Advanced Linear Continuous Control Systems

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Lecture – 17
Stability Analysis in State Space (Part-II)

Now we start with Stability Analysis in State Space part 2. In part 1 we have seen, how to determine the Eigen values and eigenvector and one point is clear is that we have seen that Eigen values plays important role in control system; that is stability control design for everything. This Eigen values and eigenvectors are important.

Now today's discussion is mainly concerned with how to check the stability of the system, when a given system is expressed in a state space form, that is the main objective.

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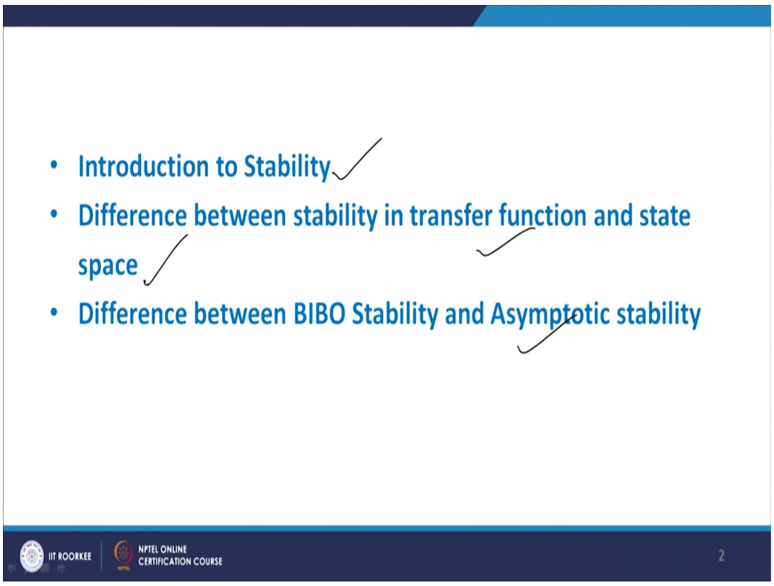

So, let us see here, in this part 2, we will study introduction to stability difference between stability in transfer function and state space and lastly we will see difference between the bounded input, bounded output stability and asymptotic stability. How about the stability?

Normally we say that stability means that when the input is bounded, the output is bounded that is called A stability; is sometimes we say that when the roots are lying on the left side of their space, the system is stable. If the roots are lying on the right side of they explain, system is said to be unstable. Now the stability can be defined in different way off.

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Let us say we have some system, suppose this is your system. Now, we have applied input to the system, so now we get the output. So, normally we find that when the system is stable, whatever the input is applied, let us say a steady state or input of one, output we come along one so; that means, when you apply input in a system also some internal things, internal states internal parameter, they also pour some response. So, but the at steady state, all the response will vanish and remaining is the force response that is input force, internal things and we get the output. So, in the output what you want? What is internal force we applied, that we obtain the output?

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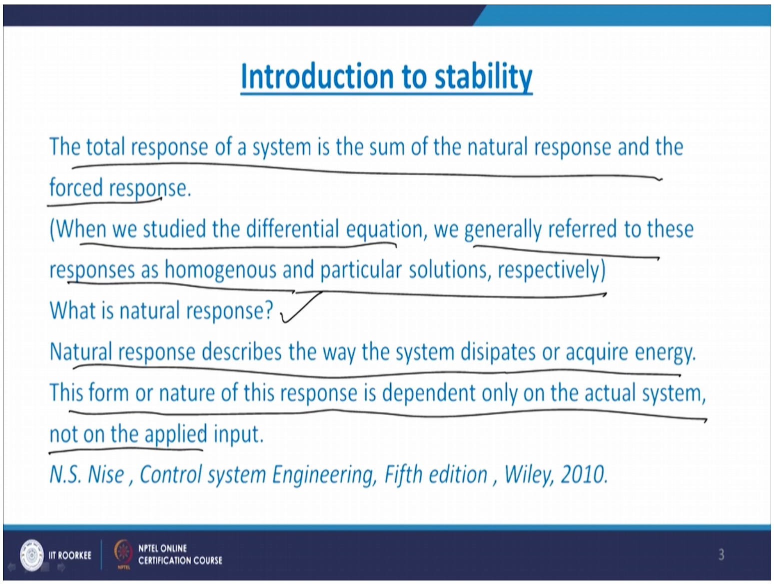

So, I will try to explain the stability in this way. The total response of a system is a sum of natural response and a forced response, that means, and I told you that for the system; what are the input is applied, that is a force and what are the thing is provided by the system, that is called natural response.

When we studied the differential equation, we generally referred to this response as homogenous and particular solution respectively that is called homogenous and particular solutions.

What is natural response? Natural response describes the way the system dissipates or acquires energy. This form or nature of this response is dependent only on the actual system, not on the applied input; that means, that natural response is depend upon the system itself, does not depend upon the input.

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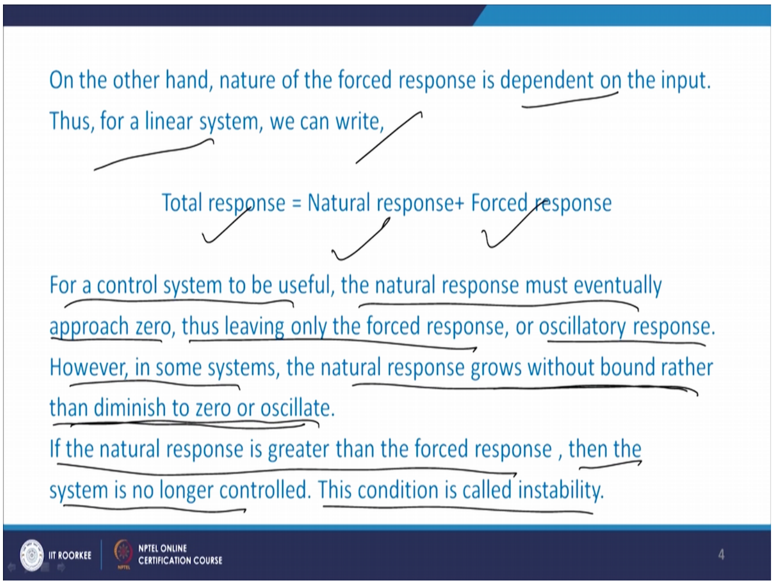

On the other hand, the nature of the force response is dependent on the input. Thus for the linear system, we can write the total response equal to natural response and forced response. For a control system to be useful the natural response must eventually approaches to 0; that means, another word issue vanish, thus leaving only, the force response or some sort of oscillatory response.

However, in some system natural response grow without bound rather than, diminish to 0 or oscillate; that is sometimes what happened? When we apply input, the natural response that will grow and growth simultaneous to state into infinity and it is not coming back to the steady state that is the direction of the 4 state. So, in that case the system becomes unstable. So, what is the reason? However, in some system the natural response goes without bound rather than diminish to 0 or oscillate.

So, if the natural response is greater than the forced response, then system is no longer control, this condition is called instability; that means, your force response must be greater than the natural response for the stability of the system. If the force response is less it is; if force response if it is if the nature response is greater than the force system is unstable, we always want force response should be more than the natural response.

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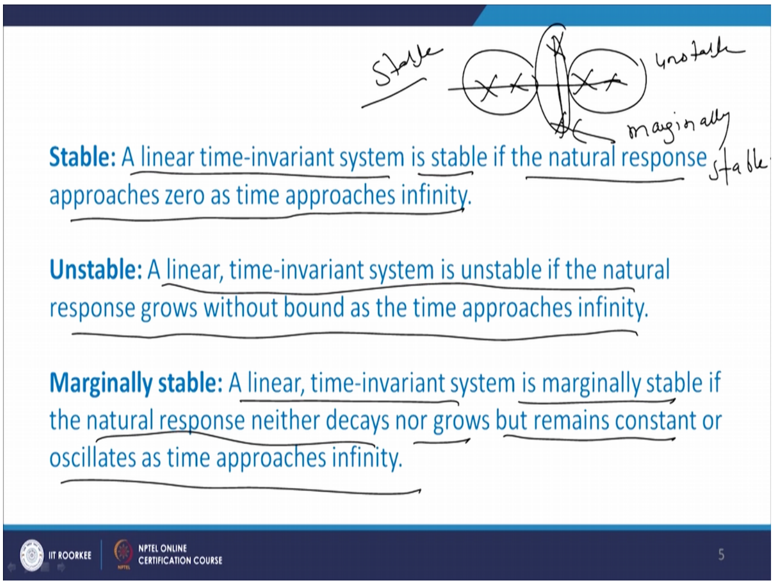

Now, we can different stability in this way also. A linear time invariant system is stable; if the natural response approaches is 0 as time approaches to infinity unstable. A linear time invariant system is unstable, if the natural response grows without bound as the time approaches infinity. And third marginally stable, linear time invariant system is marginally stable, if the natural response neither decays nor grows, but remains constant or oscillate as time approaches infinity.

Now, these are different definition of the stability. In a classical studies we have seen the stability with respect to poles; that means, you have seen that if the pole lying on the left hand side, this indicates stable. If they are lying on this side this indicate unstable. But it lies here, that is called the marginally stable, marginally stable.

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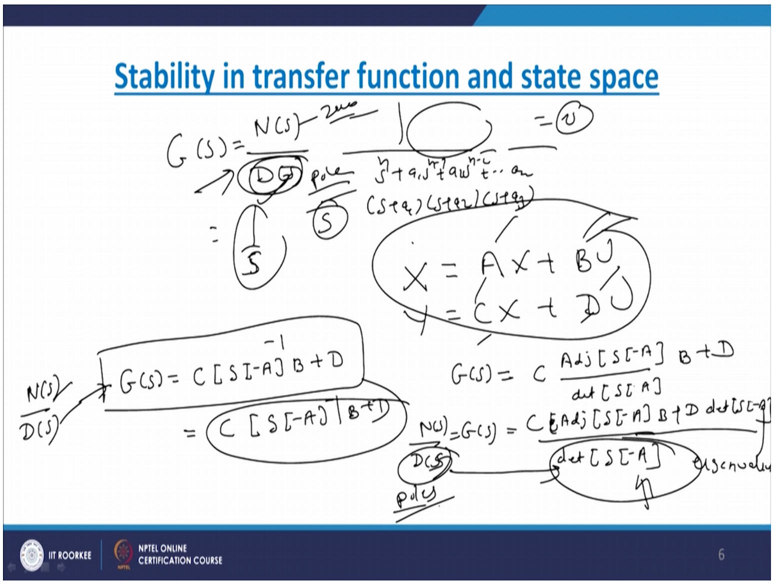

Now stability in transfer function and state space, now how to check the stability were in transfer function and a state space. I think you have already seen the how to check the stability in transfer function in a classical part.

Now, again I am reviewing this part. Suppose if I have written G S equal to N S by D S. So, N of S it is N S is numerator and D S is denominator and it is written in the most of a textbook that in order to check the stability of this, we have to check D S. The question has come in the mind, Why D, Why not N S? D S concern with the poles and N S is concern with 0.

So, why we are taking care of pole non 0's, are 0's are present in the system? Because definitely the 0 is in the system but we are concentrating only pole, why it is so? This is mainly because you see it, the pole is there, so if you take any equations S raise to N a 1 S raise to n minus 1 a 1 S raise to n minus 2 up to a n or if you solve it, we get characteristic equation that is in terms of S. Let us say, S plus a 1 S plus a 2 S plus a 3.

So, what is what will happen? A space specific value of S equals to say minus a 1. So, this part equals to 0 and system may be unstable whereas, about the numerator this 0; that means, the denominator plays a vital role in case of the stability of the system. Now we can understand this concept this way also. The denominator part that is say let us say 1 by S is nothing, but the 1 by S is nothing, but your indictor and we say S it is nothing, but differentiator.

So, instigator generally we call as a low pass filter. Low pass filter means that low frequency that is at the steady state and S is a differentiator that is remain in a transient part and we are concern with the steady state part, so what will happen and the steady state. The effect of derivatives very less, but effect of integrator will be remain because this integrator is a low pass filter. Therefore, in that view we can say that in order to check the stability of the system, if it is in a transfer function form, we have to calculate the roots of the denominator all the poles of these D S will gives you the stability of the system.

Now, coming to the state space; how to check the stability of the system is state space? Going to see here; we have X dot equal to A X plus B U; this is the plant and output equals to Y equals to C X plus D U. Now in order to check the stability, which element you take? We can take A B C D, on which element the stability depends. So, in order to get this point what will you do, we can write the equation as G of S equal to the transfer function equal to C S I minus A inverse B plus D. This equation we have already seen then we are convert the given system which is in a state space form in into a transfer function domain G S.

So, G S and this one so this is G S is nothing, but the transfer function and this is nothing, but your modern control or state space. So this are relationship between G S and this one. So, what is G S? This G S equal to numerator by denominator and stability of the G S it depend upon this now what we do, now we solve this equation this C S I minus A inverse B plus D we have to solve this, so how you solve this? So, this S I minus inverse we can write down in terms of adjoint and in terms of determinant.

Therefore, this equation we can write down as now the G S C adjoint of S I minus A divided by the determinant of S I minus A B plus D and again we solve this, what will get this equation as? We can get it determinant of S I minus A equal to C adjoint of S I minus A B plus D multiplied by determinant of S I minus A. Now the G S equals to C S I inverse B plus D and now S I inverse in terms of adjoint plus determinant and after you solve it, what you have got? We have got G S and G S equals to N S and D S.

So, D S in transfer function is corresponding to this determinant, S I minus A. Therefore, for D S, there poles where as in case of state space, it become S I minus A. Therefore, whenever we check the stability of the system, we are only taking care of A matrix with never bother about this B matrix, C matrix and D matrix; particularly stability is concerned as we are seen N S D S and this part, so D S is equivalent to S I minus A.

So, normally we call the determinant of S I minus A if you solve it, we called as Eigen values and D S means solve the characteristic equations, we will get the poles of D S. So, poles in terms of transfer function and in whenever we say the state space we call as Eigen value.

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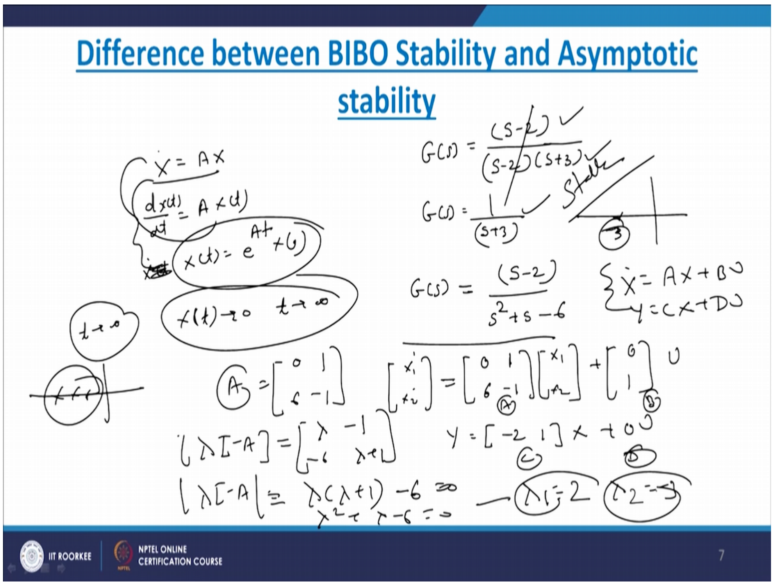

Now difference between the bounded input, bounded output stability and asymptotic stability, normally this BIBO word, we are used in case of transfer function approach. When input is bounded, your output is definitely bounded. Now about asymptotic stability; it means if you having X dot equal to A X X dot equals to A X. So, if you solve it what will get d x by d t into A X of t, therefore, this X dot t or we can say X of t equals to e raise to A t X of 0 this is the state equation.

So, the more details of this issues we will see later on, but now for time being because we are to learn what is the difference between BIBO and asymptotic stability I have to written this equation. So, X of t if you solve it, we get X of t equals to e raise to A t X of 0.

Now assume that your time t; t tending at t tending to infinity and all the Eigen values of A lying on left side that is A is all the elements of your A minus. So, what happen? X of t tending to 0 at t tending to infinity that is the stability that is called we can say asymptotic stability that is for A system X dot equals to A X at t tending to infinity, X of t tending to 0 system is asymptotically stable. I think from this discussion, we got one point at BIBO and asymptotic stability are the same things. But can this definition is true for all type of systems, can you say that for all system bounded input bounded stable and is also called as asymptotic to stable system?

Let us see, so now I am taking one example say the G of S equal to S minus 2 divided by S minus 2 S plus 3, so this is our system. So, this is 0 and this are poles. So, there are 2 poles and one 0.

Now definitely when you want to check the stability of the system, we say that S minus 2 in the numerator, S minus 2 denominator. We just cancel it and after cancelations, what we will get? G of S equal to 1 1 of by S plus 3 so definitely S minus S plus 3, so Eigen value on the left hand side that is minus 3, so this system is stable because mathematically this system is true; S minus 2 S minus 2 has been cancelled.

Now we try to think from the state space point of view further purpose. Now we solve this so if we solve this G of S that is equal to S minus 2 A square plus S minus 6 so G S we have solved. Now we want to write this system into state space form, we have seen the different types of form that is a controllable canonical form, then diagonal of form, Jordan form, the various form of various type of forms we have seen even observable form you have seen.

So, now we write this G S in any of the form. Now write this G S in terms of state space model, state space model means we want X dot equal to A X plus B U, Y equals to C X plus D U. The most simple form is the companion form; that is controllable canonical form where we have taken output as a first variable remaining variables are the derivative of the output, so this part we have already seen last time. So, now, what we do? Whatever thing we have learn previously, we have up we want to apply to this problem.

Now, the state space model of this plant, we can write down as X 1 dot X 2 dot because the order of the denominator is 2. So, the order of this number of states we are taken 2 X 1 dot X 2 dot equal to 0 1 6 minus 1. Here X 1 X 2 plus 0 1 U and what is the output? It is minus 2 1 into X 0 into U.

Now we have got A matrix, we have got B matrix and we have got C matrix and the D matrix A B C D and previously we have seen that whenever we want to check the stability of the system, when the system is expressed in a state space form, we are taking care of A matrix. We are not taking the A B C D. Therefore, here also we will find that here A B C D.

So, definitely for checking the stability in state space, I will take care of the A matrix. Therefore, this A A equal to 0 1 6 minus 1. Now we calculate the Eigen values of this so minus lambda minus lambda I minus A equal to this lambda minus 1 minus 6 lambda plus 1.

Now determine the determinant of this, determine this lambda plus 1 minus 6 equal to 0. So, lambda square plus lambda minus 6 equals to 0 and if you solve it, so we will get Eigen values are lambda 1 equals to 2 equals to minus 3. So, lambda 1 2 lambda 2 minus 3; so, see here in transfer function, we have cancelled pole 0's. We have we have got 1 by S plus 3 and here we have got Eigen values lambda 1 equals to 2 lambda 1 equals to minus 3 and we have used all basic concept here.

We are not adding some more additional thing, based on the basic concept, we have got this results. Here the system is stable that is in order to the system. This system is stable your transfer function, but the same system we found that it is unstable, because here lambda 1 equals 2 lambda equals to minus 3. So, why this is happened? Because whatever logic we applying whatever may be form may be classical and advanced logic should be same and you should follow the same thing.

So, this is happened because we have done pole 0 cancellations. So, you just see here that whenever there is pole 0 cancellation is involved, the definition between BIBO and asymptotic stability will never match. They will only match when there is no cancellation, when the cancellation, it will never match.

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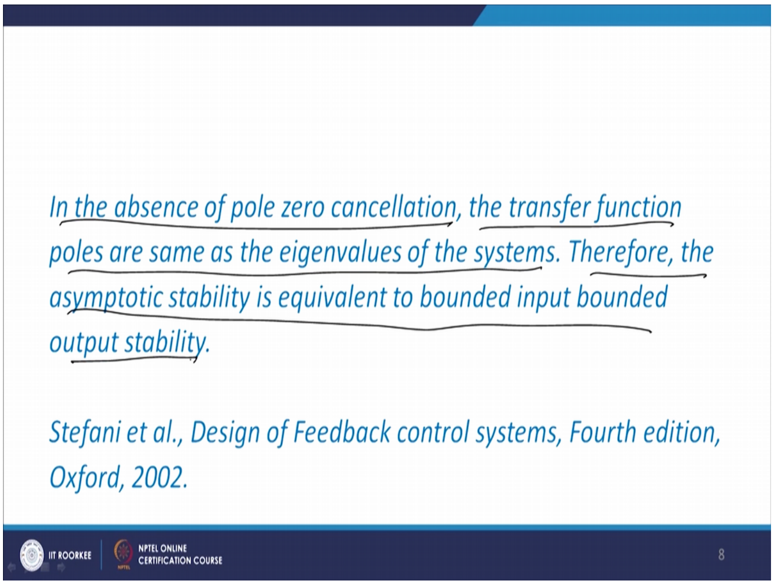

So, here it is written in the absence of pole 0 cancellation. The transfer function poles are same as the Eigen values of the system. Therefore, the asymptotic stability is equivalent to bounded input, bounded output stability. Now these are some references Nise, Stefani.

Thank you.