Advanced Linear Continuous Control Systems
Dr. Yogesh Vijay Hote
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture - 24
Steady State Error for State Space System

Now, we start with the Steady State Error for State Space System.

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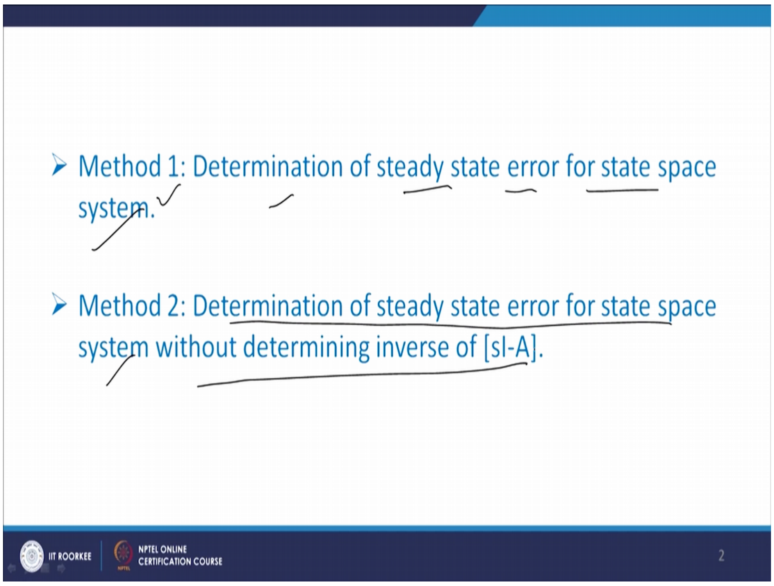

In this we will study two methods, method one determination of steady state error for state space system, method two determination of steady state error for state space system without determining inverse of sI minus A. In classical approach we have seen the calculation of steady state error for type 0, type 1, type 2.

So, why this need to calculate the steady state error?

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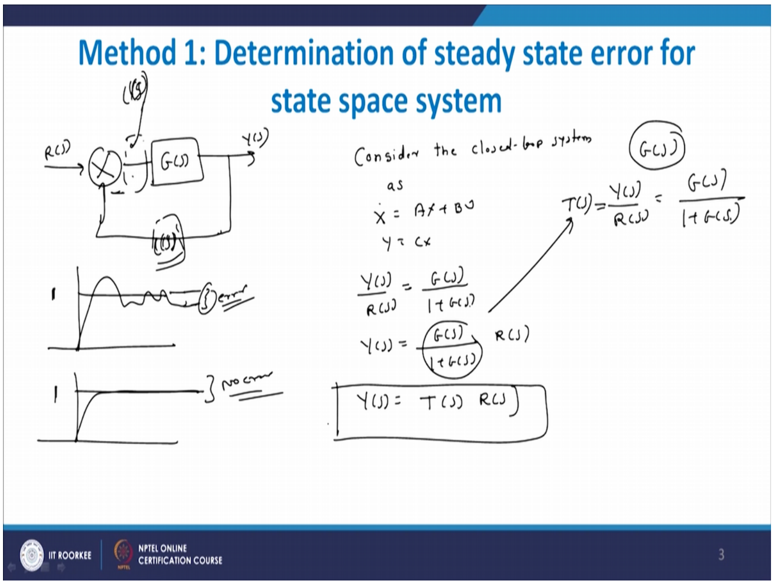

Suppose if you take classical example or classical control plant G s and. Now, here is a feedback R s into Y s. So, our basic purpose that we are giving the input that is R s that is a desired and we are getting for the system we are we are getting Y of s, and we are checking the response as we are applied input is like this. And if you getting response like these and then we are saying that there is a steady state error, this is error. Sometimes if you apply input step input and we get like this. So, we say that there is no error.

So, for any plant we have apply input getting output, we are saying where there is a error or not, but why we are doing all these things. Because my desired aim is important in control system, we have some desired aim, desired input or desire output we want ok. So, the desire output we have fixed first and then we see the plant it is from the plant we are getting a desired output or not. And if we are not getting a desired output then what we are doing we are designing the controller. That means, here also in a classical control system we are designing controller and here this particular part if you see this part we use a controller C of s, this C of s can be also be in the feedback path also if required.

So that means, the calculation of steady state error is very very much important. But now, we are working on advanced linear control systems that is working on the state space system, and whenever state space definitely final aim is to design a controller and the and if you cannot calculate the error it means that our controller design may not be so useful therefore, we have to calculate the steady state error for a state space model.

Now, we will see how to calculate the steady state error for a state space system. So, now, we write a state space system, I assume that it is in closed loop form. So, now, consider, consider the closed loop system closed loop system as X dot equal to AX plus BU, Y equal to CX. Now, we have plant G s this plant and if you take Y of s, Y s by R s equal to G s upon 1 plus G s; there is a a closed loop plant that is a Y s by R s equals to closed loop system this Y s by R s G s upon 1 plus G s, H s H s we have taken 1 so it is negative feedback. And this we can write as T of s closed loop transfer function.

Now, here same thing we can write down Y of s R s equal to G s upon 1 plus G s, and this can be written as Y of s equal to G s upon 1 plus G of s into R of s. And then this Y of s as I told G s, 1 plus G s it can written as T s. So, the T of s into R of s these a first equation which we got Y s equal to T s into R s the our purpose is to get the steady state error that is our purpose is to get the E of s. So, how you write this E of s?

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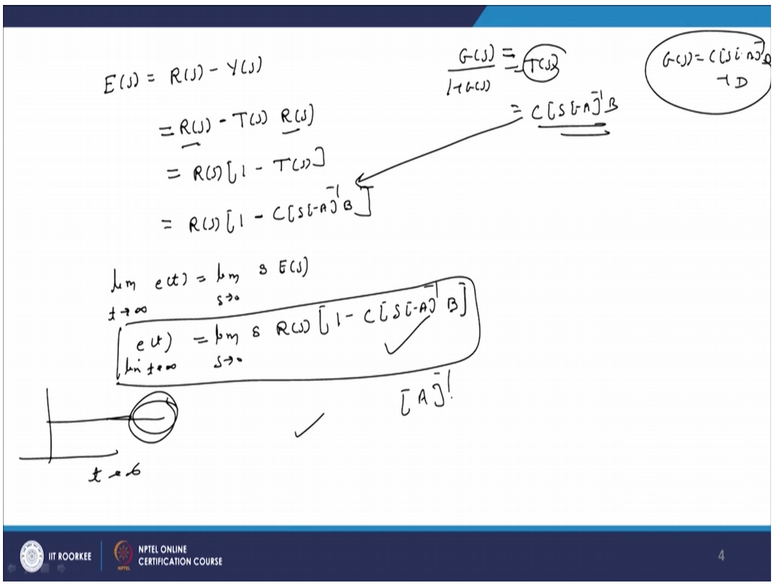

This E of s equal to reference input that is R of s minus Y of s and Y of s we have already calculate. So, just shifting this Y of s here or placing this Y of s here we will get R of s minus T of s into R of s.

Now, this R of s, R of s is there so we can take it common, R of s 1 minus T of s. Now, we have got the equation like this. In earlier classes we have seen the conversion of the given transfer function into state space even state space into transfer functions. But. Now, here our input is not a numerator and denominator here our input is A, B, C and D, here T of s is written in terms of G s if it is a part of the classical therefore, what we can do this T s we replace in terms of ABCD matrix. So, how do you replace.

So, here R of s T s, so that can be written as C sI A inverse B assume that D is 0, that is we have seen that any transfer function G of s. Now, this that is 1 plus G of s there is T of s. Now, this is transfer function. Actually when last time you have taking this example we have written in terms of G of s that is C sI A inverse B plus D.

Now, here these G s transfer function, so T s is also transfer function. So, here T of s is we written in the same way C sI A inverse B. So, this is like this. Now, we have got E of s but we have to determined the response and response in the time domain that is Y of t like this.

So, therefore, everything this Y of t, E of s, R of s, it in terms of r of t, e of t, Y of t so here also u of s we can write down in terms of E of t. Now, how to get it? So, here we can use the concept of the final value theorem therefore, this limit t tend to infinity e of t equal to limit s tend to 0 of s into E of s. And finally, this can be written as limit s tends to 0 of s into R of s 1 minus C sI minus A inverse B that is steady state error is like this that is limit t tending to infinity. Limit t tend to infinity e of t equals to limit s tend to 0 of S R s 1 plus C sI A inverse B.

Now, why we have taken t tend to infinity? Because in time domain steady state we write t tend to infinity that is time is infinite that means, when we take the response any plant this is t. So, when t tending to infinity that is we are moving last ten, n state steady state earlier part is a transient part.

So, here main aim is to determine the steady state error at t tend to infinity that is at steady state what is your output. So, this is the formula for getting the steady state error limit t tend to infinity of s tends to 0 as R of s, 1 C sI A inverse B and you will find that this error is depend upon the what input we have and again this C sI inverse B. So, now, this we have determined that is e of t where determined.

But here you also can observe that here we have to calculate sI inverse and also we have seen that again to calculate the sI A inverse I am difficult that is why we are go for the Leverrier algorithm. So, here also we can use the Leverrier algorithm. And you see that in all the techniques we have studied earlier this sI inverse and inverse plays a very very much important role.

But. now, our aim is that can it possible that we can eliminate to calculate the sI minus A inverse. So, partially we can do it, that is. Now, we will develop a method or we can solve a example in a such a manner that that there is no need to calculate the inverse of sI minus A that is our aim is that to eliminate the use of sI minus A inverse partially that is our aim is that we had to calculate only A inverse in getting the steady state error. Can it possible? That is our aim is can we get e of t without calculating sI minus A inverse. So, this is possible.

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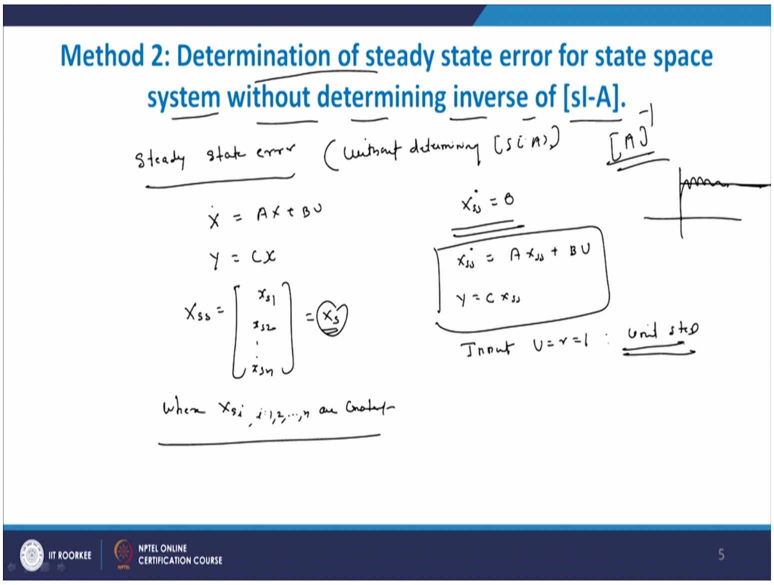

So, we can see this method. So, method two, determination of steady state error for state space system with the how determining in the inverse of sI minus A. Now, here this method is explaining. So, we will find that here also we need to get the inverse, but it is not sI minus A here in this method only there is need to get the inverse that is A inverse only, this part sI, sI part we can remove.

Now, we see this method. So, now, steady state error steady state error. So, without determining the, without determining sI minus A; now, we take the same plant X dot equal to AX plus Bu, Y equal to CX.

Now, we have a steady state because steady state is quite. So, here the steady state means X s of s we can write down as X s 1, X s 2, X s n, that came it as X into s. That means, when we are saying that at t tend to infinity our states also will reach a steady state. So, here all the state information we have written X s 1, X s 2, X s n. So, we can write down as this as also X s, this X s or X s also there is no any issue.

Now, here they state are a steady state definitely they will also constant therefore, where this X sI where I varies for 1 2 n are constant they are constant. So, when they are constant these are constant if you take differentiation of this is what will happen we can write down as is X ss dot equal to 0. So, we can write down as X s dot A X ss plus B into U, Y equals to C X ss. Now, this particular equation is nothing, but the steady state equation of a state space model here X ss thinner, but steady state derivative of steady state is always 0 just see here if you take any response if response is like these transient and finally, steady state. So, this is a constant. So, derivative of constant becomes 0. So, here also X s dot equals to 0 we are we have got X s dot equals to 0.

And. Now, about the input, so input is here input let us say u equals to r equals to 1 that is unit step. Now, here we are concerned with the unit step input steady state equation and then we will see what result we will get.

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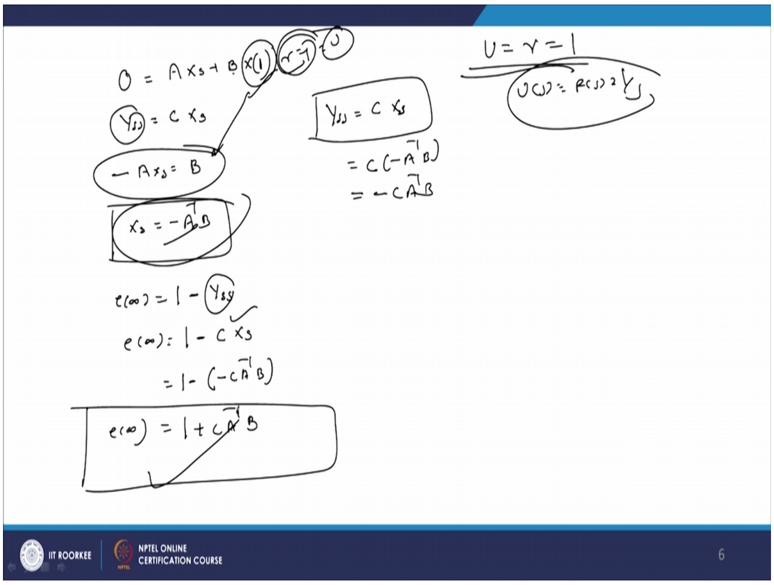

.Now, here 0 into A into X s plus B u is there that is you have a steady put of one and Y s is equal to C into X s that is here unit step input there is r equals to 1 here this can be called as a u also.

So, Y of s is a steady state output. Now, if you solve these equations, so what will get, this is minus A into X s equal to B. So, A minus X s equal to B as I told you that this X of ss it can be written as X s therefore, same thing we have written 0 X s B into r. So, B into 1 step input there is a r equals to 1 and therefore, what will get Y ss equals to C into X s.

Now, here this X of s equals to minus A inverse B, X of s is nothing but minus A inverse B. Now, what is your requirement we required error at t tend to infinity. So, here this e e of infinity the reference input is 1. So, 1 minus Y ss steady state error that is reference input one minus Y ss of s, this is equation in time domain and that is for r equals to 1 we have a solving we got A X of B. And finally, X of s equals to A minus B. Now, this steady state error equals to reference input 1 minus output Y what is the role of this here.

Now, here will come the role of this because the steady state output. So, I am considering this Y ss is nothing but your Y ss into C into X s Y of s. And what is X s? This X s nothing but C minus A inverse B that is minus C a inverse B and therefore, the steady state error we can write down as e of infinity 1 minus C into X s that is equals to 1 minus what is C X s equal to minus C A inverse B. So, we can write down it as C X s equals to A inverse B. So, we can write down as 1 plus C A inverse B to steady state error 1 plus C A inverse B and we will find that in this case the steady state error there is no term of s has come it completely depend upon the C A and B matrix and we can directly get the steady state error.

So, what advantage will get? Because advantage is simple because here we have to use A inverse, but whereas, if you see the earlier method we have to see calculate sI minus A inverse. So, this part has been removed. So, this we have calculated for step input similarly you can go for input parabolic input or for any other input you can try this approach and we can get the formula for steady state error.

So, this specifically for steady state error for a step input that is U equals to r that is sometimes you take input as U sometimes as r equals to 1 where as U of s equal to R of s equals to 1 by s. So, here we take an r as 1. So, B into 1 we have got a B and therefore, we cannot got minus A of X into B here just see this part. And finally, we have got e infinity equals to 1 minus Y ss. And what is Y ss? It is C X s and how to get C X X s, X s we have got by this and we replace in this we have got this particular.

Now, all this methodology we have seen that is calculation of error. Now, we solve an example based on this that is we use solve an example by both the method by method one and method two and see that whether we are getting the same result or not.

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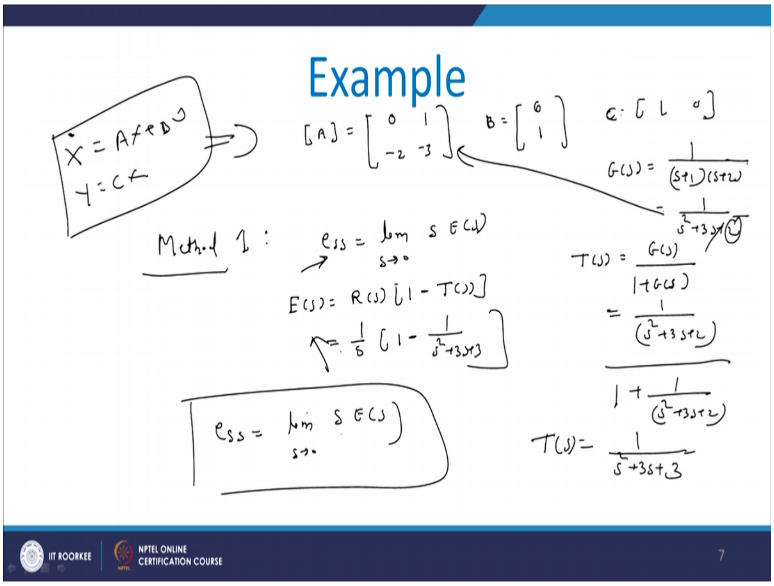

Now, we take an example let us say this matrix A is written as 0 1 minus 2 minus 3, B matrix as 0 1, and C matrix as 1 0. And this is particularly for your model X dot equal to AX plus BU, Y equals to CX this is a model.

And again one property we have got this no D it means that your; if you convert into a transfer function form the degree of denominator is greater than the degree of the numerator that is observations there is no D is involved. Now, our aim is to calculate the steady state error. So, first of all we use method number 1, so method 1, method 1. So, method 1, that is steady state error equal to limit S tend to 0 of S into E of s. And what is E of s? We have seen this E of s is equal to R of s 1 1 minus C sI inverse B that is E of s equals to R s 1 minus T s. So, we use this formula here that is E of s equal to R of s 1 minus T of s. Now, this is the platter ABCD.

Assume that this plant is an open loop form that is we have G of s equal to 1 upon S plus 1, S plus 2 that is equal to 1 upon S square plus 3 S plus 2 and therefore, this is the particular model. So, this is a G s. Now, here we want T s that means, we have to determine the closed loop cached equations. So, this T of s equal to G of s divided by 1 plus G s. So, here 1 upon S square plus 3 S plus 2 divided by 1 plus S square plus 3 S plus 2; so if you solve it, so what will get? This, 1 plus S square plus 3 S plus 2 and this will cancel, 1 remain is the numerator. So, your S square plus 3 S plus 2 plus 1, this is 3, that is this is a T s.

So, we will find that particular open loop for this example with two whereas, a closed loop this 2 has been increased by 1, this value of the last coefficient has been increased by 1. But these, T s we have to consider we do not have to consider this because our error is depend upon the T s. And what is R s? R s is the step input the step input is 1 by S, 1 minus what is T s 1 by S square plus 3 S plus 3.

Now, we have to use final formula this like this e ss. Now, this e of ss equals to limit s tend to 0 of S into E of s this is a formula. So, now, E of s is like this. Now, S we have to multiply in the see what result will get. So, here steady state error that equals to S multiplied by 1 by S 1 minus S square plus 3 S plus 3.

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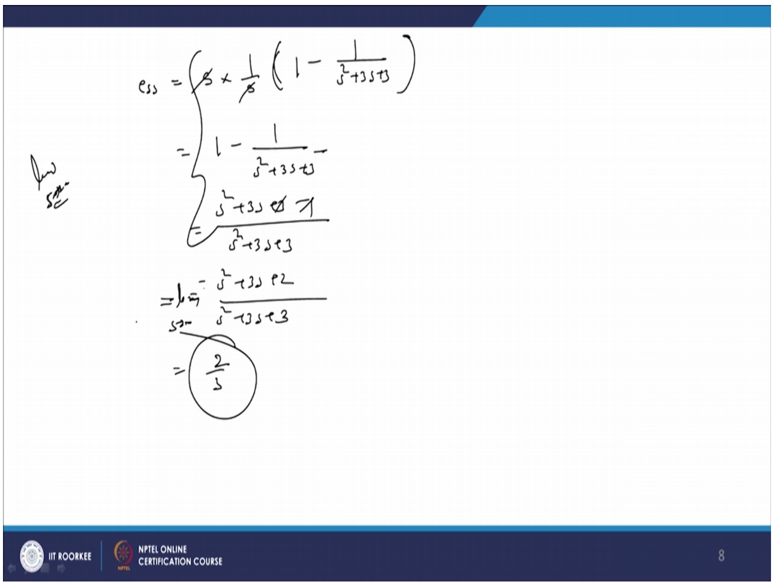

Now, this S S will cancel. So, we will get 1 minus 1 upon S square plus 3 S plus 3. Now, if you solve it S square plus 3 S plus 3 S square plus 3 S plus 3 minus 1 this can to 3 minus 1 2. So, S square plus 3 S plus 2 divided by S square plus 3 S plus 3 and here the steady state error is nothing but this limit S tends to 0, because all this portion is concerned with the limit S tend to 0. So, if you replace S equal to 0, so what will get? 2 by 3. Now, this is the steady state error which we have got for this example when consider in the system in a closed loop and by means of this we have got result.

Now, our main purpose is to get the steady state error when or our main purpose is to solve this example by second approach and we have to see whether we are getting same result or not.

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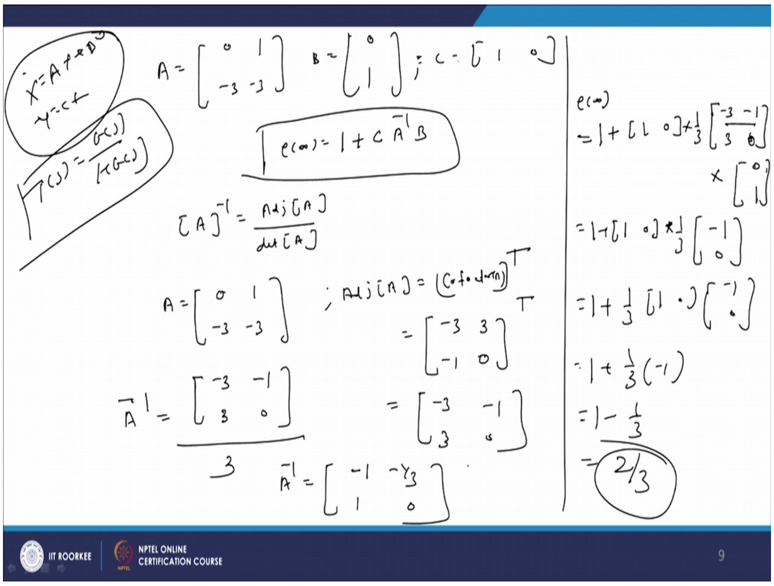

Now, the same example we are taking A equal to 0 1 minus 3 minus 3, B 0 1, C is C is 1 0; ABC. Now, ABC is there, but you will find that the a which we have taken earlier it is 0 1 minus 2 minus three whereas, here we are taking a has 0 1 minus 3 minus 3. Why? Because here our main concentration on the closed loop plant that is our model X dot equal to AX plus BU, Y equals to CX it must be in a closed loop for using the method 1, that is why here we are shown the state space model for a closed loop plant that is we write an equation ABCD from T s that is ABC and that plant is G s upon 1 plus G s.

Now, we have to use the formula. The formula for this is e of infinity equals to 1 plus C, A inverse B this formula to be used. Now, here to get the A inverse we can use the formula for adjoint of A divided by determinant of A, many times we have seen this particular formula; so here, adjoint of A, so A matrix is 0, 1, minus 3, minus 3.

So, adjoint of A then in nothing but the transpose of the cofactor the cofactor transpose there is cofactor of A transpose. So, here we will get minus 3 0, here we will get 3, we will get minus 1 transpose. So, we will get as minus 3 minus 1 3 0 and then the determinant of this, so if you calculate the determinant of this the determinant will get it as plus 3 so that means, the A inverse equal to minus 3 minus 1 3 0 and determinant of this is 0 into minus 3 0. So, it is plus 3 and therefore, this A inverse that is equal to here minus 1 minus 1 by 3 1 0.

Now, the main purpose is to get calculation of 1 plus C A inverse into B, here we can solve it like this e of infinity equal to 1 plus what is the value of C, C is 1 0. What is A? A can be written as 1 by 3 in same minus 3 minus 1 3 0 multiplied by, what is B in 0 1. So, here 1 plus 1 0 multiplied by 1 by 3. Now, if you solve this, this multiplied by this, so we will get minus 1 0. Now, here 1 plus 1 by 3, now, here 1 0 minus 1 0 so we will get 1 plus 1 by 3 into minus 1, here 1 minus 1 by 3 we will get 2 by 3.

So, here we have got the steady state error for this same example this example as 2 by 3 and even by a classical approach we have got 2 by 3. That means, we are getting the same steady state error. So, we had to see based on the problem which method to be applied. So, here we will find out for a lower order this method is means first method, but as the order increases I think some this method will be better. As when seen the various formulas earlier that is to get the steady state error here 1 minus C sI A inverse that can be used or simply we can go for the transfer function model of this plant.

So, sometimes if you having the this G s as a plant for G s that is we having the numerator and the denominator then we have to use the concept of this Laplace and we have to get this sI minus A inverse, But if you are knowing directly getting the transfer functions then sometimes this method can be very very much useful.

So, we have to see from the data available to you, based on the available informations we have you have to choose the method. Sometimes we cannot say that this is best or this is best it depends on which type of example which type of data is to available to you, but the last end of this that we have to simplify the calculation. So, here we have seen the two, one example and for using this both this example we have got the steady state error and we have see that 2 by 3 by the second method, and here also you will find the steady state error 2 by 3 and here also 2 by 3. So, both the way 2 by 3 and 2 by 3 we have got the result.

And this error is very much important in a controller design. So, when we will going for the controller design, controllability, observability we will see it is a factor.

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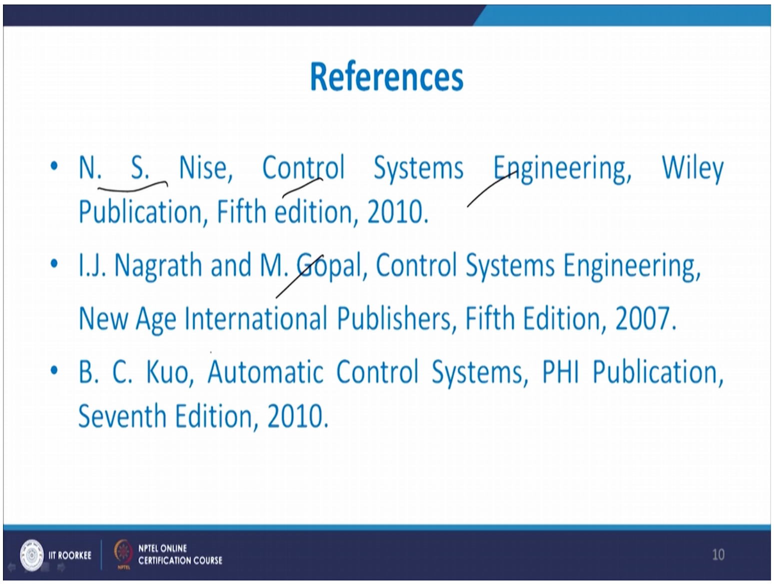

Now, the about a reference, particularly will find that the method two which I have explained to you will we will find in these particular books or some more example you can find here. And here also some more information we can get in Nagrath and B.C Kuo also there are many books which we can referred and here only one example I have taken. But similarly you go for the many more examples, solve these and see that which method is best so that you can clarify.

Thank you.