**Advanced Linear Continuous Control Systems**
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**Lecture- 28**
**MATLAB Programming with State Space**

Now we start with MATLAB Programming with State Space. The meaning is that whatever we have studied earlier; particularly modeling, stability analysis, state transition matrix, state equation. So, if you are having a software then how to determine this within the software? Now we software we are we have to use that is the MATLAB.

Because MATLAB is very important tool which can helps us in the analysis of the system. Now here we are going to study modelling of state space.

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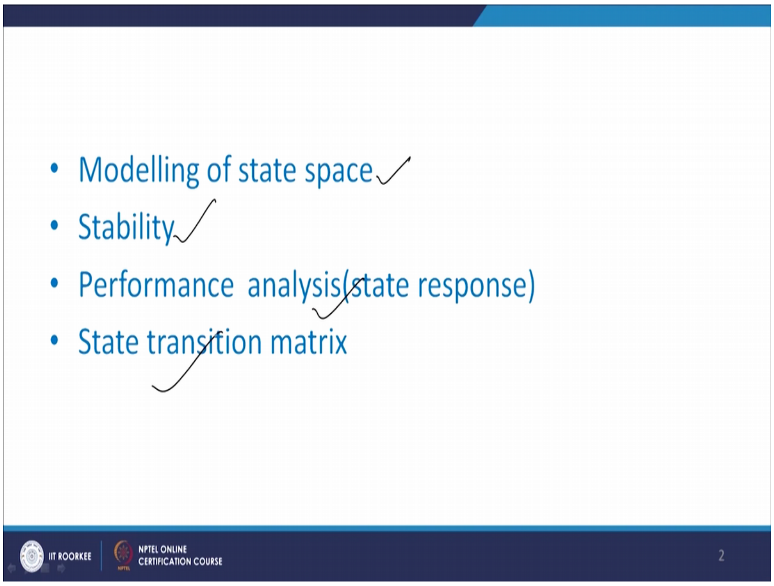


Second one is stability, then performance analysis and state transition matrix.

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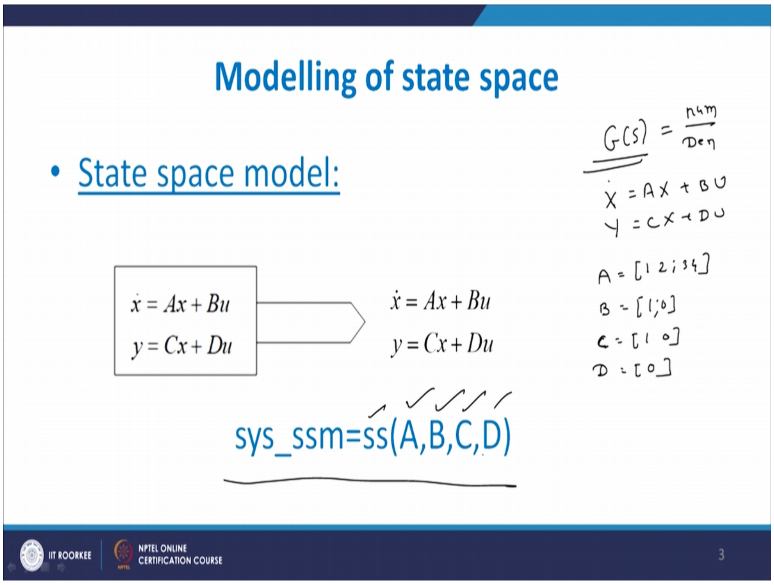


The modelling of state space, I think in classical control we have seen a transfer function that is G of S; which has be represented by numerator by denominator. But now here is the model which has been represented in terms of x dot equal to A X plus B U and Y equal to C X plus D U.

Suppose we know A, B, C and D and we have to replace or we have to put all this A, B, C, D in MATLAB file, so what we can dowe can write a command A, B, C and D with ss; ss means state space; the repositories if you go to the MATLAB window we have to write A elements say A equal to 1 2 3 4.

Then we have to write B elements say 1 0; then C 1 0 and say D equal to say 0. So, here A, B, C, D and if and after word you have to write this command you will get A B C D directly.

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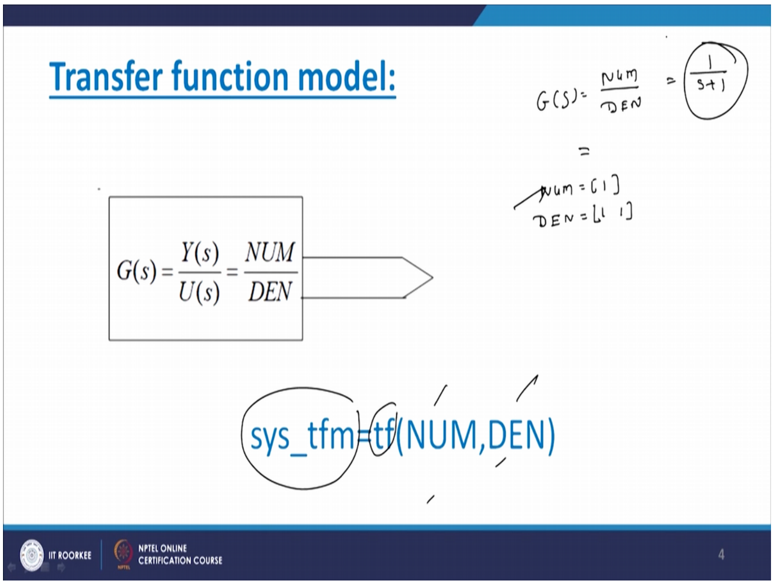


Now about the transfer function transfer function as where seen G of S equal to numerator by denominator; suppose we have transfer function 1 divide by S plus 1 this is simple transfer function and this we have to write in a MATLAB.

So, in that case you have to write the command see numerator NUM 1 and now have to write denominator DEN. Now here S plus 1, we write 1 1 and finally, the complete system transfer function that is SYS tfm; that is transfer function model equal to t f. So, this is a command tf now this numerator and then denominator; we will get a transfer function like this using a MATLAB.

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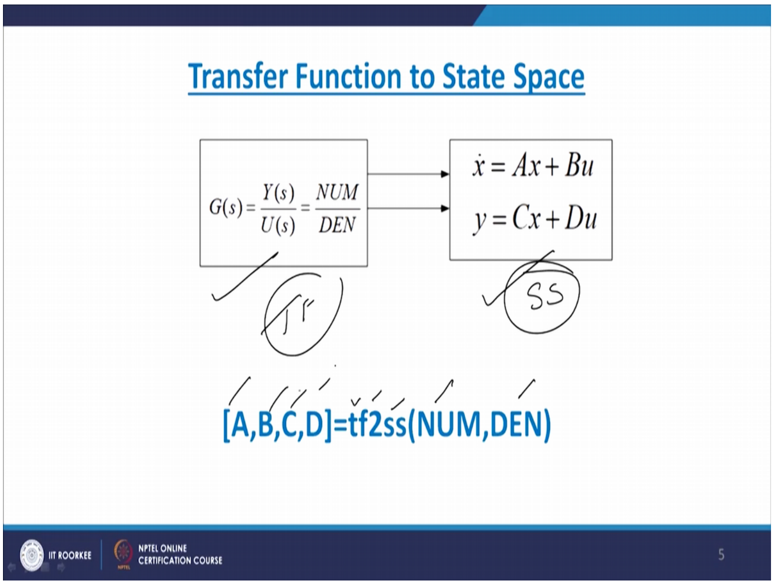


Now another part transfer function to state space, we have seen earlier that if you are having a transfer function model we can easily converted into companion form, canonical form, including diagonal form Jordan; Jordan form.

Now, what we want? We want to convert given transfer function into a state space. So, see here this is G S; this is transfer function model and this is a state space, this is transfer function and now this is a state space ss model. What we want? Transfer function to state space; so we start with writing command tf; the transfer function 2 ss; tf 2 ss that means, we are getting a state space model and what is the input in this case?

Input is numerator and denominator so we will get A, B, C, D; that means, format given transfer function we can easily determine A matrix, B matrix, C matrix and D matrix of a state space model. Now another part state space to transfer function.

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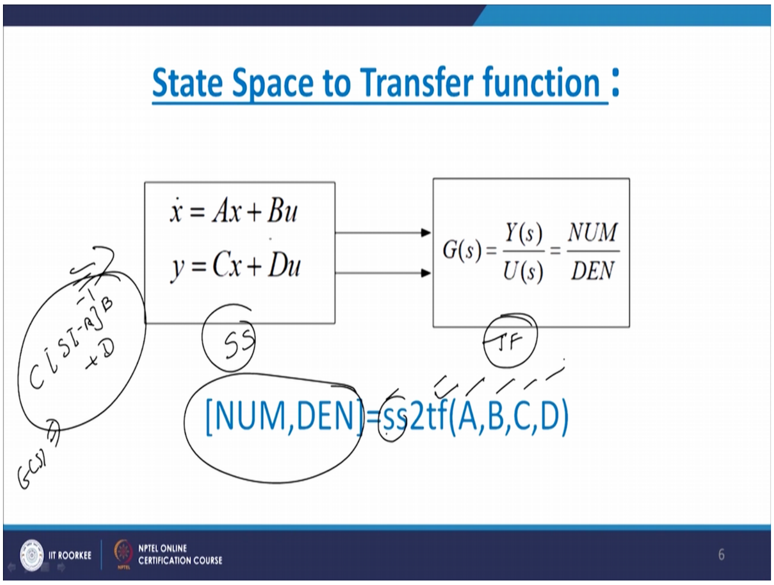


Now, this is state space model; now, here is a transfer function we have seen various algorithms for determining transfer function from a given state space models; that is if you want to do it by manual calculations, so what we can write C SI minus A inverse B plus D, this is a transfer function that is a G of S or we can use the leveller algorithm.

But if the same thing if you required in a MATLAB if the command which we have to use that is ss 2 tf of this here ss 2 tf that is state space to transfer function. And here we will get output in terms of transformation therefore; here we are entered A B C and D; now we take another example.

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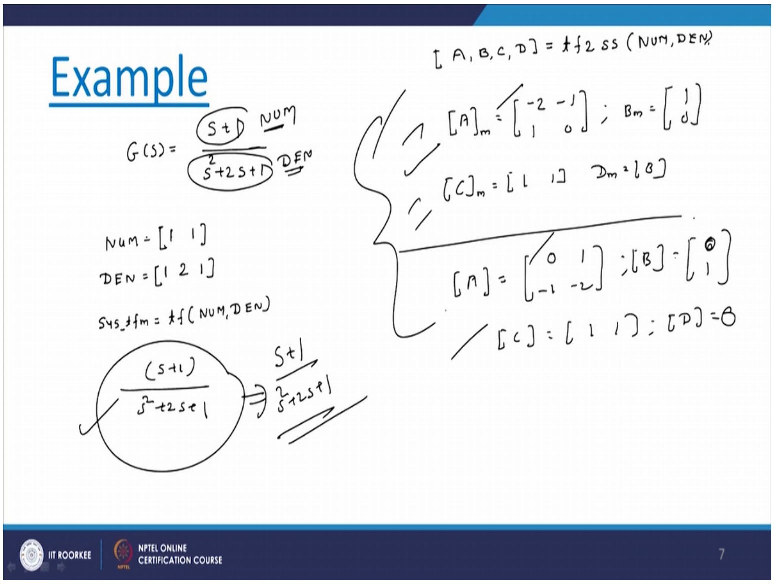


Now, we take the example as G of S equal to S plus 1 divided by S square plus 2 S plus 1.

Now, this is a transfer function model now first of all what we want the transfer function we have to write in a MATLAB; in MATLAB environment. So, this S plus 1 this indicate numerator and this S square plus 2 S plus 1 indicate denominator numerator and denominator.

So, numerator we can write down in MATLAB; NUM then this S indicate 1 and this 1; so, this is numerator. Now, about the denominator DEN that is equal to 1 2 1. So, numerator we entered, denominator we entered and as a discuss this is SYS transfer function model; that is equal to tf, the numerator comma denominator this gives you in MATLAB the state space model as a S plus 1 S square plus 2 S plus 1.

Now, we have got a transfer function model; now what we want? This transfer functions we have to convert into a state space model that is the transfer function which is got by MATLAB by MATLAB we get transfer function model as S plus 1 S square plus 2 S plus 1 these model we have to convert into a state space form.

Now, as we have seen their command the command which can use here tf 2 ss. So, here we can write down a command as A comma B, C, D; that is tf 2 ss numerator NUM denominator. So, if you use this command the results which we will obtained using

MATLAB that is A matrix as A model that is minus 2, minus 1, 1 0 then B model 1 0 and C model 1 1 and D model as 0.

So, here by means of this command tf 2 ss we have got a state space model like this, but here if you see the model this model is of say a different from A B C D, but if you do it manually we can write down this model as A equal to 0 1 minus 1 minus 2; B matrix as 0 1, C matrix as 1 1 and D matrix; this is 0; that means, using MATLAB you got this result.

But you can also get result by this way manually, but if you determined the Eigen values of this, Eigen values of this will be the same values; that means, property cannot change if you convert the given transfer function model into a state space model. So, you will find out this A A m; B m, C m, D m here A B C D both have the same thing properties remain same.

Now, about the multi input multi output system, because in fact, is system is generally of multi input multiple output form. Then in that case how to determine the model; that is how to determine in the transfer function model from the given state space model, so, the command which we have to use that is numerator, denominator.

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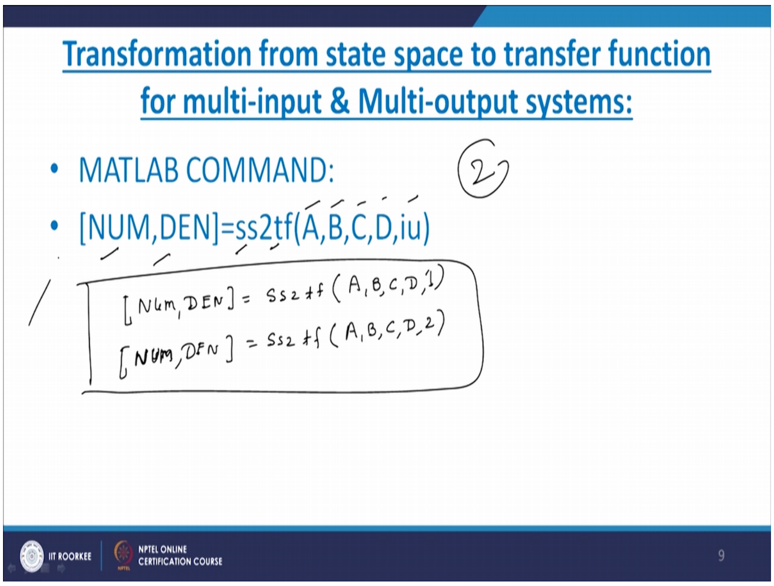


This ss this is state space to transfer function A B C D and iu, iu means particularly input; that means, which input you are taking let us say 2 input so here one or first intense for second state will get 2; that means, this command for 2 inputs system can written as numerator denominator ss 2 tf A comma B comma C comma D input 1 input 1 for 2 input system we write input 1. Then for second input numerator denominator ss 2 tf A comma B comma C comma D comma 2. So, these are the commands for 2 inputs system.

Now, how to use this command I will take the example. So, example this is for 2 input 2 output system.

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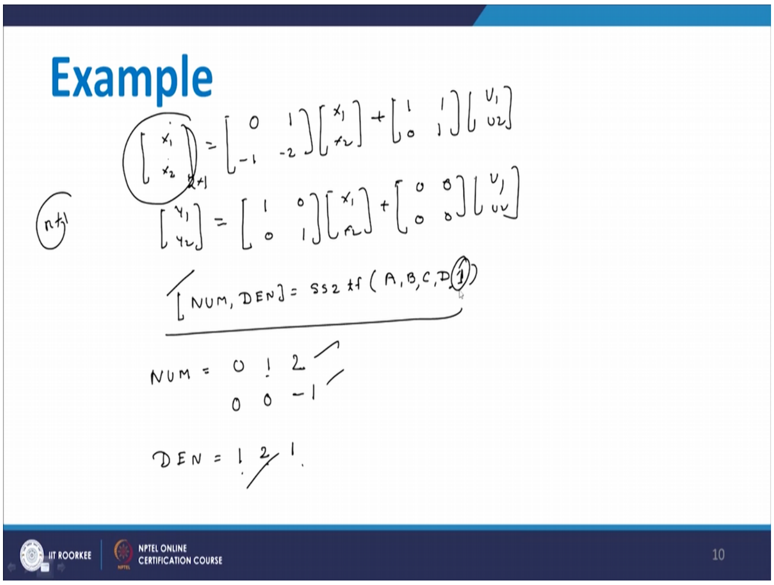


And we will see in what way we will get the result using MATLAB; now here X 1 dot, X 2 dot; 0 1, minus 1 minus 2 X 1, X 2 plus 1 1 0 1 U 1 U 2.

Now, this is a state model using that there are 2 inputs now about the output is Y 1 Y 2; 1 0 0 1; X 1 X 2 plus 0 0 0 0; U 1 U 2. So, we will find that this is a 2 input 2 output system and the most important point we see that whatever with the multi input multiple output system.

But if you see the order of this state space model 2 row 1 column always column is 1. If there are n states it will be n cross 1 and this will change the state model or state equations particularly states remains same; if you always the first order that is the advantage of the state space model. Now this state space model we have to write in the MATLAB environmental; so, how to write out?

So, as discussed earlier this command we have to use. So, here we can write down numerator; NUM, DEN denominator ss 2 tf; A comma B comma C comma D comma 1. Now so, if you write this command in a MATLAB we will get the result and you will find that the result we will get, numerator you get as 0 1 2; 0 0 minus 1 and denominator you will get 1 2 1.

Now, here we have got numerator 0 1 2, 0 0 minus 1 and denominator 1 2 1 when you have input 1; input as 1.

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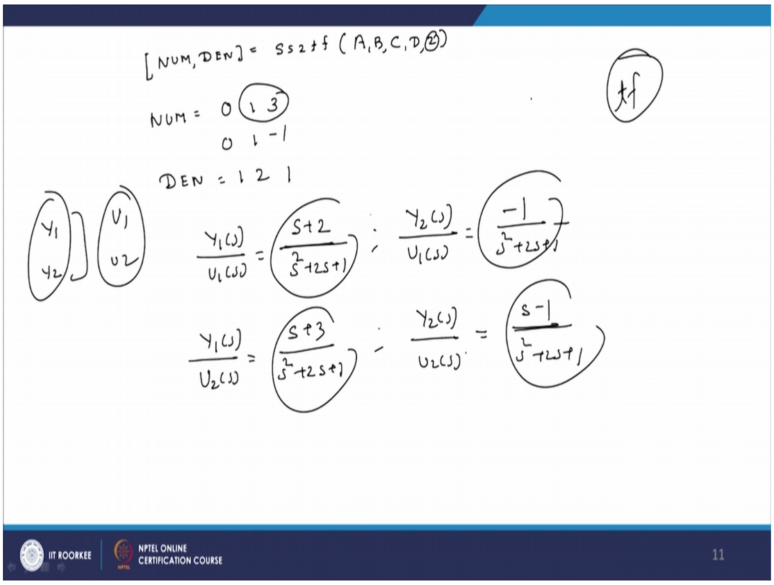


Now here when you use the command numerator, denominator as ss 2 tf A comma B comma C comma D comma 2. Now here we are applying the input 2 and therefore, we will get the numerator as 0 1 3 0 1 minus 1 and denominator we will get as 1 2 1.

So, here we will find that there are there are 2 inputs and 2 output system. So, here output is outputs are Y 1, Y 2 and inputs are U 1 and U 2; U 1 U 2 are the input Y 1, Y 2 are the output. So, here we will get 4 transfer functions; the first transfer function because of the input U 1 of S. So, here we can write the first transfer function Y 1 of S of U 1 of S to be if you see here 1 2.

So, we can write S plus 2 divided by the denominator S square plus 2 S plus 1, S square plus 2 S plus 1. So, here input U 1 is applied that time output is Y 1, similarly output is Y

2 of S and then same input U 1 of S; we can write down this as minus 1 minus 1 divided by S square plus 2 S plus 1.

Now, coming to the this part here your input is 2, so output Y 1 of S because of input U 2 of S; this is 1 3 that is S plus 3 divided by S square plus 2 S plus 1. Similarly output Y 2 of S because of input U 2 of S; we can write S minus 1, S square plus 2 S plus 1. So, these are the 4 transfer function we can obtain by using MATLAB.

And here we have to use the command t f; t f of numerator and denominator we will get 4 transfer functions. So, it means that even though our state space model involves multi input multi output system; we can easily determine the transfer function models.

Now, about the canonical state space representation, in earlier part if we have seen different type of transfer function; that is controllable canonical form then controllable observable form then we have seen a diagonal form, Jordan form; different types of forms we have seen.

So, these type of form can be possible to determine using MATLAB.

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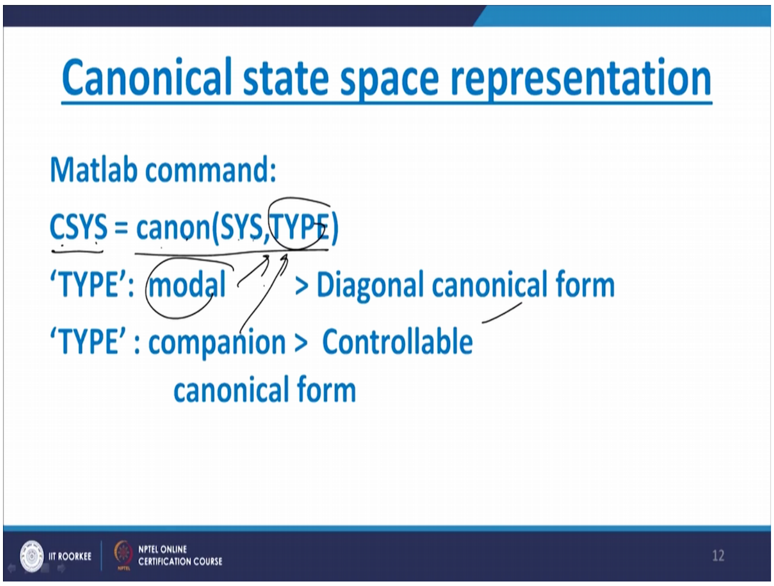


So, there is a one command is available in the MATLAB that is if you write the commands. CSYS canon SYS, TYPE this gives you the information about the diagonal form, as well as the canonical form that is controllable canonical form or we also call as companion form.

So, here this is type, so if you want diagonal canonical form that is if you want the A matrix involves diagonal elements. So, in that case here we can write the command as CSYS canon SYS and we write command as a model here that gives you diagonal canonical form.

But if you required that this given system we have to represent in a companion form or a controllable canonical form; we have to write the command as CSYS canon SYS and this we can write. So, we can get the different types of state space representation and we have already seen that transfer function is unique, but there are different types of state space model can be determined. It means that the Eigen values of given matrix A can never be change whatever may be the transformation we can use.

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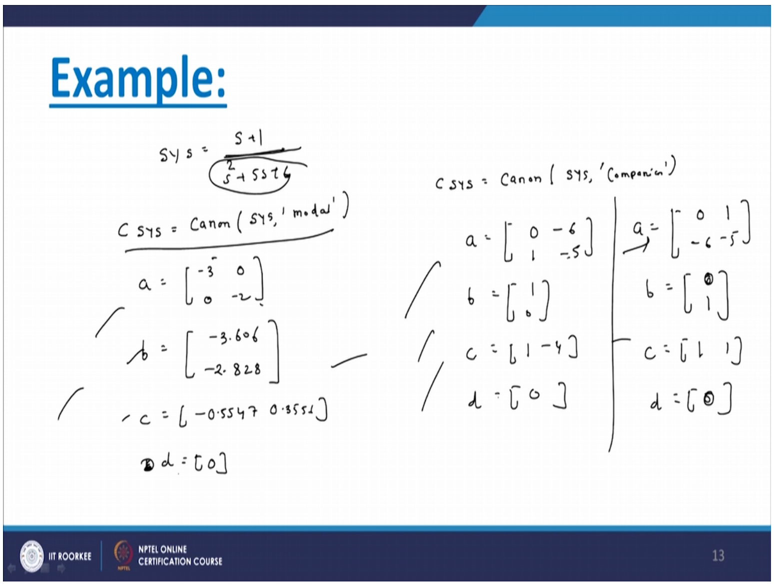


Now, we solve an example suppose SYS equal to S plus 1 divide by S square plus 5 S plus 6. Now this system we have to represent in a diagonal form using MATLAB. So, the command which can use CSYS canon canon SYS comma model; so if you write the commands CSYS canon SYS model and you can get the result as say a matrix as minus 3 0 0 minus 2.

We will get b matrix as minus 3.606 minus 2.828 and c matrix minus 0.5547, 0.3556 and d matrix is 0. That means, you will get a, b, c, d that is in particularly the newer system; it is in diagonal form. You see that for this S square plus 5 S plus 6; there are 2 Eigen values, one is minus 2 and another is minus 3 defined as they will come in a diagonal form if use this particular command.

Now, we want this system in a companion form; the command which can use CSYS canon canon SYS and here command is companion. So, if use this command you will get the result again in terms of a, b, c, d matrix. So, here we will get result a matrix as 0 minus 6, 1, minus 5, b matrix as 1 0, c matrix as 1 minus 4 and d matrix as 0.

So, here if you see this command if you got a b c d matrix and a has form 0 minus 6 1 minus 5, but I think we will find that this is somewhat little somewhat different than we have determine the companion form.

That is if you go through the actual model and if you manually determined the companion form or we call as controllable canonical form you will get a matrix as 0, 1 minus 6 minus 5, b matrix as 0 1, c matrix as 1 1 and d matrix equal to 0. So, here we will find that if you make the transpose of this you will you get this part.

So, this is also a one type of companion form and we will find that the all these 3 form have the same properties. So, sometimes whenever we come across requirement of the forms; we can use this type of form and most important is that all this form we can use to the MATLAB; we can get it there is no need for any calculation, we will find that the order of the system increases to convert one form to another is quite difficult.

But luckily we have MATLAB software, we can use for analysis of the systems, but in that case is more important thing that we want some variable because then your variables are there; we can easily carry out all this type of things. Now the stability in a state space model is a very important point.

And we have seen how to determine the stability of a given state space model, when we have seen the stability we have seen how to determine the stability of a transfer function model.

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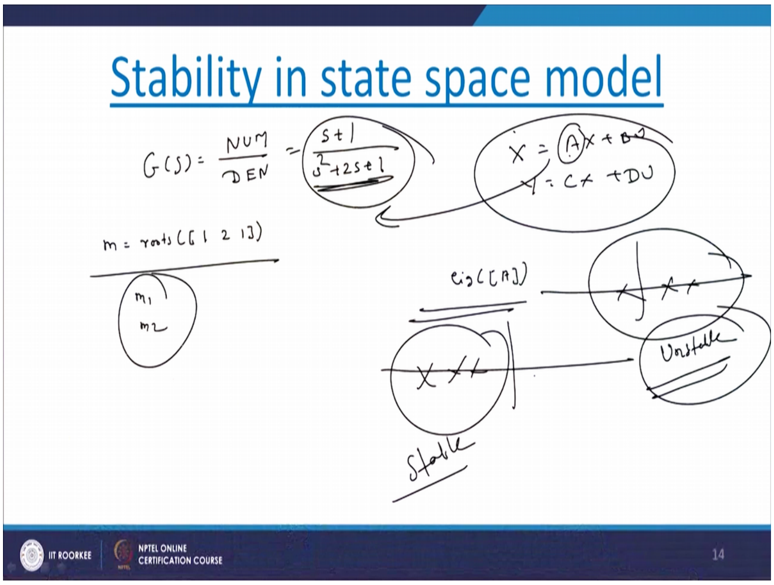


That is if G of S is there numerator by denominator say S plus 1 upon S square plus 2 S plus 1.

This is your transfer function model; if you want to determine the stability of this model S square plus S square plus 2 S plus 1; then how to determine? We can write a command as m equal to roots 1, 2, 1 bracket complete; here will get roots as this is taken can say we have a second order model we get m 1 and m 2 and base on this m 1 and m 2 we can decide the stability of this model; particularly this model is it in open loop model.

But now we want to determine this stability of the system in a state space model the state space model is X dot equal to A X plus B U; Y equal to C X plus D U and we have seen that the stability of this state space model it depends upon the A. And we have proved that why stability of a system in the state space model A is same as this denominator.

But now if you want to use MATLAB we have to use the command Eigen, Eigen of A if we use this command we will give we will get the Eigen values of matrix A and if the Eigen values of this matrix A lying on the left hand side; then we called the system as a stable system. But if use this command if you get any one of the Eigen value on the right hand side on the S plane we get system unstable.

So that means, stability we are getting by means of a simple Eigen command whereas in case of transfer function approach we can use the command a roots, we can get the stability of the system. Now the response in the state space model; in a classical control normally we are using simple step command or sometimes impulse command to get the response of the system that we have seen that if you take a transfer function as a numerator by 1 and denominator as say 1 2 1.

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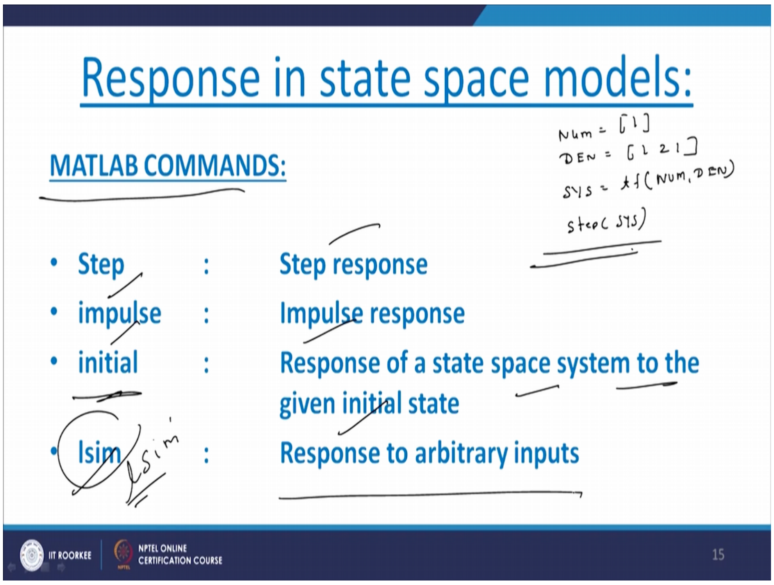


So, how you do it? SYS equal to transfer function numerator, denominator. So, this is a transfer function and simply we are writing down the commands step SYS that gives you the step response of the system. Suppose if you want impulse, we can get the response of the system particular in transfer function form.

So, this type of step command as well as impulse command we can also use in the state space model, but sometimes we cannot define our inputs. Input can be any arbitrary or we can have any arbitrary inputs. So, in that case how to determine the response of the system using MATLAB and this is a every time required.

It is because we have a state space model, then the state space model a different inputs will come sometime this term as will come. So, the system has been have different nature therefore, your inputs will also change therefore, this step command cannot be use every time. So, therefore if you see the MATLAB refresher there are various commands in available which can helps us to get the response of a system.

So, here the these are the commands are step step response, impulse is impulse response, then this is a initial command the response of state space system to the given initial state and lsim, this command is call lsim this is for response to any arbitrary inputs. Now using any example you can see how to determine the response of a system when any arbitrary input is given.

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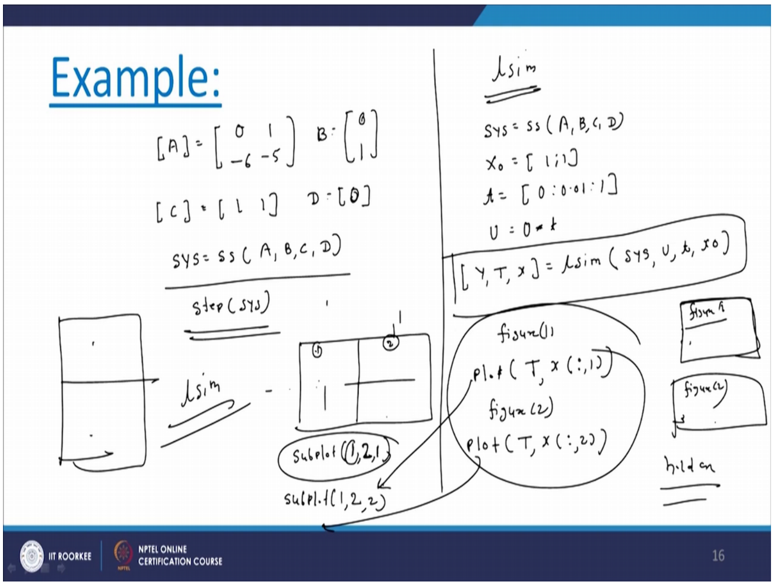


Now, your system is represent by say matrix A is 0 1 minus 6 minus 5 this is A matrix. B matrix as 0 1, C matrix as 1 1 and D matrix is 0.

Now, here we want to determine the response of a system for any arbitrary input. Now first of all we see how to (Refer Time: 30:36) the response of this system, let us say step input. As step input normally we are using for transfer function model will find that for this model also we can use the step command to get the response.

So, here what we can do? We can write say SYS equal to SS state space we can write only A comma B comma C comma D. Then we can write the command as step SYS that gives you the step response of the system.

Now, our main purpose is to use in any arbitrary input and I just discuss earlier we can use the command lsim now how to use it, so here we can write for lsim, how to use lsim command? So, here SYS equal to SS A comma B comma C comma D; then we have to defined some initial conditions see initial condition can be defined as a 1 1.

Let us say we define the time as say 0, 0.01, 1; then input this is a input U you can take any input. Now here I can take say 0 multiplied by say t; you can take sin t, cos t any you can take, but it has been represent with time t; so U 0 into t and here Y comma T comma x, we can write on a lsim SYS here input U time t and here is the we can say x naught is the initial condition, so this command we can use.

And now we want the state response. So, here we can write down in the way because here we have tourist states there are 2 states. So, we can write one figure 1, figure 1 plot T comma x 1 that gives the response for state 1. Similarly, we can write down figure 2 and we can write the command as plot T x.

Now, this gives you the response of the system because of the different states. If you want output response, we write the command output Y with respect to time T. So, in this way we can get the response of states as well as the output, but now here we have got response in 2 different figures. Now here if use the MATLAB first of all one window will come this shows a figure 1.

And afterward we will get figure 2 as another window. So, 2 different windows will get and we can get result, but sometimes what we want? We want result in a one particular window like this. If you want this is a one particular window and we want result like this the first result should like this one and second result should be like this.

So, in that case we can use the command is subplot. So, if you write the command like this subplot; now here 1, 1 indicate 1 row 2 column first this is a first row like this; now here is 2 column and here is a first positions.

So, if you write this command subplot and after that if you write this, you will get the result this will printed here. That means this result if you there if you printed here and if you want another result subplot 1 comma 2 phase 1 row, second column and the second this position second position. And again if you write this command you will get the result of this here.

That means results we can plot in any of the window or same window or different window. Similarly we can also we can also get result in this like this window this position this position by means of this subplot comma. Even we can use also hold on command; if you use the hold on command or the same window we can get the 2 results.

But sometimes if you want to better understanding of the system, we can go for this results, this response or this response I think this is far better because the same result we are getting in a one particular window; now, about the state transition matrix.

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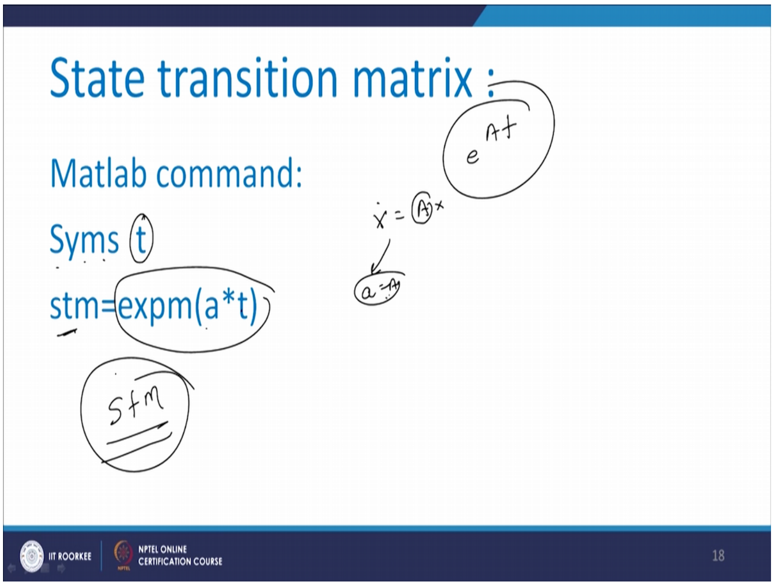


That is e raised A t; we have seen the different methods to get the state transition matrix, we have seen a power series method, then we have seen the Laplace transform approach, we have seen diagonalization method and also we have seen it Cayley-Hamilton theorem. And aim of all these method is to get the state transition matrix, but again we have see in the calculation wise this is quite difficult.

If you increase the order is quite difficult therefore, we have MATLAB this within a single command; you can get the state transition matrix. But here the main thing is that this is the point t; if you want result with respect to time t which is the generalized form; we can write the command Syms t and this is stm, expm like this a multiplied by t; a star t.

If use this a command you will get thus directly in state transition matrix; a into t that means, whatever is the matrix that like X dot equal to say A X. Now this is A we can let us say write down in terms of a where a equals to small equals to capital A; if I write like this write a star t we will get the state transition matrix.

So, here we have seen the all the MATLAB commands which can be helpful to get the state transition matrix, then response different types of modeling. So, this studies whatever you are studied that is the first part of the MATLAB.

Now, we will go for the second part of the MATLAB when we will study the concept of controllability, concept of observity, then control design include pole placement design, observe design. So that there in that case we will go for the part 2 and we will see again how this command helpful in that case.

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Now, some references we can use these references and try to solve as many example as possible using this commands.

Thank you.