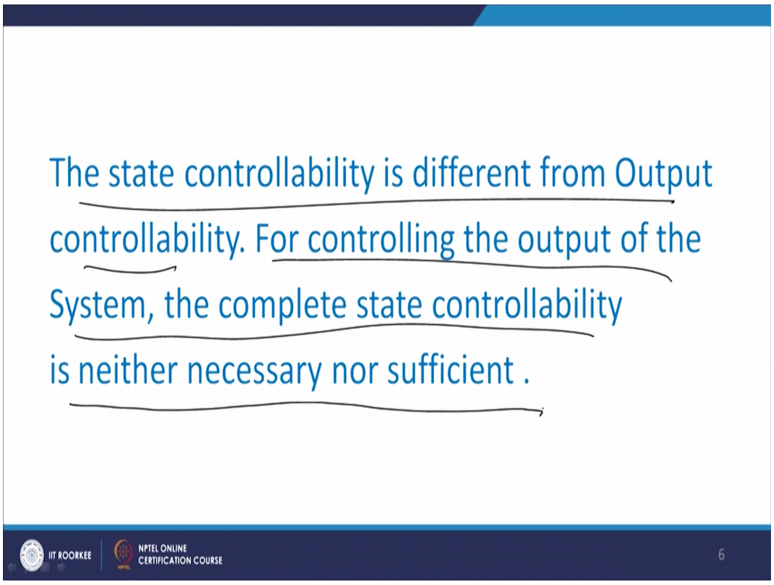

Now, we will try to solve one example and we see that that particular system is output controllable or not at. The state controllability is also we can write in this way; the state controllability is different from output controllability. For controlling the output of the system, the complete state controllability is neither necessary nor sufficient; that means both output controllability or state controllability or independent. But as I told for complete analysis of the system, we need to check both state controllability as well as the output controllability, because our practical says we have a output. Therefore, output is very important.

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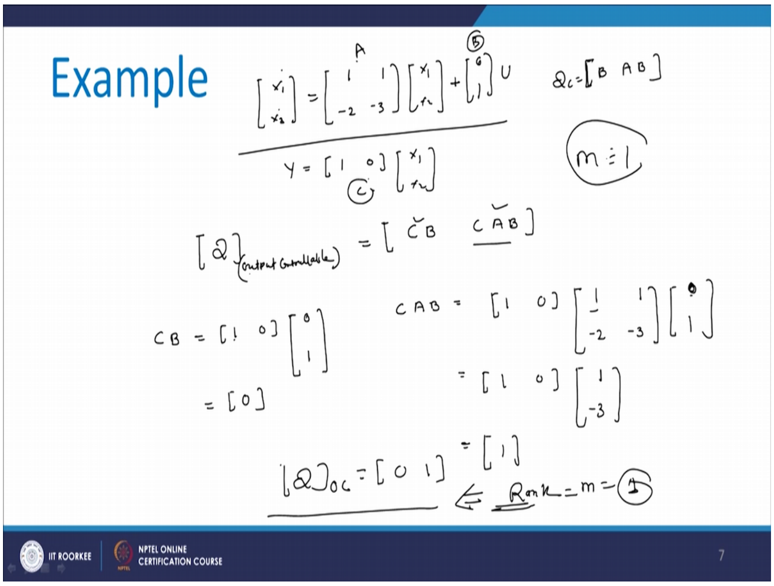

Now we start with an example and example is of second order system. And for the system, we like we will cheek the controllability output controllability of the system and for this example: we have already seen how to check the state controllability. Now just see it, Say X 1 dot X 2 dot say, 1 1 minus 2 minus 3 X 1 X 2 say 0 1.

Now, this is a state space system, but in this case this is a state equations. And if you want to check the controllability of the system, this is A and this is B; we can easily check the controllability by taking Q C B A into B.

But now here we required output controllability and when I as I told you, output controllability means the output C matrix plays an important role. Therefore for this problem, we should also know the output equation. Therefore, the output for this say 1 0 X 1 X 2, now this: now it is A B and C has been defined.

Now, just see the formula for output controllability. The output controllability is C B C A B C A raise to n minus 1 B. Now in this particular problem, your n is 2. Therefore, A order of A will be 1. Therefore, we have to take C B and C A B. Now Q output controllable. This is given as C B and C A B.

Now, now you determined C B as well as C A B. Now just see C B. What is a C matrix? C matrix is 1 0. And what is B matrix? The B matrix is 0 1, now after solving 1 into 0 0 0 into 1 0. So, now, C B is 0. Now come into C A B C A B. So, C is 1 0 and A is 1 1 minus 2 minus 3 and B is 0 1.

Now, we have to solve this equation. Now here 1 multiplied by 0 0, 1 multiplied by by 1 will get 1. So, we can write down this as 1 and here minus 2 multiplied by 0 0, minus 3 multiplied by 1 we will get minus 3, and after solving this 1 multiplied by 1 and 0 multiplied by minus 3. So, we will get 1 and therefore, this Q matrix that is O C; that is output control matrix that is O is output, C is controllable. We can write down as 0 1 and from this will find that the rank of this rank this is equals to m that is equals to 1 because of 0 it becomes 1.

So, rank is 1 and what is m here? m is also 1. That is our row we can say 1. So, therefore, it is satisfied our conditions that is this condition as well discuss these conditions of rank m is satisfied in this particular example. Therefore, this system is output controllable. Therefore, if you a given any system and we have to design a controller is a controller can be designed only when system should have this particular properties. If system does not have a output controllability as well as the sate controllability, then it is difficult to design a controller

But sometime in a practical systems and system involved so many state and we have to design a controller. So, in practical point view, we should have some more flexibility in a designing of the controller. Therefore in a control, there is one concept is present that concept is called stabilizability. This is also very important concept from control point of view and this concept is also related to our controllability.

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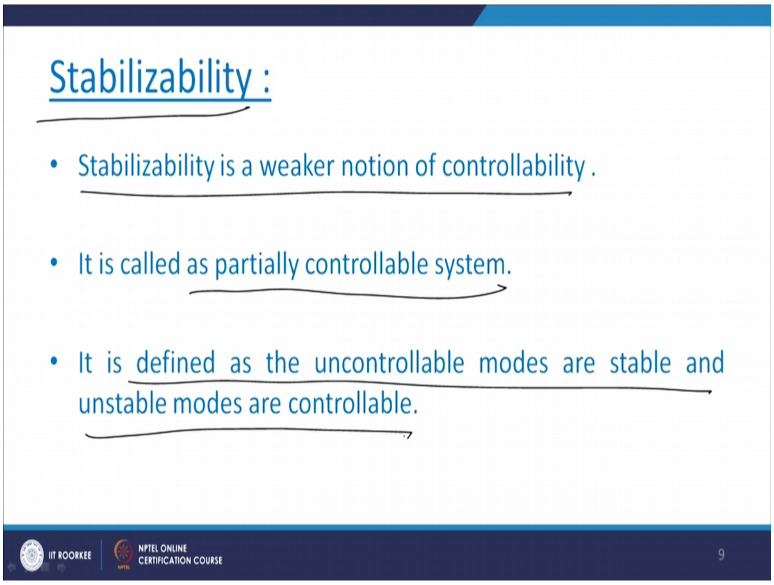

Now, how it is related? See here. Stabilizability is a weaker notion of controllability; that is controllability is there this, a weaker part that is partially controllable we can say. That is stabilizability means we can say partially controllable system. That is not a fully, but partially controllable system. See here partially controllable system. It is defined as the uncontrollable modes are stable and unstable modes are controllable that is called stabilizability.

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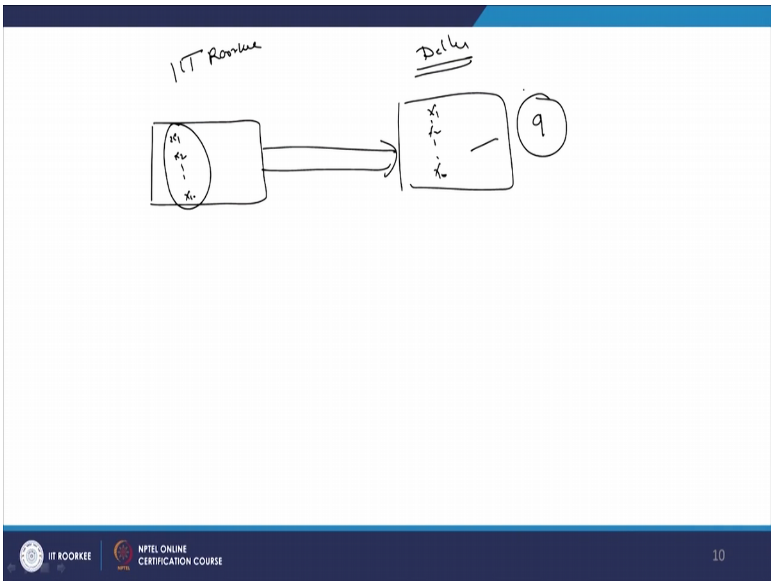

Now, how we think about this stabilizability concept. Again we come to the example which we have attain earlier; that is here there are 10 states or we can say there are 10 students here. They are going to Delhi from Roorkee.

Now, here this is IIT Roorkee and it is Delhi and that students are going for some industrial trip at Delhi and therefore, there are states again X 1 X 2 X 10 and we have seen that if all states which are at IIT Roorkee, all student which are be there at IIT Roorkee. When they reach to Delhi, we are observed. Let us say all 10 states or 10 students are available or we are observed or we can say they are present there.

Now we are saying that your system is state controllable. But any of the state is missing, we have saying that it is not a controllable system. But as student has reach to Delhi and as a teacher, I have to take to the in safe for industry, but I also that one is missing. There state is missing (Refer Time: 15:41) As I told earlier that when any of the student has been miss in that industrial tour, it is difficult for us to teacher to carry out the student for industry; that means, he has to come back and he has to see where is this.

So, if the similar situation happened, if any of one of the student has been miss in the industrial trip. So, what is the best option? I will see where is that student. Suppose I have got information that students which has been reached to Delhi for industrial true trip, but at the same time he has left to some his relatives. If that information I got and I talk to him that he is there. But he told me that because of some health problem he is not coming to this one; that means, I have got idea that the student even though reach the Delhi, but it is not observing.

Therefore, but he is somehow stable, then what I can do? Out of this 10, I will take nine students to the industry knowing that what is not there not there; he is relatives house his health is not good. So, in such case we can say that system is in stabilizable mode or this is called as weaker notion of controllability or we can say partially controllable system.

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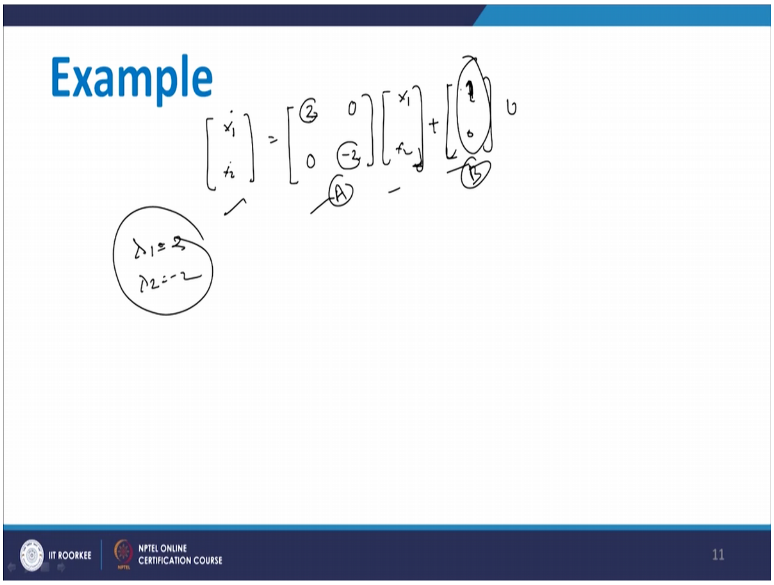

Now, we see this part when example. See here this X 1 dot X 2 dot; now here 2 0 0 minus 2, say X 1 X 2 1 0 into U. Now this is our system X 1 dot A X plus B U.

And now we are talking about the stabilizability and as I discuss that stabilizability is that important weaker notion of the controllability. As looking after this system 2 0 0 minus 2 and here we got 1 0 and this part is coming in Gilbert test and is the Gilbert test, if you have seen that if the given system; if it is it diagonal form or particular a matrix in the diagonal form, we have to see the B matrix and if any of the element in this B matrix, this is the matrix is A. if the any of the element in this B matrix is 0. Then system is not controllable that part we have say the this is Gilbert test.

Just looking after the example, we observe that this system is not controllable, but now I have to work on the system. So, just looking after the system we observe that see here this is 2; this 2 is a unstable system. Because if you take the stability of the system will find that the Eigen values at lambda 1 equals to 2 and lambda 2 equals to say minus 2.

So, here the system is unstable system, out of this we find that the 2 indicate the unstable part of the system, but that unstable part is there if you see the one that is a controllable. That is unstable part is there, but it is controllable, but if you see this is the minus 2 which is the stable part. This is the stable part, but 0 is uncontrollable part; that is if the system having this type of property that is this unstable part is controllable and stable mode is uncontrollable, if this is the then this is called stabilizability of the system.

As I discussed it define as uncontrollable modes are stable and unstable modes are controllable. So, here we have seen the same part this is the unstable mode. It is controllable where stable mode is uncontrollable. So, this is called stabilizability of the system.

Now coming to the, another issue that is called pole 0 cancellation; actually pole 0 cancellation this is the part which is concerned with the state transfer function because when for transfer function when was pole say 0, but at the state space, there is no such issues. We got X dot equals to A X plus B U. We got A B C D matrix.

So, the concept of pole 0 is also related to this controllability and observity issues. This is because even though system, it is in a transfer function. We are converted into state space. Suppose if you have a system which is given in a transfer function domain and if there is a pole 0 cancellations. So, just cancel is a pole 0. Here can we say that system is not controllable.

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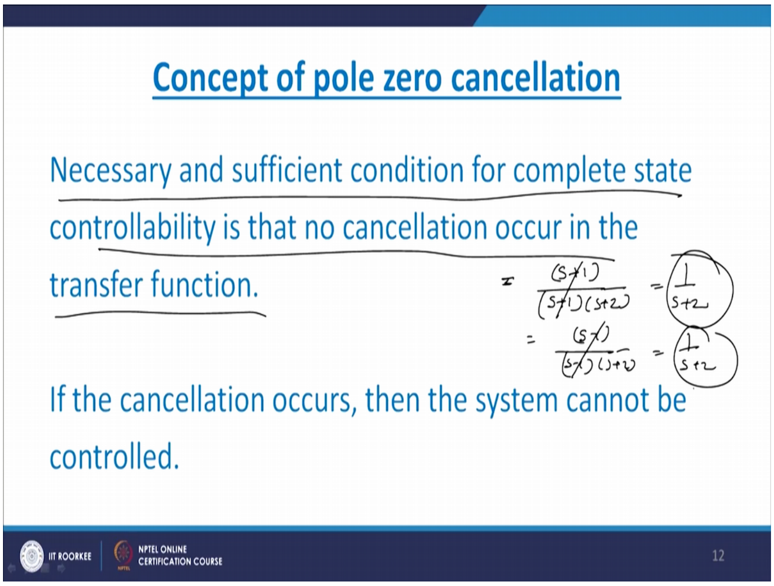

So, here the system is not controllable. So but a, if I am we are saying system is not control, what is the reason behind it? Can we prove it? So, here one philosophy is associated with the pole 0 cancellation. So, it is say that necessary and sufficient condition. For complete state controllability is that no cancellation occur in the transfer function; that is if you take any transfer function if there is any cancellation of pole, say 0 system cannot be controllable according to the theory. That is if you take say S plus 1 S plus 1 and S plus 2. If this is a plant actually mathematically, we cancelling this and we in case a S plus 2.

So, mathematically S plus 1 and S plus 2 will cancel. We got S plus 2 everything is stable; that means, that this that is system is stable or controllable everything we can observed; that means, mathematically is it true. But from control point of view, this is not true. It again if you can say that this is this although the mathematics is important in the control system. But from practical point of view we have control point of view, every concept of mathematics cannot be applicable directly for the control system analysis.

Here this is the same issue S plus 1 S plus 1 a cancel 1 plus S plus 2 and the most sever condition will come, when we can say S minus 1 here S minus 1 S plus 2. So, this is the system we cancel. Here 1 upon S plus 2. So, this system is stable, but this cancellations cause problem to the controllability of the system even a stability of the system.

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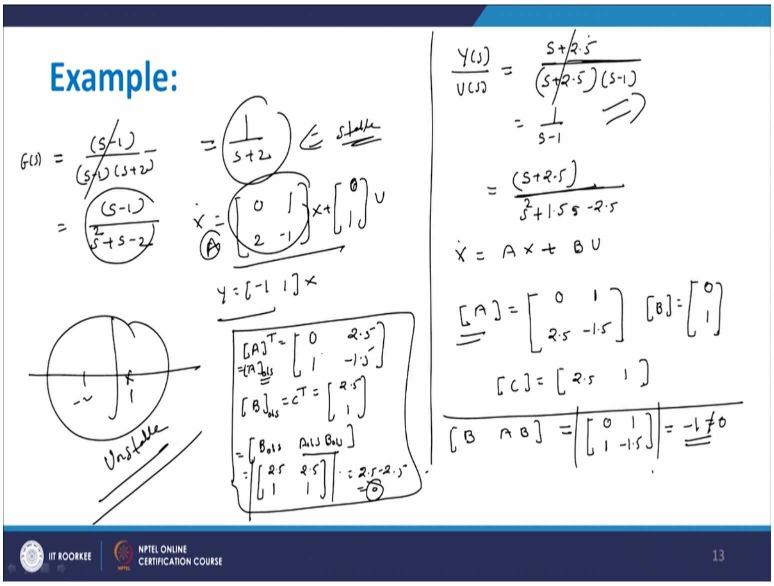

In a earlier part we have already seen say S minus 1 S minus 1 say S plus 2, this is say G of S. Now, if you solve it, will get S minus 1, S square plus S minus 2: no problem mathematical point of view, if you talk about mathematical point of view; this S minus 1 S minus 1 cancel, we will get S plus 2 and we cancelled the system is stable and same thing, if you are taking care of state space analysis. If you develop the model of this plant in a state space, this X dot equal to a X means here 0 1 2 minus 1 X plus 0 1 into U and Y equals to minus 1 1 X.

So, the same plant if you are not done any pole 0 cancellations, we get S minus 1 S square plus S minus 2. Now here 0 1 1 minus 2 2 minus 1 and if you check the stability of this model a, so we will find that there is also 1 eigenvalue on the right hand sides. Let us say at 1 and here is minus 2 and now this is unstable.

So, we find that if you do the pole 0 cancellation, the original system is unstable, but actually it becomes unstable. So, there is in stability issue, but similarly this also issue with the controllability of the system. If there is a pole 0 cancellation, we say that the system is not a controllable. Now we see an another example. Suppose if you write Y S by U U of S equal to S plus 2.5 S plus 2.5 S minus 1.

Now, this is a plant and what we have seen that this is S plus 2.5 S plus 2.5 S minus 1. So, this S plus 2.5 and this we can cancel easily and we will get S upon 1 S minus 1. So, now, according to the concept we are as I discuss, the system is not controllable, but can you do really true to just check it. What we do? We can use our conventional approach.

For the conventional approach what we can do here. We solve this equation S plus 2.5 divided by S square plus 1.5 S minus 2.5. So, after solving this we got this. Here we have cancelled 1 by S minus 1 and mathematically we have done like this. And now we are talking about the controllability concept without any pole 0 cancellation, whatever system where keep as it is

Now, what we can do? Now we determine is state space model of the system. The state space model of the system is say X dot equal to a X plus B U. Now the a matrix in this case is 0 1 2.5 minus 1.5. And if you see the b matrix B matrix is 0 1 and C matrix 2.5 and 1. So, this is a controllable canonical form or we call as companion form of matrix.

Now we will try to check the controllability of the system. Now have to check the controllability of the system. Now for taking checking the controllability, what we will do? We write this as B A B matrix and if you do it, so what will get? Here B is 0 1 and this A B matrix if you multiply, will get here 1 minus 1.5 and we calculate determinant of this; if we take the determinant of this, what we will get? We will get minus 1 is not equal to 0.

So, minus 1 is not equal to 0. That means that your system is not control is controllable. So, as we discuss earlier that our system is uncontrollable because this is pole 0 cancellations. Now the thing is that this we have done in case of the controllable canonical form or companion form and we say that in this particular form, this A B and A B it does not depend upon this mathematical matrix S plus 2.5.

So, whatever may be elements in the numerator, it not affected by B and A B. Particularly if you write the given system in this controllable canonical form therefore, if you want to check the controllability when this is the pole zero transition; I think we have to involve the elements of C matrix that is a elements of C matrix because C actually do in the pole 0 cancellations. Therefore we have to do or make a elements that in the elements of B, there must be the elements of C. So, this is possible if you convert this system into the control observable form that is controllable observable canonical form.

So, observable canonical form means this a changes to A transpose and this C C transpose equals to B; that means, here I am writing down here this site. This A matrix now here actual A matrix become A transpose is a transpose is 0 2.5 1 minus 1.5 and here this B matrix so; that means, here this a transfers is nothing, but a matrix observable from obvious and again here B observable equal to C transpose equal to 2.5 1. And now here if your solid b observable, a observable to b observable matrix.

So, we find that this B observable. So, B observable matrix is 2.5 1 and now A observable is like this and we multiplied by B observable here will find that it also become this multiplication of this A observable is and B observable this. If you multiply, it we will also get 2.5 two 1. And if you take the determinant of this if you equals to 2.5 minus 2.5 equal to 0.

Now will find that this system is not controllable; that is pole 0 cancellation we are directly reflected through the mathematical approaches of checking the controllability of the system. Therefore whenever any system is given to you for designing of the controller, then in that case if this a pole 0 cancellation; we cannot design a controller. That means, if you are given system in a transfer function model and if the pole 0 cancellation, the no need to design a controller. That means, system when this controllable let me that particular state cannot be controllable; that means, that there is no need to go for the controller design.

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Now, these are some references. Now this is the part a concern with the controllability of the system, but in a control application along with controllability, we need to know the observity of the system. And what is the observity? How it is useful in a state space analysis particularly in control design that will see later on.

Thank you.