Advanced Linear Continuous Control Systems
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Lecture - 03
State Space Representation:
Companion form (Controllable Canonical Form)

Where today, we start with companion form of modeling which is also called as Controllable Canonical Form.

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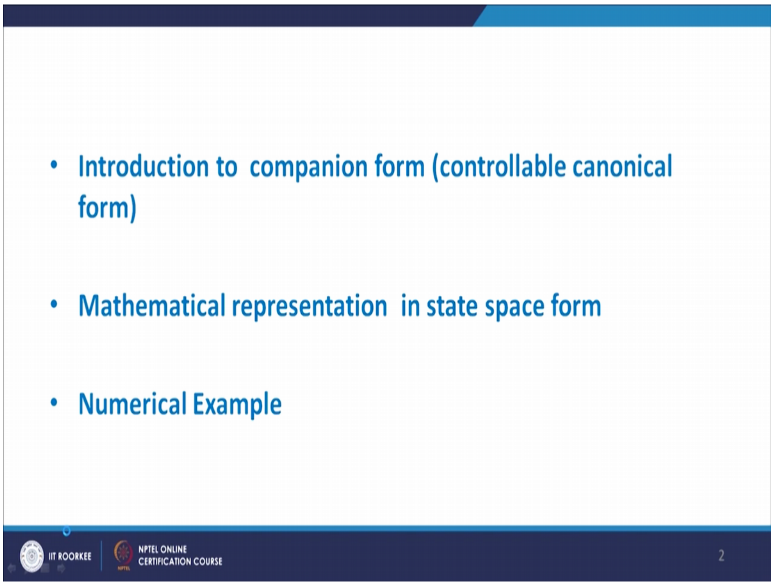

So, in this we will going to study, introduction to companion form that is called controllable canonical form, then mathematical representation in state space form and numerical example. Now, about the introduction to companion form of modeling; as last class, we have seen that any system can be represented in state space form and in that case we have concerning different physical variable. But sometimes, what happened, if you are written the equation in differential form or if you are transfer function form, then in that case how will you get the state space model.

So, there are different methods are available to convert given system into state space form or we can say there are different forms are there, that is called companion form, canonical form. In a companion from means, if there are two types, that is called controllable canonical form and observable canonical form. Then there are other method which in which the given system has been converted into a canonical form; in that canonical form, the particularly diagonal canonical form and Jordon form. Now, the about the controllable canonical form, what is the importance of this particular form? The main importance of this form is that that is very simple in nature. In this case, what we have to consider as the output as a first variable and remaining variables are the derivative of the output and if you do this one, we can easily get the state space model.

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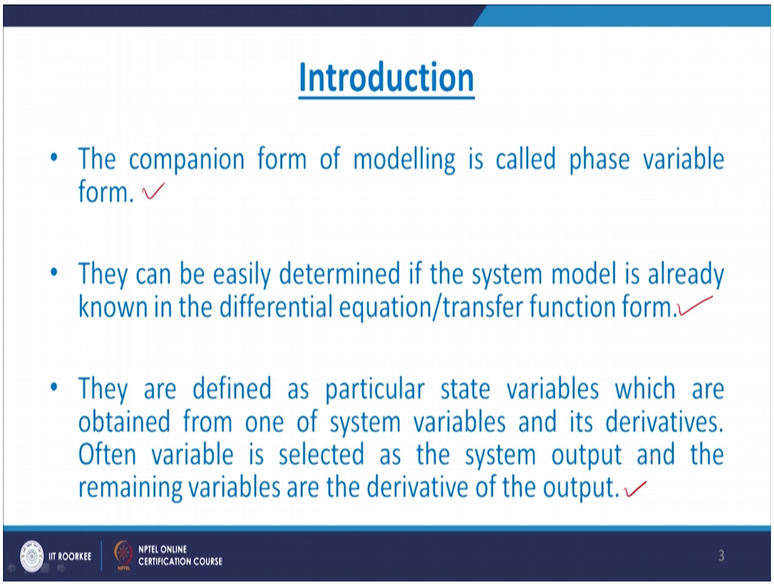

So; that means, here, the companion form, companion form of modeling is called phase variable form. They can be easily determined if the system model is already known in the differential equation or transfer function form; third point, they are defined as particular state variable which are obtained from one of the system variables and it is derivatives; often variable is selected as system output and remaining variables are the derivative of the output. It is the basic concepts; that means, what we are do here? We have to take output as a first variable and remaining variables are the derivative of the output; that is a strategy.

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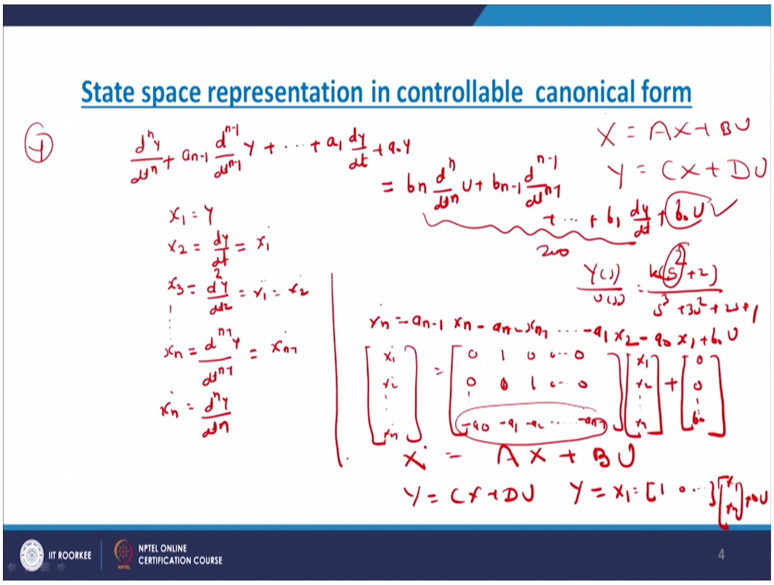

Now, we have to represent the given system in a state space form. So, state space form is, we want to system to be in this form X dot equals to A X plus B U, Y equals to C X plus D U, this we want.

So, now to get this form, we first of all write a differential equation. So, that equation that equation is nth order differential equation, so, we write this nth order differential equation as d n y by d t n. So, here nth order differentiation of y plus a of n minus 1. Now a differentiation, so n is there, we write n minus 1, if d t of n minus 1 of y plus and this process repeated plus we write a 1 into d y by d t plus a 0 into y that is equal to, so, this function we can write equal to b n, but differentiation of n of d t n u plus b of n minus 1, differentiation n minus 1 dt of n minus 1 and this process is repeated plus b 1 into d u by dt plus b 0 into u.

So, this is your nth order differential equations. Now, in this case this a 1, a 1, a 0 are the variables. Here also there are some variables. Now, this equations or this differentiate equation where to write down in a state space form. Now, here we are consider two cases. The one case that that, there is no differentiation terms involved, that is the are all the differentiate terms in the output we are to make 0; that means, these terms I want to make 0. The derivative of u, I we will make 0, all in these form is present, these form is present.

So, what is this type of system; that means, if you write in the transfer function, let us say I can write transfer function as y s by u s equal to s square plus 2 by k, so, k divide by s cube plus 3 s square plus 2 s plus 1. Now, this is a part I have written. Now, here what we are doing here I am removing this s square part, only constant part we are keeping ok. So, now, your equation is this one, d raised to n dt n plus a n minus 1 d of n minus 1 dt n minus 1 upto a 0 y equals to b 0 into u. Now, we go for our strategy. The our strategies says, you have to consider output as a first variable. So, when we say output as a first variables, so, here output is your y, your output is y. Now first variable, we can write here x 1 equals to y.

So, x 1 equals to y is our first variable and now what about says, remaining variables are the derivative of the output; remaining variables are the derivative of the output. So, what we can write? Now another variable is x 2, we can write as d y by d t. This is a differentiations. There is equals to, so, we take it is x 1 dot. So, x 2 equals to d y by d t equals to x 1 dot. Now about the remaining variable, so, remaining variables, I can take another one variables x 3, x 3 is d square y by d t square. So, now, here that is nothing but equals to x 1 double dot. There is equals to x 2 dot. So, now, here x 2 dot equals to x 3, x 1 dot equals to x 1 equals to y. Now, this process we can repeated. So, x 2, 3, we can write now here x n.

Now this x n equals to, so, here 3 raised to 2. So, we write d of n minus 1 y dt of n minus 1. So, that is equals to let us see 3 used by 1. So, here we can write down as x n minus 1 dot, there x n equals to x minus 1 dot. And similarly, now we have to write x n dot, so, what is x n dot, x n dot equals to d n y of d t of n. Now finally, we have to represent this equations in a state space form, now here x n dot, we are written now, we what we have to write this is d n by y dt n.

Now here, x n dot this x n dot equals to we can write this equation as a n minus 1 and what is d ; differentiation of minus 1 dt of n minus 1 is given as that is equals to x n. This is minus, then minus because if you view this side a of n minus 2, x of n minus 1 and this process is repeated and finally, will get here minus a 1, here a 1 d y by dt. So, d y by dt equals to a 1, x 2 and minus a 0 into y y is x 1 and this part we are keep 0. You just see here and now we write b 0 into u.

So, we got x n dot equals to a of n minus 1, x n minus a of n minus 2 x n 1, a 1 x 2, a 0 x 1, b 0 into u. Now, there are variables are x 1 dot u see here x 2 dot x n dot and now we have to write in a state space form. So, what is the state space form is, x dot equals to a x plus bu. So, we write as x 1 dot, x 2 dot up to x n dot. That is equal to, now what is x 1 dot, your x 1 dot equals to x 2. Just see here, this x 1 dot equals to, so, you will write 0, 1 and remaining elements are 0. So, we write, remaining elements as 0, 1, 0, 0, 0, like this. Now x 1 dot, we have written, now we coming to x 2 dot. What is the x 2 dot? x 2 dot equals to x 3.

So, see here x 3. So, this is x 2 dot x 3. So, we write 0 1 sorry 0, 0, 1, 0, 0 like this and this process repeated and now we coming to x n dot. Now, here x n dot equals to see here a 0 x 1, a 1, a 1, 2 like this a of n minus 1 x n. So, what way we can represent this. That is equals to minus a 0 minus a 1 minus a 2 and this process repeated up to minus a of n minus 1. And now, here x 1, x 2 up to x n plus, we write this as 0, 0 and particularly x n dot, we get here b 0 into u. So, this, you have x dot. Now here, a x plus b u. So, b is this 1. So, now, state equation we are determining, now we have to write output equations. So, output is y equals to, y equals to c x plus du and in order to write this y, so, what is y; y equals to x 1.

So, y equals to x 1, it means we write as 1, 0, 0, 0. Here, we can write x 1 to x n and 0 into u. So, x, y equals to x 1. So, this is the state space model we have determined for given nth order differentiation equation. And the most important points is that, here all the derivative terms we have to 0. So, the derivative comes are 0 are there, it means that there is no 0 is involved in the transfer function model. All the nominator constant and denominator constant and therefore, now what we are do, we are take one example and try to solve it. So, for that purpose, I have to taken simple example.

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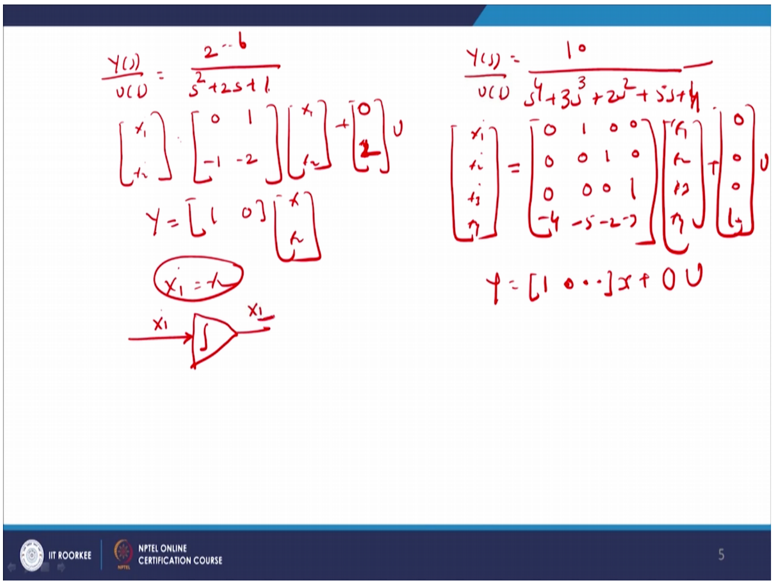

I can write as y s by u s, this is transfer function model. Now, I have written as 2 s square plus 2 s plus 1, this is the transfer function and you know transfer function, Laplace transfer output to Laplace transfer of input initial conditions to be 0. We have already seen in this transfer function. Now, here we have to write this in a state space form particularly, a companion form, phase variable form, other words it is called controllable canonical form control; This also a word which is common for this type of problem.

So, here we want to write it is state space form. So, what, so, here we find it is 2; 2 means order 2. Therefore, the state space model we get having the matrix order is also 2 by 2 cross 2. So, here how many state will takes? So, here we take 2 state, x 1 dot and x 2 dot. Now here, we can write easily. The x 1 dot, we can generalized from 0, 1. This is x 1 and x 2. So, x 1 dot, you can find equals to x 2 and here b into u.

So, here 0, next, now what is x 2 dot, x 2 dot if you see the previous slide, you will previous slide where x n dot equals to a 0, a 1, a 2, n, that is if you say these are nothing but the coefficient of the denominator. So, we can write down directly, we will find at, we can write down directly minus 1 and minus 2 and here, this is 2, b 2, this is b. So, this is a state equations; Now, what is the output equations and here output y as 1, 0 into x.

So, here we get 0, this is 2, this is b u just see here. You got a equation b. So, here that is why we written here 2 and here y equals to 1, 0, x 1, x 2, this is a state space model. Similarly, we can take another example also y s equals to, say 10 divide by s 4 3 s cube plus 2 s square plus 5 s plus 4. This is the fourth order model. Now, we have to write down in a state space form. How will you write, 4? So, we how many state will take we take 4 state, x 1 dot, x 2 dot, x 3 dot and x 4 dot; that is equal to see here what I have write in down here.

So, here x 1 dot equals to, so, y 0, 1, 0, 0; this can I should out to be 0, 0, 1, 0; then here 0, 0, 0, 1 and write the element in the reverse order, 4. So, why write minus 4, second we write minus 5, third we write minus 2, minus 3 and again this is x 1, x 2, x 3, x 4 plus and now here 10 coming 0, 0, 0, 10 and to u and output, output is same output equals to 1, 0, 0, 0 x and 0 into u because d is 0. So, this is, we can say the representation of any system in a state space model. Now, what we have to do here?

Now, we have to represent these particulars system in terms of n in Simulink diagram because normally, all the system, we have to analyze through some software. Software mean, we are mean software mat lab therefore, we have to simulate through an integrators. So, differ integrators, we have to use. So, how to represent the given system into the state space model but in terms of Simulink diagram? So, here, we have to use the concept of integrator as I told. So, here I when I am saying x 1 dot equal to x 2 ; that means, here I have use integrator like this and here, here, x 1 dot is there up to integration it become x 1.

Now, we have to represent the given system into a into a form that is called that is we can say different integrators are involve. So, how will you do it? So, for that purpose, now what we did, so, that we can write here.

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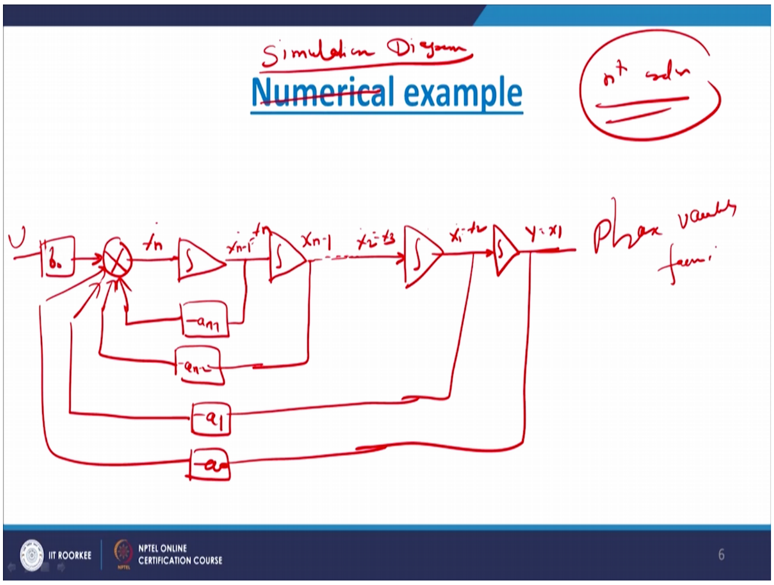

This is your simulation diagram, simulation diagram, we can say not a example because example we already seen; therefore, we can stimulation diagram. So, here how many integrators we required? Now here when the plant is nth order, nth order, we required n integrators. So, therefore, what we are doing here, we are doing integrators. So, what you can do here I using first integrator here, this is integrator. So, this integrator is say x 1 dot that is equals to x 2 and that is equals to y equals to x 1.

So, x 1 dot, when we integrate we get x 1 that that equals to x 1 and now here we will add another integrator. So, here integrator we use. So, here what we want, we want here x 2. So, when we get x 2 dot, x 2 dot for integrate with this will gets 2. So, here what we divided x 2 dot equals to x 3. And this, we repeat or move this process like this and finally,, so, we are coming to an integrator that is x of n minus 1. Now here, x minus 1, the how will get x minus 1; that means, in these case when x minus 1 dot integrate will get x n minus 1 therefore, we have write x of n minus 1 dot, that is equal to nothing x n and now what we look at, required another integrator here and now here is x n dot and now x n dot, we write like this.

So, x n dot, we have seen that is equals to b u. So, we write this is b and this is because we seen in a previous slide here. So, this is b u. So, here b 0 into u, this is the this u we have to taken here. I can say small b, small b and now this x n dot equals to minus a of n minus x n. Then here, another one minus a of n minus 2 x of n minus 1, third write minus a 1 into x 2 and here minus a 0 into n. So here, all x 1 dot equals to x 2 dot equals to x 3 and finally, x n dot equals to b u and whatever equation we are taken here that has been simulated and here output y equals to x n.

So, this is the Simulink a diagram of a controllable canonical form or also call as we can say phase variable form variable form. In this case, we will find at there are no 0's are involved that is no derivative of the u has been involved. But in real time, 0's plays very important role or derivative of the input plays important role. So, how to develop the state space model of a system when there exist 0's as well as poles; 0's means what I saying that 0's means that derivative of the input. So, how to do it is? So, that part we will see in the class.

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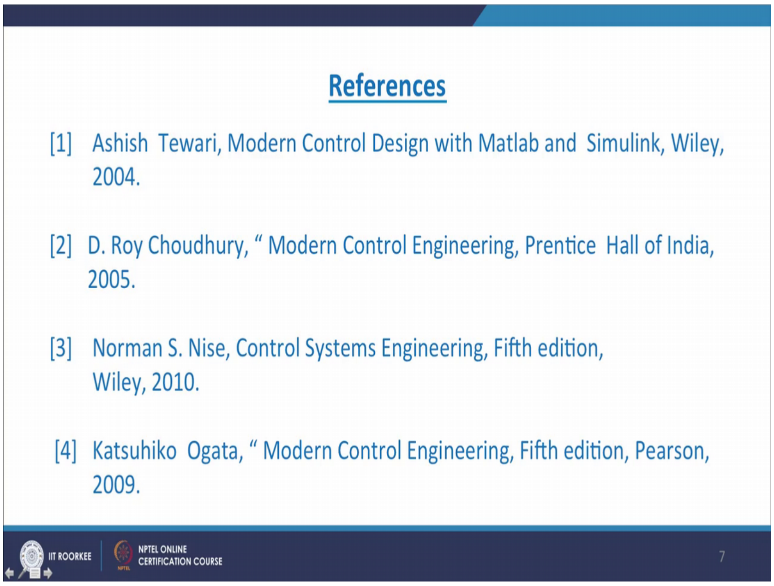

Now, these are some references.

Thank you.