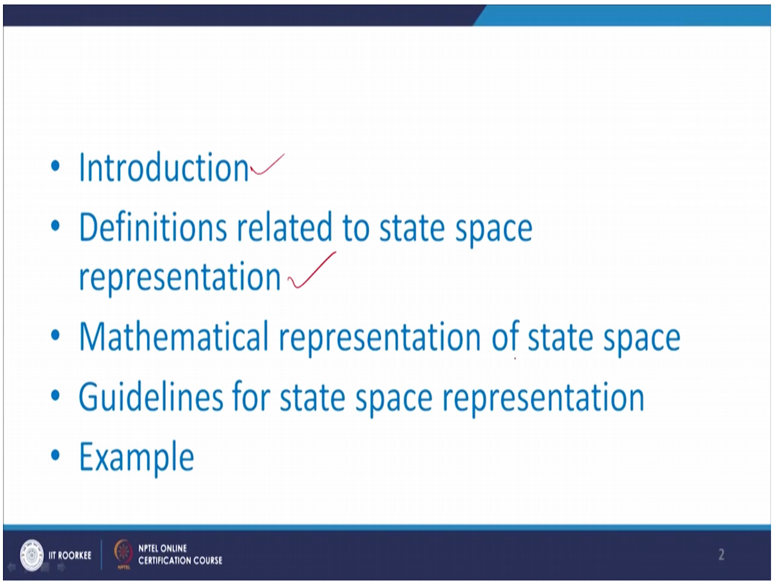
Advanced Linear Continuous Control Systems

Dr. Yogesh Vijaya Hote
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture – 02
State Space Representation

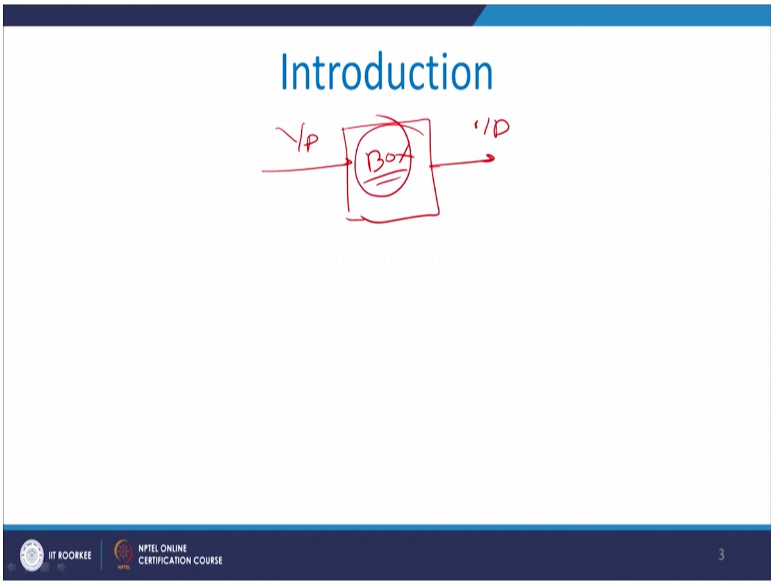
Now, we start with State Space Representation. In this we will study introduction, definitions related to state space representations.

(Refer Slide Time: 00:38)



Mathematical representation of state space, guidelines for state space representation and example, now coming to introduction section, in last class we have seen the advantages of state space approach over the classical approach and, here we seen that in case of classical approach. We are representing model in terms of transfer function transfer function Laplace transform output. To Laplace transform input initial condition 10 into 0. Now, here state space form when state space is coming how to represent the model.

(Refer Slide Time: 01:25)



So, most important point is that in case of state space model, we have input output and let us say this a box and in this particular box, there are some states are present some information is present and that information a is to be utilized to get the proper output; that means, here in case of state space we have input and some information in the box.

So, that information is the box I am saying that it is called state space, because that information will provide a output along with input, where as if you say the classical approach this box the thing in the box it is a black box, applying input and getting output. The thing is that that how to select the variable, because in the box there are number of variables are there for example, suppose last time I given a example of doctor and, patient goes to doctor and doctor is saying that your some problem in the eyes.

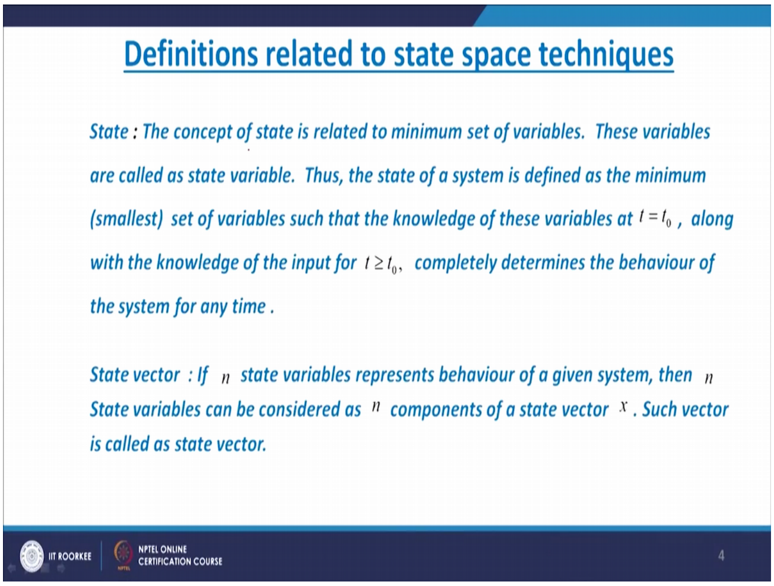
So, we can do go for some test relating to eyes, but patient is saying that no sir I had to do the heart test that means, meaning that heart means that heart test is that more variables are required so; that means, we have problem in the eye, but you are testing the heart along with the eye actually there is no problem.

But we are taking the more variables, but state space say that no state space say that you take the variable which is actually required do not take much, another example I can say, suppose say our head of department ask student coordinator of which class to attend the meeting. So, let us say there first year, second year, third forth year.

The four students have the approach to head regarding the meeting, but these another student they are saying that they also want to come, but although they can come but the main purpose that four representative, they can solve the problem whatever the discussion will be taken place in a meeting of a with the students with student representative, they can convey the message, but another is coming it means that although you can come, but that naturally we not required this one.

So, therefore, when is state space is that we should be confined to minimum 1, which can govern the performance of the systems. Therefore, the state is nothing, but that has some stakeholder in that particular system and, based on that informations along with input we can get the output, at different states are possible. So, x 1 x 2 x n to they have been present in terms of a vector that is called state vector.

(Refer Slide Time: 04:17)

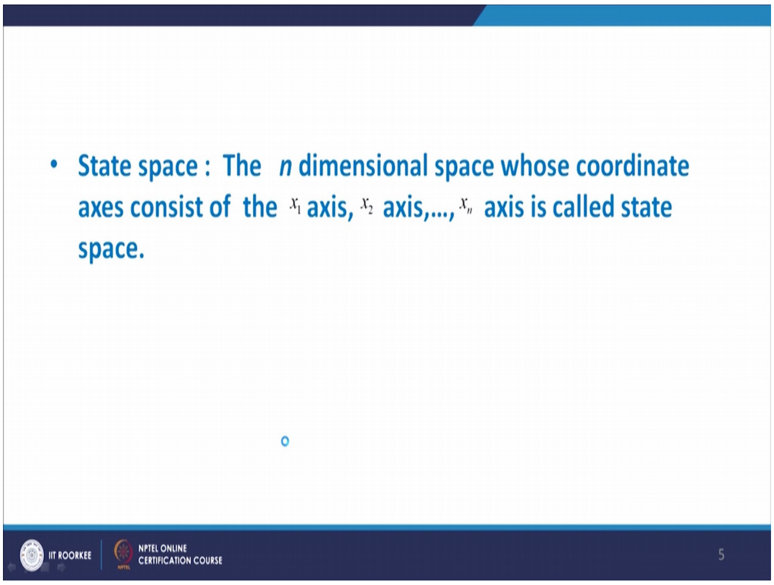


So, I read the definition properly, how do you define the state. This state the concept of state is related to minimum state of variable that I has told you that, you are a variable should minimize should be more, these variables are called as state variables.

That the state of a system is defined as the minimum, or smallest set of variable such that the knowledge of these variables at t equal t naught along with the knowledge of the input for t greater than equal to t naught, completely determine the behavior of the system for any time; that means, your output depends upon not only on the input but also internal state of the system.

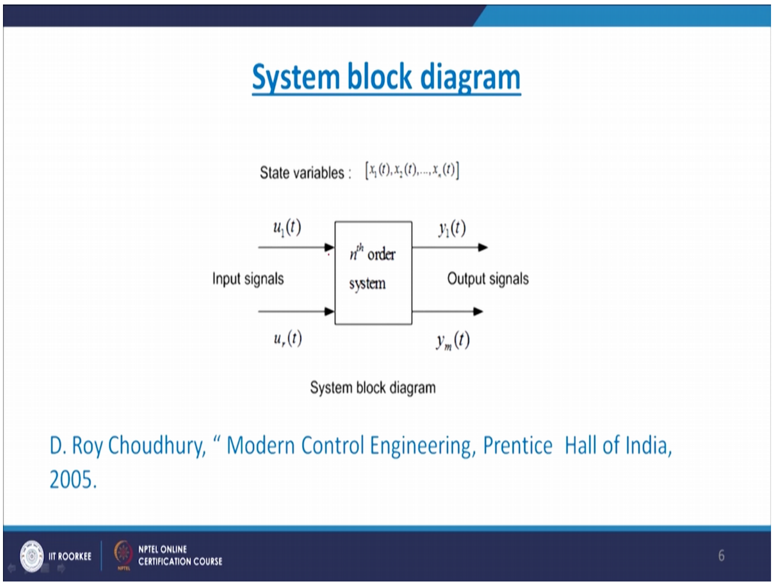
So, that is state now state vector, if n state variables represents behavior of a given system, then n state variables can be considered as n component of state vector x, such vector is called as state vector; that means, as I told you that there are many state vector say x 1 x 2 x n so, combine it become a x so, x minus x to x l that is called state vector.

(Refer Slide Time: 05:22)



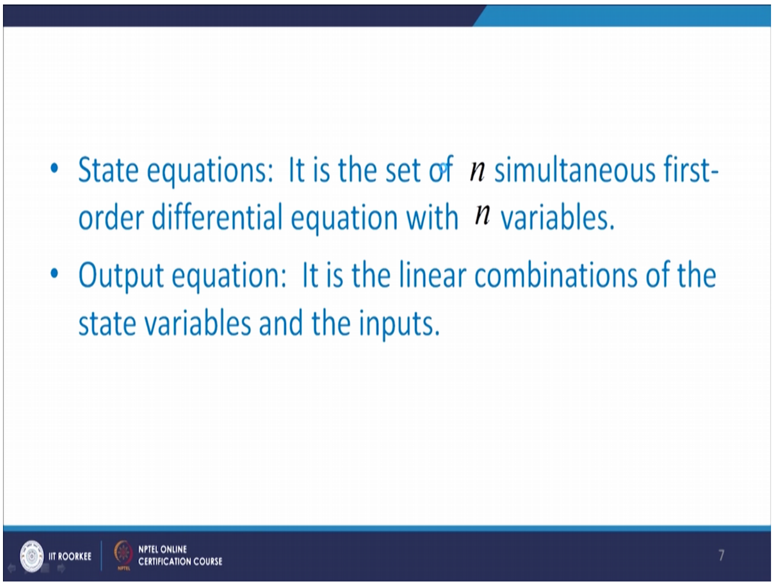
Now, state space the n dimensional space, whose coordinate axis consist of the x 1 axis, x 2 axis and x n axis is called state space. Now, representation as I told you that this is the plant nth order of systems.

(Refer Slide Time: 05:38)



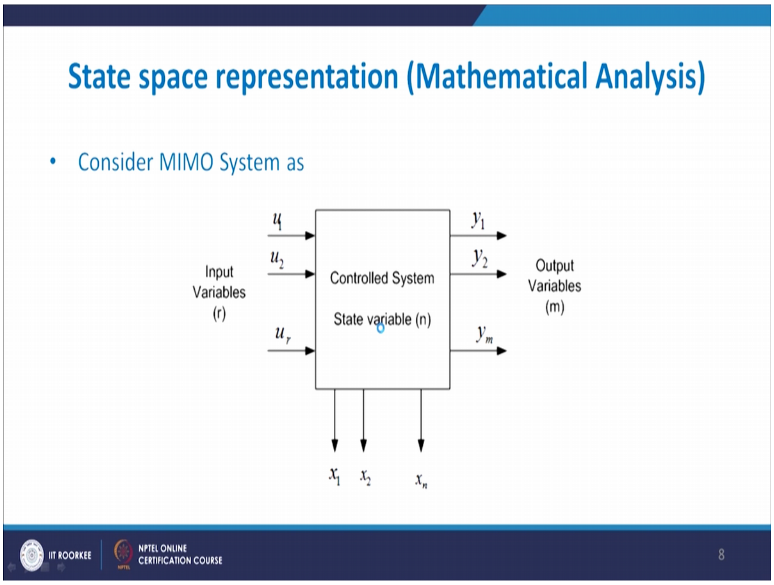
Now, the inputs are (Refer Time: 05:41) here input signals u 1, u 2, u r, then this is output y 1 up to y m m is the output, r is input, and this is nth order of system and this system there is a involvement of state x1, x 2, x n.

(Refer Slide Time: 05:56)



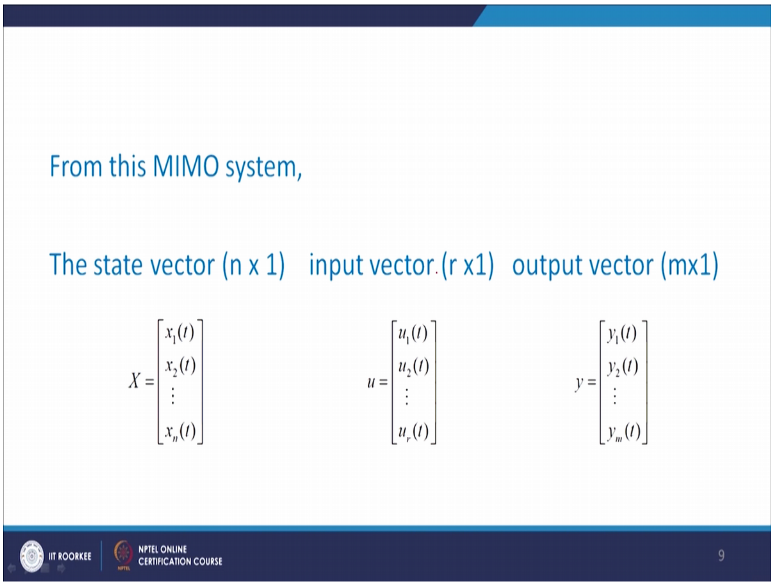
So, for this we can also define state equation, it is the set of n simultaneous, first order differential equation with n variable and output equation it is the linear combination of the state variables and the inputs.

(Refer Slide Time: 06:13)



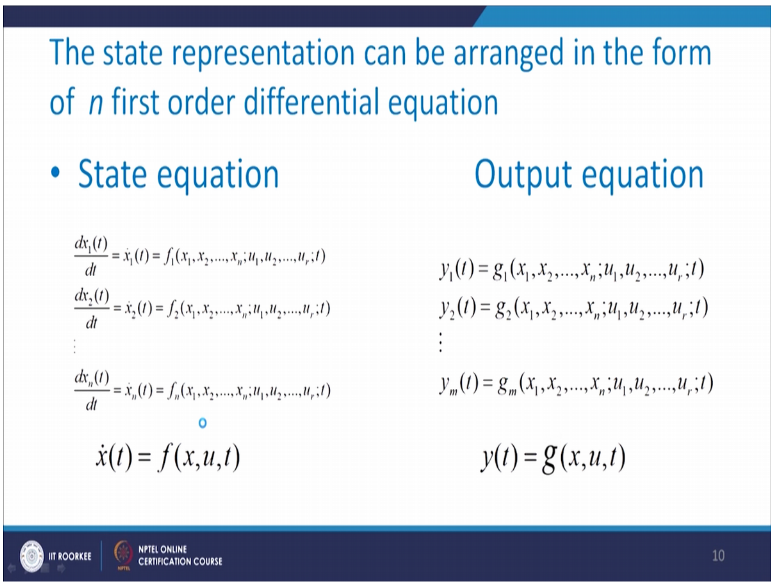
Now, again I am giving the same diagram, now here again input variables u 1 to u r y 1 to y m and state variable x1, x 2 x n output variable n input variable r.

(Refer Slide Time: 06:25)



So, here their state vectors we can represent X so, x states are n. So, we write x1 x 2 x n n cross 1, input vector r cross 1 that is u 1 u 2 r and output vector m cross 1, y 1 y 2 m; that means, m indicates output r indicates input and this n indicate the number of states that is for (Refer Time: 06:55) MIMO system.

(Refer Slide Time: 06:57)

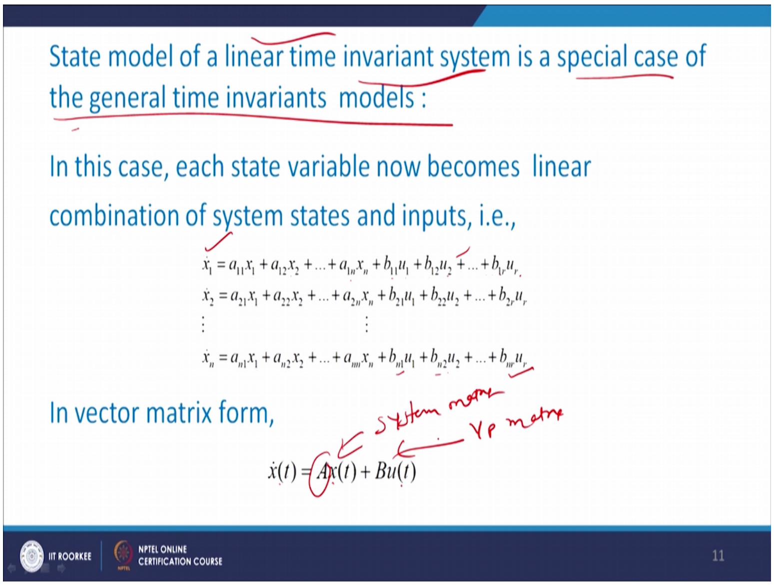


Now, we have to represent this equations so, here in state space we have to represent in a first order differential equation form that means, here you will find that the first order equations. So, dx one by dt equal to x 1 dot t, which is a function of x 1 x 2 x n u 1 u 2 u r t; that means, x 1 x 1 dot function of all these variables.

Similarly if you take s 2 d x 2 by dt dt that is x 2 dot t, which is also a function of x1 x 2 x n u n u 2 u r t. Similarly dx n by dt that is equal to x n dot, it is also a function of x 1 x 2 x n u 1 u 2 u r t finally, this is represent as s dot t equal to function of x u t. Now, similarly the output equations, output equation is also function of x and u therefore, it is represent as y 1 2 equal to g 1 function of x1 x 2 x n u 1 u 2 u r t y 2 equals to g 2 function of x1 x 2 x n u 1 u 2 u r t.

And finally, y m t so, m the output as I told earlier function of g m x 1 x 2 x n u 1 u 2 u r t. So, that is state equation it is output equations, as I told you earlier also the most of the systems are non-linear sometimes, we have to represent in a linear form that is called state space form or that means, we are discussing state space form; that means, whatever is the suppose if you write in terms of linear time invariant system.

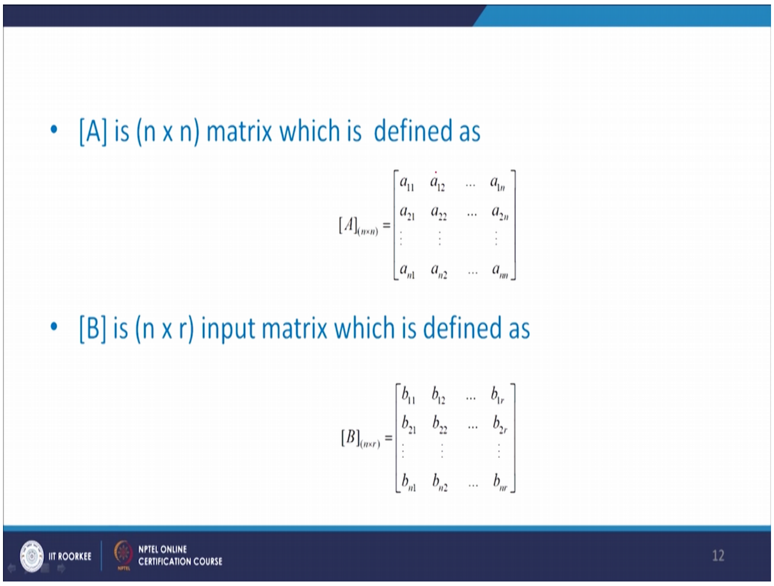
(Refer Slide Time: 08:46)



So, here this can be written as that is in this case is each state variables now, becomes a linear combination of the system state and input that is you have x 1 dot equals to a 1 1 x 1 x 1 a 1 to x 2 a 1 1 x n b 1 1 u 1 b 1 to u 2 b 1 r into u r. So, this is nothing, but they are nothing, but the special case of general of time invariants system, that is linear time invariants system form.

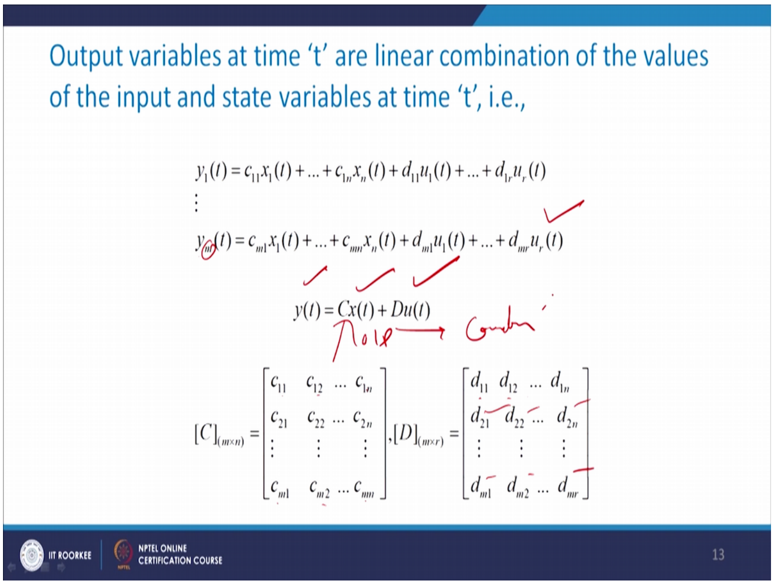
So, here we write x n dot equals to a of n 1 x 1 a of n 1 x 2 a 1 m x n b n 1 b n 2 and b m r. And therefore, in vector matrix form is written as x dot equals to A x plus B u, where this A, we called system matrix and were B is your input matrix. So, s dot equal to A x plus B u this is your state equations.

(Refer Slide Time: 09:48)



So, what are the elements in a we write down, we written like this a 1 1 a 1 2 a 1 n a 2 1 a 2 2 a 2 n, then a 1 a n 2 n and B is also a form n into r form b 1 1 b 1 2 b 1 r b 2 1 b 2 2 b 2 r, then b n 1 b n 2 b n r.

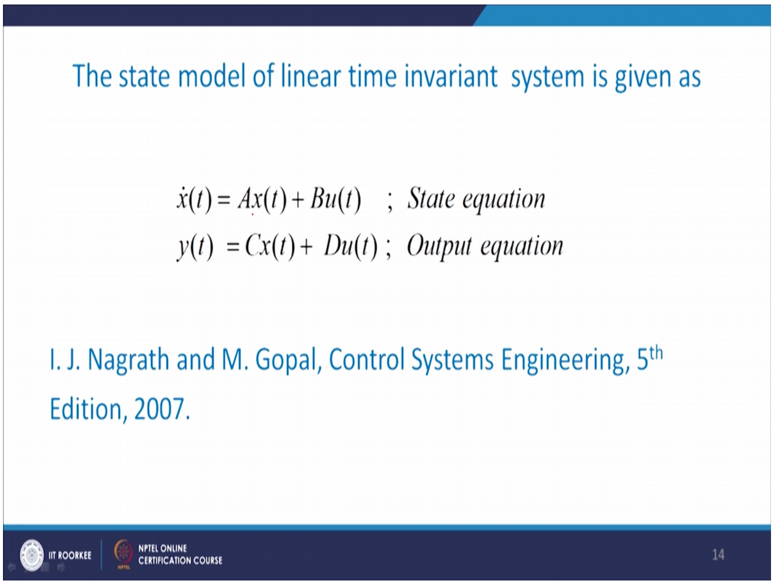
(Refer Slide Time: 10:08)



Now, coming to the output equations as the output is also a combination of your input as well as state variables therefore, we also written output in terms of c 1 1 x 1 2 c 1 n x n t d 1 1 u 1 t d n r. Similarly we can write y 2 y 3 and finally, we reach as the output is m.

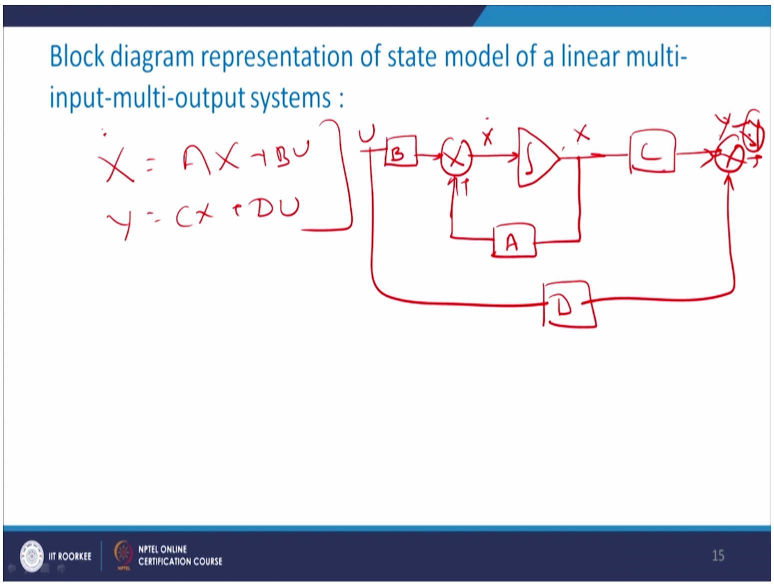
So, we are written y m t equals to c m 1 x n t c m n x n t d m 1 u 1 t d m r u r t that is y t y of t equals to C x of t and D u of t and the elements the coefficient are c 1 1 c 1 2 c 1 n, up to c m 1 c m 2 c m n, then d 1 1 d 1 2 d 1 n d 2 1 d 2 2 d 2 n d m 1 d m 2 d m r. So, here y equals to C x plus D u. So, C is nothing but your output matrix and D is the coupling, coupling between input and output.

(Refer Slide Time: 11:12)



So, this final state model is x dot equal to A x plus B u y equals to C x plus D u and as I have told earlier A is system matrix, B is input matrix, C is output matrix, D is coupling matrix coupling between input and output. And you know in future you learn the stability issues performance, we will see the importance of each matrixes A B C D right, now only understand at this is the state model and this are the significance of A B C D as I told A is system, B input, C output, D is the coupling.

(Refer Slide Time: 11:57)



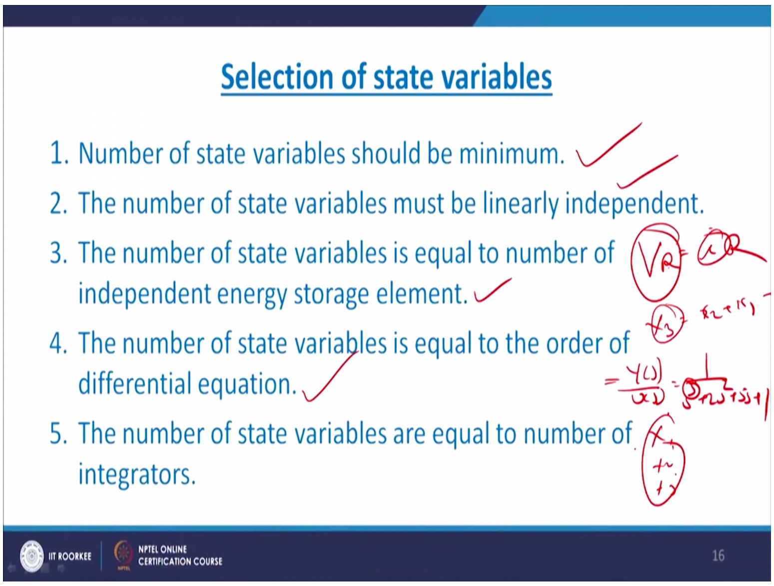
Now, we have to represent this system in terms of block diagram that is now you have system we have to X dot equals to A X plus B U Y equals to C X plus D U. Now, this system we have to represent in terms of block diagram. Now, here X dot that is first order differential equation that this one thing is clear here.

That whatever may be the order of the system, state equation always a first order whereas, if you see the transfer function is not possible, you have different order system sixth order tenth order 100 order, but here is through that whatever may be the system, your order is always one.

Therefore, defiantly we have to use how many indicators, we have use only one indicators therefore, here I am using only one integrator here. Now, here one integrator have is now here I write summation block, now here X dot. Now if we integrate it becomes X, now X dot equals to A X, this S dot equals to A X plus B into U.

This is B X dot equal to A X plus B U and what is Y? Y equals to see here, this is Y, I can say this is Y Y equals to C X and now here D U so, here I can cut Y equals to this is Y Y equal to C X plus D U. This is a block diagram representation of a state space model of a linear multi input multi output systems X dot equal to A X plus B U and, what is Y Y is equal to C X plus D U.

(Refer Slide Time: 14:09)



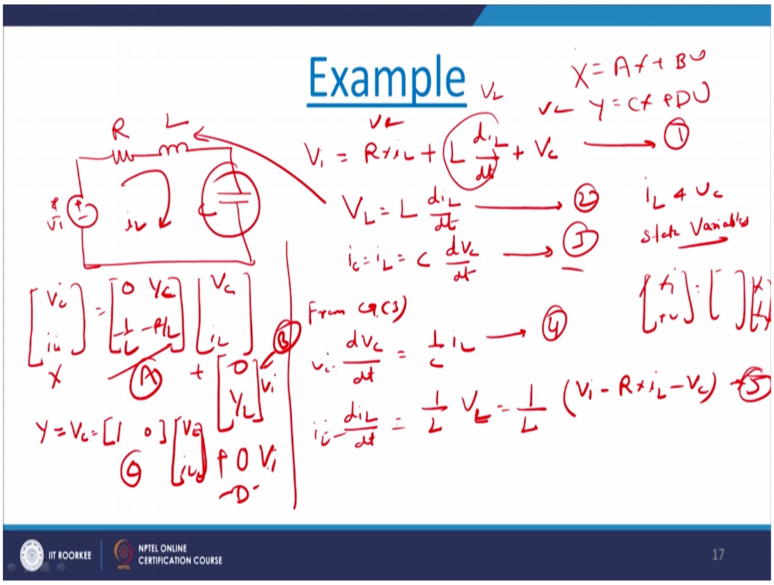
Now, the selection option this is very important in state space analysis, we (Refer Time: 14:13) should we take. So, the general logic is been given here, to select the state is that we have to select the state which as energy storage elements. So, whenever the problem come to you have to see that how many energy storage element are there. So, number of energy storage elements equals to number of states; that means, that if you select minimum energy storage elements energy storage elements; that means, that always minimum 1.

So, here first point is that number of state should be minimum, number of state variable must be linearly independent, it means that is let us say V R equal to i into R, suppose someone as selected V R at the state variable, but V R is depend on i. So, we can select i as a state variables. Similarly if you write X 3 X 2 plus X 5 that means, that if you select X 3, then cannot select X 2 as the state variable; that means, number of state variable must be linearly independent.

Then third point the number of state variables is equal to number independent energy storage element, as I discuss earlier also that is whatever be the variables equals to state variables are equal to the number of independent energy storage element. Forth point number of state variables equals to the order of the differential equations that means, you have difference equation, when we write in terms of Laplace form that is transform function form, that is when I been writing down Y is equals to U S equals to 1 upon X cube plus 2 x square plus 5 x plus one \

Now, here order is 3 it means that you know actual system in state space, you have selected 3 as a state variables. And number of state variables are equal to number of integral; that means, that if 3 state variables and, they are in terms of let us say x 1 dot x 2 dot x 3 that means, that is equal to the number of integrators. Now, we will see the example and through example, we will see how to develop state space model and one thing is clear.

(Refer Slide Time: 16:43)



Finally, my model be required in terms of X dot equals to A X plus B U Y equals to C X plus D U, this model we want. So, in order to understand it properly I am taking very simple example, that is we are studied in the first theory electrical engineering that is simple R L circuit so, I am drawing simple R L C circuit.

So, I am writing down R E L C, this is simple circuit this is input V i this is R this is L and this is C, I can write plus minus here then this is input V i input V i now as soon as this current is flowing to circuit i L. Now, we have to write and as I told you that our purpose is to develop state space model.

So, as far the theory of electrical engineering, first of all we write the equation of the circuit differential equation. So, how you write it, we can write down as V i input R into i L as current i L, we have taken plus L d i L by dt plus here see I can see voltage across capacitor c, V i equal to R into i L a d L plus dt plus V c.

So, we write equation number 1. Now, our purpose is to develop the state space model, as I told you that number of energy storage element, you see how many energy storage element are L and c they are energy storage element, why because if you see the inductor.

We can write down for this inductor V L equal to L d i L by dt, this is the equation number 2 so, C L L d i L by dt that will show the energy storage i L. So, now, we have got first variable i L, the second turned flow into this capacitor. So, i C which is same as i L, we can take it as C d V C by dt, this is the equation number 3. So, we found at that i L and V C these are the steps.

So, i L and V C we consider as a state variables state variables. And finally, our purpose we should write the equation in total X dot equal to A plus by B U that means, when I am writing down this equations, that is it should be in terms of V C and and i L. So, what we can do now so, from equation 3, from equation 3 we can write down as d V C by dt equals to 1 by C into i L.

Because your steady (Refer Time: 20:03) that is whatever X will come that is your model will be if you take any variable X 1 dot X 2 dot, you get here X 1 X 2. So, here i L we already got. Now, we write this equation 4. Now, coming to the equation number 2 in equation number 2.

We write dl by dt equals to 1 by L into V L so, we will find at here we get V L, but our state is V C ; that means, this particular part V L, we have to write down in terms of V C so, how we write down in terms of V C that means, we can write down as.

So, here we will find as this is this part equals to V, so, V L equals to V i minus R i into i L minus V C therefore, what you write, this 1 by L this is V L, we can write down as input V i minus R into i L minus into V c. Why we have done like this?.

Because here we have got V l and our state is in terms of V C state is terms of V C and i L so, only V L so, we want V C that is why we have original differential equation therefore, we make adjustment like this, this is V i this is V R V R and V C.

So, V L written in terms of V L and V C so, V R R into R into V C into A into B C so, now, we write equation number 5. Now, what we want we want state space model. So, what you want what we can do, we can write this as V C dot, this d V C by dt and here i L dot.

And now this is V C and i L that is X dot X, now V C dot equals to here V C dot equals to 1 by C into i L; that means, the part concern with V C 0 the 0 into V C, what is i L equals to 1 by C here 1 by C written and what is the i L dot i L dot this is i L dot, this is V C dot.

So, i L dot that is equal to concern with this here this year V C into 1 by L; that means, minus 1 by L into V C, then here about i L so, minus R by L. So, we write minus R by L plus you see here V C dot equal to minus 1 by C into i L so, we write 0.

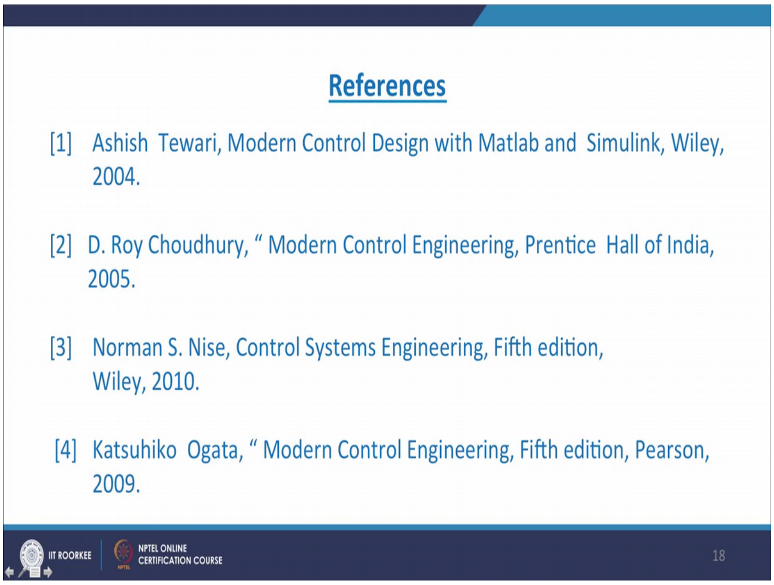
And here i L dot here in terms of input voltage so, input is 1 by L into V and, what is output your output is Y that is equals to V C so, V C means here we write 1 0 V C into i L and what about the input, input is the D D is 0 so, we can write D 0 into V i input.

Now, this is your state space model of an electrical circuit, which is simple nature which involve resistor inductor and capacitor. And taking two energy storage elements and we have got this is the state space model and, and we see this is X dot is equals to A this A matrix this correspond to B matrix, then this C matrix and D is 0 matrix, A B C matrix.

So, this is the one example I have taken. Similarly different example we can try and, we can develop the state space model of the plant. In earlier we are also determine the transfer function model of such type of circuits, but transfer function if you determine you get second order, but at the states space you get only the first order system.

So, whenever any system is given to you, you have to see energy storage element, then format equations and developed the state space model of the plant. But many times what happened, if you are getting some differential equations in that differential equations, there is need to develop the model that means, there are no physical variables are involved. So; that means so, variables is to be assume which is not actual variables so, for that variables we can also gets with performance of the system.

(Refer Slide Time: 25:30)



Therefore the study of all this type of different variables will going to study, in the next class. Now, you have some references.

Thank you.