Advanced Linear Continuous Control Systems
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Lecture – 26
State Transition Matrix (Part-II)

Now, we start with State Transition Matrix part 2. So, computation of a state transition matrix using diagonalization method; for any control system application, modeling, then state transition matrix, state equations, controllability, observability, control design, observed design. These are very important particularly in advance control system, advance linear control system. These are very important issues.

So, in the all this issue will find that diagonalization is a method which is useful in a modeling. It is useful in a state transition matrix. It is useful in controllability, observability and even it is useful in the controller design. Therefore, this diagonalization has wide applications. Now today we are discussing, the how to use this diagonalization concept for getting the state transition matrix. Earlier we have seen how to determine the state transition matrix using power series method also by Laplace approach; Laplace transform approach.

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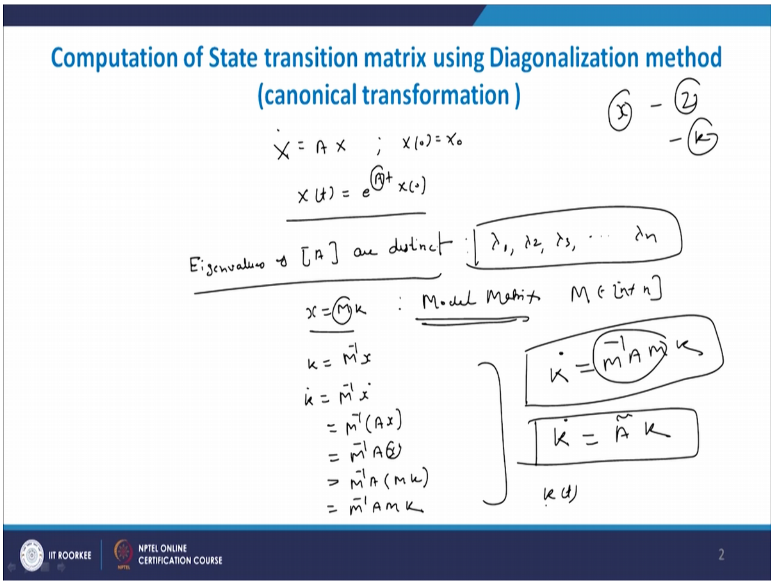

Now, we will see how to determine the state transition matrix using diagonalization method. So, we start with the same state physic equation. So, use the state physic equation is same. X dot equal to A X naught equal to X 0 and here X of t to e raised to A t into X naught. Our purpose is to be determined e raise to A t by means of diagonalizations. So, diagonalization we have seen that it is based on the sub additional state or can be different state; that means, original system, let us say in a state X. It can be coming to state Z or it can be coming into state K any state.

So, here our system it is sate X form. Diagonalization means that it has to come into different state. So, here first of all we assume, this matrix A has distinct eigenvalues that is eigenvalues are not repeated; that means, we assume that the eigenvalues of A eigenvalues of matrix A are distinct. That is eigenvalues of A are say lambda 1, lambda 2, lambda 3 up to say lambda n; these are eigenvalues they are different and as I told you that diagonalization is based on new state vectors. So, what we are assuming here in new state vector?

Let us say K. So, we can write down a state equation. Let us say a in terms of X equals to M into say K. x is original state vector, K is a new state vector and this is M is a model matrix model matrix; is also belong to n cross n n cross n matrix. Now, we have to move further. We want equation in terms of K. So, what will write from this equation? This K equal to M inverse x, K equal to M inverse x. Now after this we can differentiate this equation K dot equal to M inverse x dot. Now this M inverse, what is x dot? x dot equals to A x. So, we can write down A into x that is M inverse A into x.

Now, what is x? This x is nothing, but M K. Now here, our state it is in terms of K in the left hand side. So, the right hand side also we should get the equation in terms of K. So, therefore, M inverse A and this x is replace by M into K. So, if we can write this as M inverse A into M into K and this is nothing, but K dot equals M inverse A M into K this is equations. And now this M inverse A M this equation it is in diagonal form. That is K dot equals to A bar into K where this A bar involved the eigenvalues lambda 1 lambda 2 and lambda n; that is in diagonal form. Diagonal form you have seen the elements are very very less. On the diagonal elements are present remaining elements 0.

Now, our main purpose is to get e raise to A t. We have raise reduced A vector, but we have to reduced to e raise A t. How removed it? So now, we have got this one; just like x dot equals to A x, we have determined x of t. So, here also because equation in terms of K dot equals to A bar into K, we can get K of t, but question is remaining e raise to A t. Now, you correlate after word K of K nu of A t.

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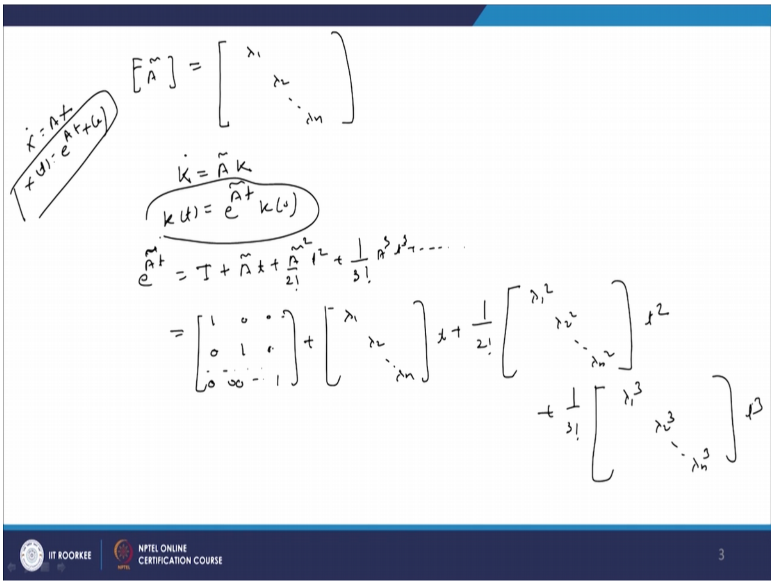

Now in this particular case, A bar as I told earlier this A bar is nothing, but is involvement of eigenvalues lambda 1, lambda 2 up to say lambda n and I am writing down this equation again; that is K dot equal to A bar into K. So, as I told x dot equal to A x, we are writing down x of t equal to e raise to A t into x naught. Similarly here also, we can write down as K of t equal to e raise to A bar t into K into not to K of 2. K of t equal to A bar t equal to K into 0.

Now, what is A? What is A A bar t? So, this we can write downed it power is expansion as I plus A bar t plus A bar is factorial 2 A t square plus 1 by factorial t A cube t cube like this. Now, this equation can be written as 1 0 0 say 0 1 0 say 0 0 1 with this plus these equations is A bar. What is A bar? A bar is your lambda 1, lambda 2, lambda n. So, you replace this here; lambda 1, lambda 2 up to lambda n that into t plus.

Now, here A bar square A bar square; that means, lambda 1, lambda 2, lambda n that is we have to make square of this elements. So, we will get lambda 1 square, lambda 2 square to say lambda n square into t square plus 1 plus factorial 3 equals to lambda 1 cube, lambda 2 cube up to say lambda n cube into t cube. So, this we have done.

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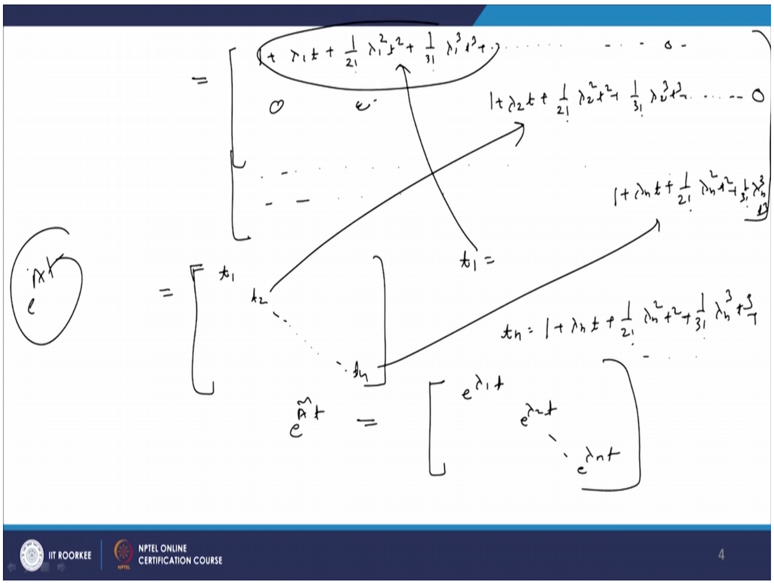

Now we can add elements of each rows and see what result will get. So, if we add this elements, we will get 1 plus lambda 1 into t plus 1 plus factorial 2 lambda 1 square into t square plus 1 plus factorial 3 lambda 1 cube into t cube and we will find that all the elements are 0. Now, coming to the second row; in second row in this second elements, in the second row will get the elements at 1 plus lambda 2 t plus 1 plus factorial 2 lambda 2 square t square plus 1 plus factorial 3 lambda 2 cube t cube and rest of the element 0; that means, in the second row in the second element we have element miss variables, rest of the elements are 0's; second element rest of the elements 0.

Now, this process repeated and at the last row; last row last column, you will get the elements as same 1 plus lambda n t plus 1 by factorial 2 1 lambda n square into t square plus 1 plus factorial 3 lambda n cube into t cube; that means, it is replace properly, we can show like this. This is t 1 t 2 and up to your t n and here this t 1 is nothing, but this part. This t 2 is the nothing, but like this and t n like this like this; t n here 1 plus lambda n t plus 1 plus factorial 2 lambda n square t square plus 1 plus factorial 3 lambda n cube into t cube and in this process of data; that means, we will get the elements in the diagonal form only.

Now, what do further? Now this we can replace in terms of lambda 1, this we can represent this of lambda 2 and this we can represent as lambda n; that means, finally, this equation that is e raise to A bar t can be written as e raise to lambda 1 t, e arise to lambda

2 t up to e raise lambda 1 into t; that means, we have staring with this A bar K into A bar K, we got A bar t. We have got this us and we are added after addition, we got this and this particular part written as e raise to lambda 1 t, this e raise to lambda 2 lambda 2, t e raise to lambda n t. But our main purpose against e raise to A t, but now here is to A raise to A bar t. Now how will we get it?

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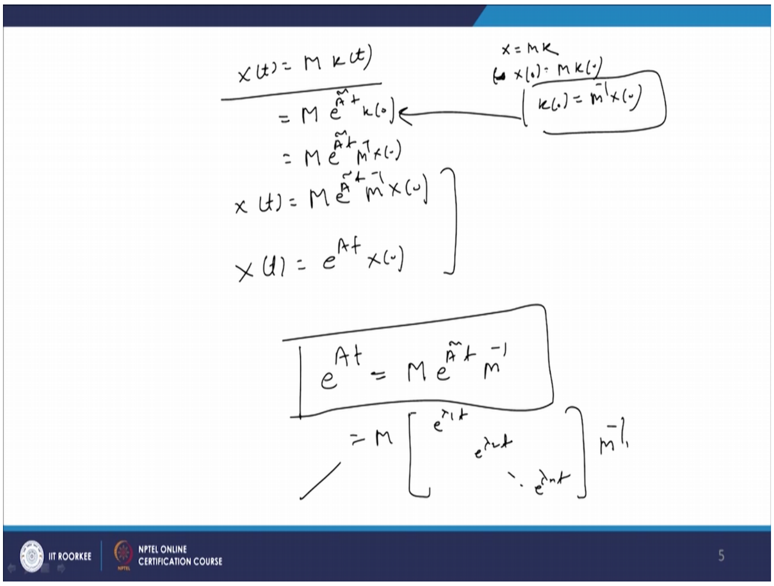

Now here, x of t equal to M into K of t. Let us see here. Here x M K there is x of t equals to M K of t and as I told you that after doing all this adjustment, we have e raise to A bar t, but our concern with the x of t. So, here this is M and what is K of t. This K of t equals to e A bar t into K of 0. This is see here this K of t equal to A bar t into K of 0, here we are replace M e raise to A bar t into x of K of 0.

Now, now here x equals to M into K; that means, if you write x of 0 equal to M K of 0; that means, K of 0 equal to M inverse into x of 0. So, here we replace this x of 0 equals to M inverse into x of 0. So, we can write down as M e A bar t equal to M inverse x into 0; that means, x of t M e A bar t M inverse into x of 0 and we know that this x of t equal to e raise to A t into x naught. So, if you compare this equation we can get equal to M e A bar t into M inverse. We will get M inverse. So, e of to equals to M into M e raise to A bar t into M inverse.

So, this is the method of getting that state transition matrix e raise to A t. At this e A bar t it can written as e lambda 1 t, e lambda 2 t and e lambda 1 t; that is M inverse. So, now this methodology we are seen and will find that as mathematical portion. It is quite cumbersome, but then question has been come why in a practice. We will do all this type of calculations. Because today we have a software's and we can easily get digliters. After will you see, how to calculate this state transition matrix using some MATLAB software. But question has become what is the importance of this in today's time?

In today's time, if you consider with the some numerical parameters numerical values, then there is no use of such methods. Only from academic point of view for getting some result it is useful. But if you working was some particular problems, what are their correlations between different states? What are the eigenvalues corresponding to this states? So, that internal information's, we can get by this one. That type of information cannot, can be achieved by means of software. Therefore, this has better, if you see from research angle is quite useful where will try to solve some way example. This is one this approach.

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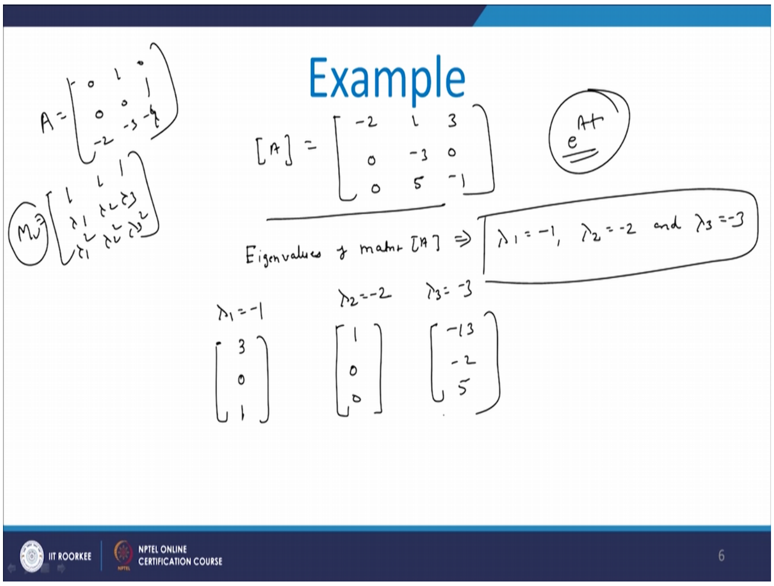

Let us say A given matrix A is written as minus 2 1 3 0 minus 3 0 0 5 minus 1. This is a matrix A and our made property determine e raise to A t. This is and where to use this daigonization approach and we always in the formulae and then use. Now will try to determine e raise to A t. As in this particular approach, you will find that it based on the model matrix M and how to get the model matrix? The model matrix depends upon the eigenvalues and also the eigenvector. Therefore, we know the problem has come to you and if you want to determine e raise to A t by daigonization, the first step way to calculate the eigenvalues of this matrix.

So, the eigenvalues of this matrix, if you calculate it the eigenvalues of this matrix A; eigenvalues of matrix A lambda 1 equal to minus 1, lambda 2 equals to minus 2 and this lambda 3 equals to minus 3. These are 3 eigenvalues. So, to could do get the M, we not only require eigenvalues; we also required the eigenvectors and we was the last time that if A given systematic A it is comparing form; that is or controllable canonical form that is A equals to 0 1 0 0 0 1 minus 2 minus 3 minus 4.

If this is the case, we was seen that the model matrix M can be written determine as 1 1 1 lambda 1, lambda 2, lambda 3; lambda 1 square lambda 2 square lambda 3 square that is called Vandermonde matrix. That is eigenvector can easily be represented, but when A general matrix is there because you see that this matrix general matrix. We cannot know what is the property of this matrix just looking after this A matrix. Thereby in that such case, but actually calculate the eigenvectors.

So, if you calculate the eigenvectors of this matrix A; eigenvectors of each Eigen, the rules we will get for eigenvector for lambda 1 equals to minus 1. It will get 3 0 1. The eigenvectors for lambda 2 equals to minus 2 1 0 0; that is the eigenvector for lambda 2 equals to minus 2 1 0 0. And third lambda 3 equals to minus 3, the eigenvectors are minus 13 minus 2 and this is 5.

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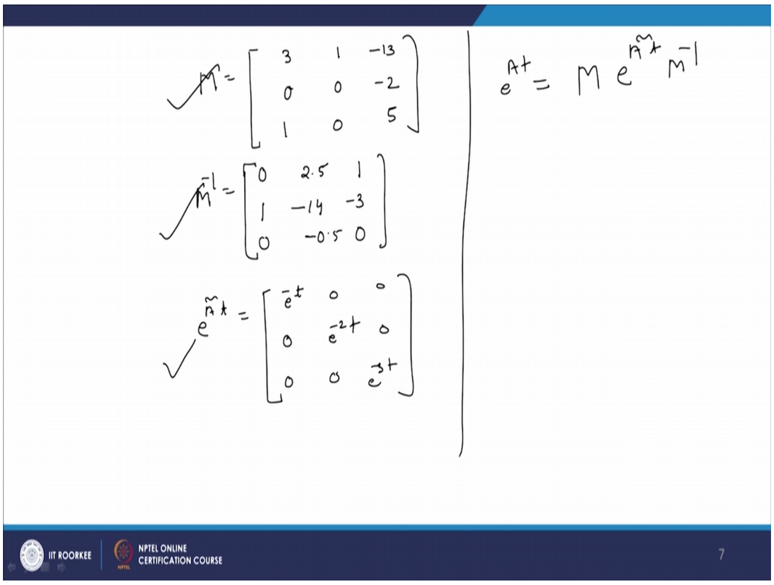

Now, these are the eigenvectors for each eigenvalues and to get the model matrix, we have to combine this eigenvectors. So, the eigenvectors we have combined and we will get the model matrix; this M equal to 3 1 minus 13 0 0 minus 2 1 0 5. So, we have got m, but now you see the formula. M e raised to A bar t into M inverse. M we now calculated. As I told by eigenvectors and eigenvalues or to get the M inverse, this also quite trouble some work and you say that the order increases quite difficult. Even for (Refer Time: 20:18) also if you do it, it will take at least statements. But the simple whether already explain the liberal go them where there is no need for calculating the determinant. By so, the trace of the matrix we can also get it.

Now here if you calculate the inverse of this matrix M, so we will get M inverse; M inverse we will get it as 0 2.5 1 1 minus 14 minus 3 0 minus 0.5 and 0. So, we have got the M inverse. The next step we are to determine this e A bar into t; e raised to A bar into t that to be determined. So, now e raise to A bar t and this e raised to A bar t is depend upon the eigenvalues; that means, in the in the diagonal element or the in a diagonal rows we can say, we have to write the element in terms of eigenvalues. So, the eigenvalues are minus 1, minus 2 and minus 3.

So, here we can write down as e raise to minus t, e raise to minus 2 t, e raised to minus 3 t and rest of the elements are 0's. So, we have got M, we have got M inverse, we have got e e bar into t and what is the final stage. Your final state is e raised to A into t that is M e A bar t into M inverse.

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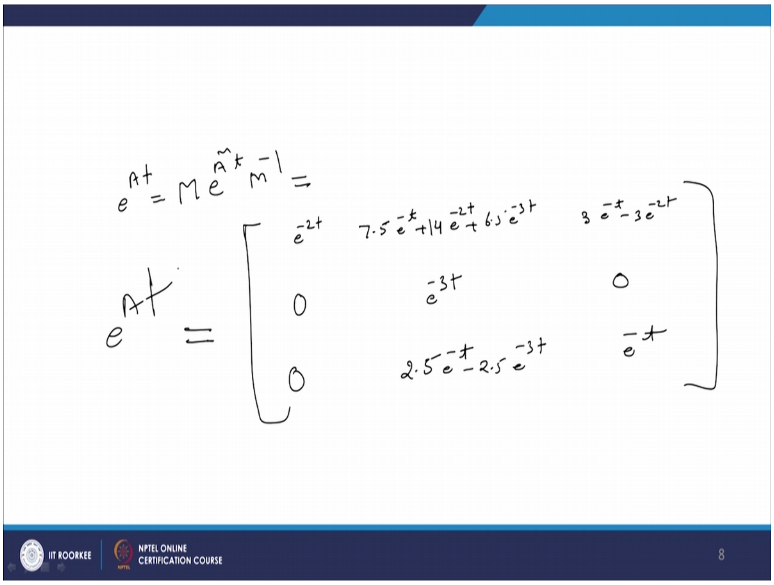

So now, if you multiply all these 3 matrices, you will get the result as that is e raised to A into t that is M e raised to A bar t into M inverse that is equal to e raise to minus 2 t 7.5 2 e raise to minus t plus e raise to minus 2 t plus 6.5 into e raise to minus 3 t. Any way 3 into e raise to minus 3 minus 3 e raise to minus 2 t, here we get 0. Here we will get e raise to minus 3 t, here we will get 0. Then third row 0 2.5 into e raise to minus t minus 2.5 e raise to minus 3 t and here e raise to minus t.

So, this is nothing, but a state transition matrix using a diagonal method or diagonal or we can say a canonical method, diagonal canonical form of method and again as you seen conclusion wise it is quite cumbersome. Therefore, as I to earlier be better to go for the Laplace transfer, but sometimes we have to go for the some internal analysis. This method is quite useful. This method as I told, it is useful for state transition matrix also state equations in the model.

We will forget this approach is also useful in control design. So, that part will see when I will explain you the how to design a state feedback controller. Now, these are some references D. Roy Choudhury, B. C. Kuo and also this Nagrath and Gopal.

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Thank you.