Advanced Linear Continuous Control Systems

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Lecture - 15
Determination of Transfer Function from State Space Model (Part-II)

Now we will start with the second part of getting the transfer function from state space model.

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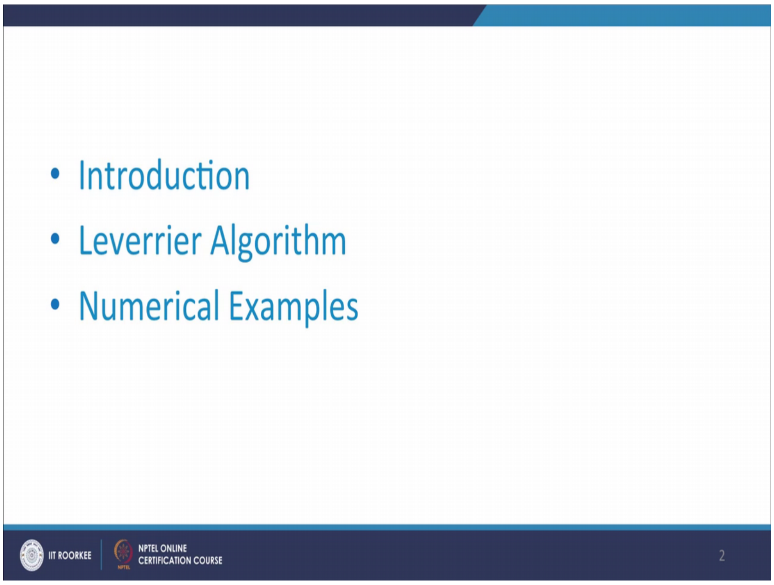

Introduction, Leverrier algorithm, numerical examples.

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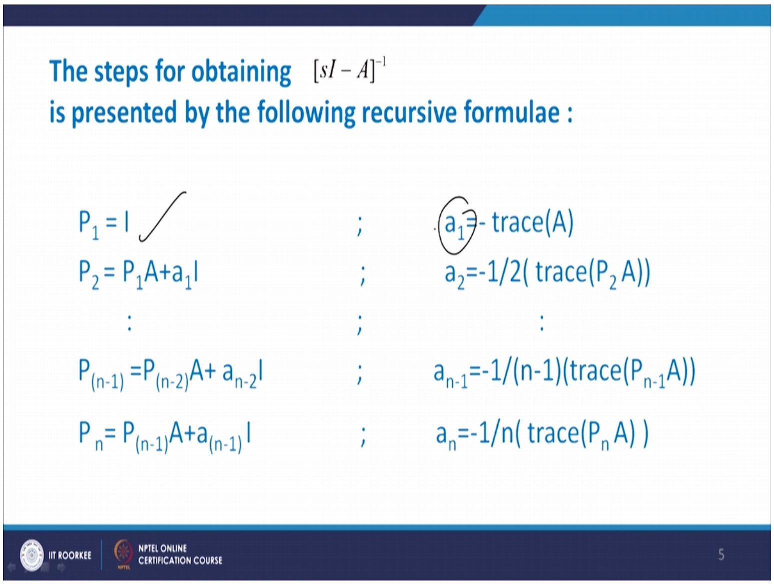

So, in previous part you have seen that, in order to get the transfer function from a state space model we have to calculate the inverse, and to calculate the inverse for higher order system is quite cumbersome. Now here we want a method which can give we can easily give the inverse of a given system matrix.

Now Leverrier has proposed that instead of determining the inverse or determinant or cofactors we can calculate the inverse of a given system matrix by using the trace of a matrix. Trace means a sum of diagonal elements, so that is the unique advantage of the method which is proposed by the Leverrier. So here there are some advantages see it as the order of the system increases the mathematical computation burden will also increases.

Second earlier the digital computer fails to determine s I minus A inverse because the Laplace operator s is a scalar. However, nowadays due to a advanced software it is possible to find a inverse of s I minus A. Third point we need an approach where there is there is need to determine inverse without determined a adjoint and determinant, so Leverrier algorithm is based on trace of the matrix that is some of diagonal elements. That is Leverrier has proposed a method we say that in order to calculate the inverse there is no need to calculate the inverse, that is no need to calculate the determinant. No need to calculate the cofactor only simple the concept of diagonal elements you can get the inverse, so that is the beauty of this Leverrier algorithm.

It that means, if I am taking this as a A matrix, A matrix say equal to a 11 a 12 a 21 a 22 now the trace of this matrix trace equal to a 11 a 22 that if that is simples addition of the elements of a given matrix. We are getting a trace and here in this method using this concept itself you can get the inverse of a given system matrix and once we get the inverse we can easily create the transfer function from a given state space model.

Now, we will see the procedure, that how to get the transfer function by means of Leverrier algorithm.

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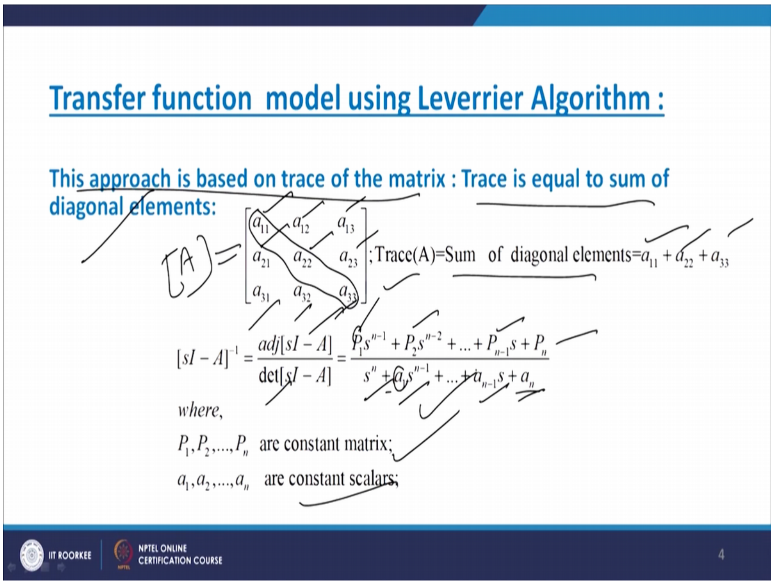

Now, this approach is based on trace of the matrix, trace is equal to sum of diagonal elements so here a 11 a 12 a 1 3 first row, a 21 a 22 a 2 3 second row, a 31 a 32 a 33 this is a third row. So this is a given matrix this is matrix A.

Now, trace of this sum of diagonal elements, that is diagonal elements are a 11 a 22 a 33 this is a diagonal elements. So here a 11 a 22 a 33 now s I minus A inverse that is written as a adjoint of s I minus A divided by determinant of s I minus A that is equal to P 1 s raised to n minus 1 P 2 s raised to n minus 2 P n minus 1 s P n. And here s raised to n a 1 s raised to n minus 1 a of n minus 1 s an and now here P 1 P 2 P of n minus 1 up to P n are constant matrices.

So, here the elements from P 1 to P n are the constant matrix and a 1 to a n are the constant scalars.

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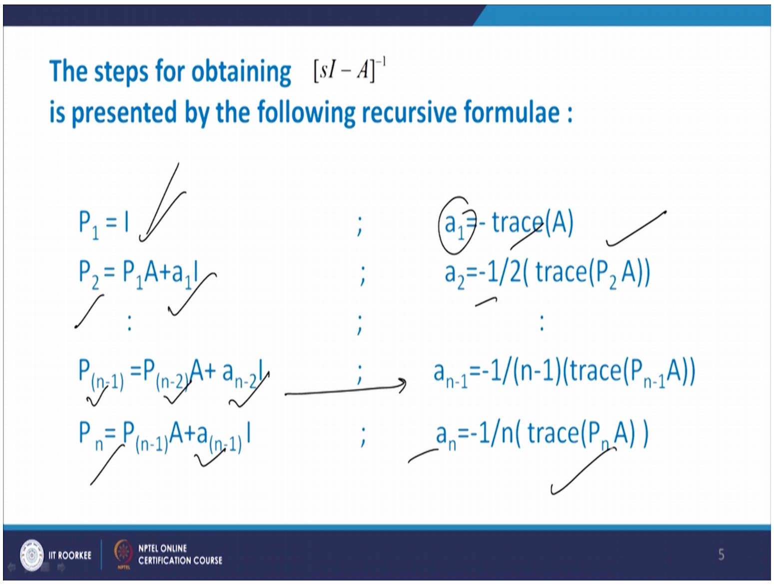

Now, the steps for a obtaining s I minus a inverse the first step we write P 1 equals to I, that is an identity matrix that is the first step. That is how do you get it so we write this P 1 this first element equal to a identity matrix. Now a 1 a 1 equals to minus trace of A that is first element that is in this case this P 1 and a 1 we have got a 1 equal to minus trace of A and P 1 is nothing, but the identity matrix.

Now this P 2 P 2 equal to P 1 into A a 1 into I and a 2 equal to minus 1 by 2 trace of P 2 into A. So we got P 2 a 2 and you will find that in this particular method there is no as such division no need for any minors simply simple multiplication and additions we have got we have got P 2 and a 2 P 1 and a a 1 I am we have just P 1 and I have assumed I and a 1 we have got minus trace of A.

Similarly, if you move further P n minus 1 equal to P n minus 2 into A a n minus 2 into I and here P n that is P n minus 1 into A a n minus 1 into I and similar to this we have got a n minus 1 equal to minus 1 n minus 1 trace of P n minus 1 into A and an equal to minus 1 by n trace P n into A; that means, all the elements of P 1 P 2 P n and a 1 a 2 a n minus 1 an we will going to determined and simply by a multiplications and simple additions. So, we will see the example, so we have taken the same example which we have taken earlier.

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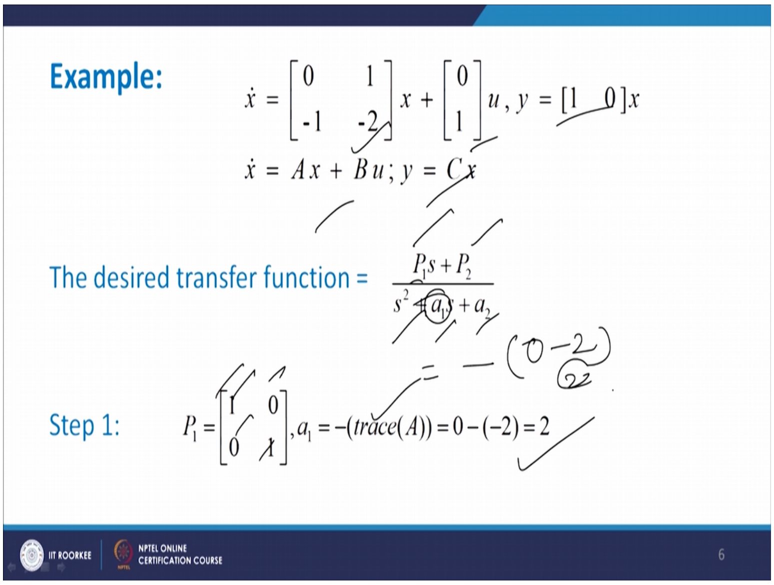

So, x dot equal to A x plus B u and what is the desired transfer function? My desired transfer function is P 1 s P 2 s square a 1 s plus a 2 this is required to be determined. Now to how to get a P 1 how to get P 2 how to get a 1 and how to get a 2 now P 1 you just see the first step P 1 equals to I that is a identity matrix.

So, you will find that this P 1 we have taken as 1 0 0 1, now to get the a 1 how do you get a 1? This a 1 equal to minus trace of A, that is trace of A. What is A? A means diagonal elements so; that means, here minus 0 minus 2 that is minus of minus 2 is plus 2 so you get ; that means, we have got the trace of a matrix.

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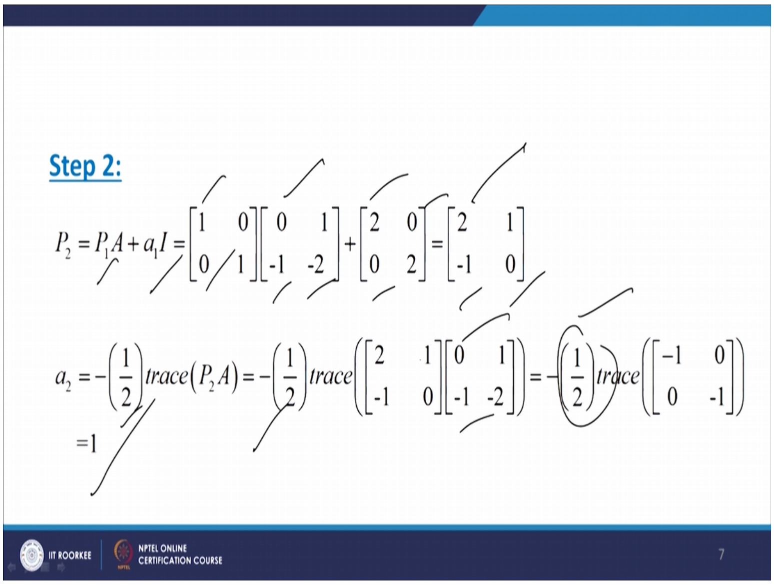

Then a step 2 this P 2 equals to P 1 a into a 1 I. Now what is P 1? P 1 is 1 0 0 1 a 1 is 0 1 1 minus 2 what is a 1 a 1 into I what is a a 1 we have got a 1, we have got 2.

So, we write as 2 0 0 2 and after addition we get 2 1 minus 1 0. Now to get a 2 a 2 equal to minus 1 by 2 just see here step that is minus 1 by n. So here n is 2 1 by 2 trace of P 2 into A, so minus 1 by 2 trace of 2 1 minus 1 0 A is 0 1 minus 2 and after multiplication of this one and a division of minus 1 by 2 we have got a 2 equal to 1.

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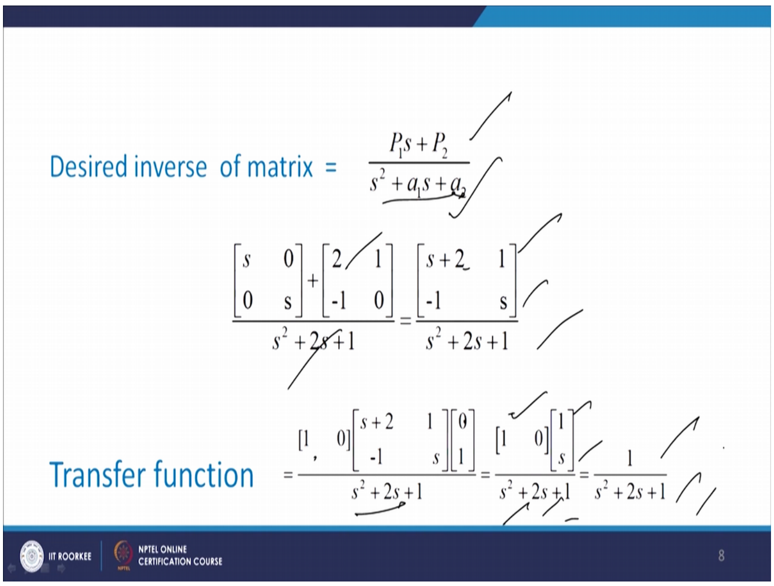

Therefore the desired inverse of matrix equal to P 1 s plus P 2 s square plus a 1 s plus a 2. So, we write the elements of all that is P 1 is the identity matrix s 0 0 s then P 2 we have written. So all these values we are written; and finally, you see here the transfer function is see s I inverse; that means, s plus 2 1 minus 1 s divide by s square plus 2 s plus 1 0 1 and after solving this one so we will find out this is after solving this we will get 1 0 1 s and about determinant we have got s square plus 2 s plus 1. And finally, we get 1 s square plus 2 s plus 1.

So, we will find that you know solving this example simply we have done some multiplications as well as additions and we have got the inverse; that means, no need for any further calculations. Now we try to solve another example which we have solved last time and we see whether we are getting this same result or not, so we have taken the model x dot equal to A x plus B u y equal to C x plus D u.

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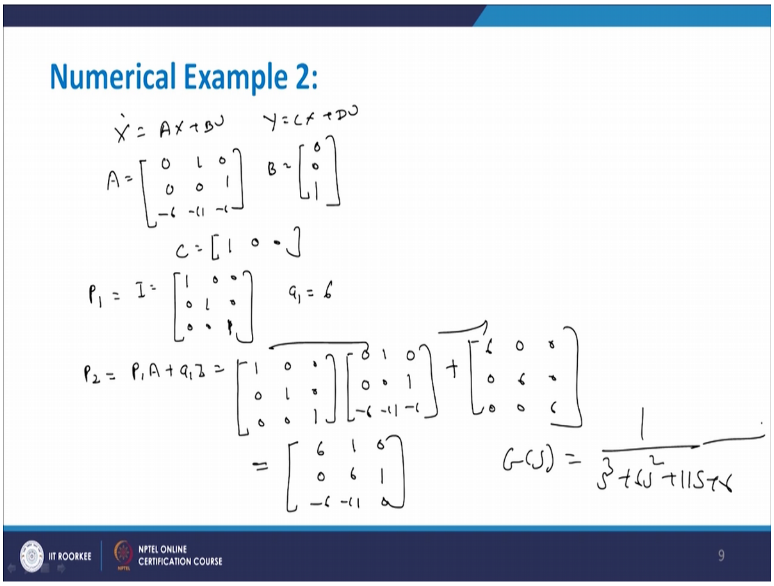

So, here if you write A as 0 1 0 0 0 1 minus 6 minus 11 minus 6 B equal to 0 0 1 and we have C that is equal to 1 0 0 A B C we have written. Now what is our first step first step to get P 1 and then a 2 and like this the first part is to get the P 1, so we write P 1 equal to I that is a identity matrix that is equal to 1 0 0, 0 1 0, 0 0 1.

Now, we have got P 1 now to get a 1 what is a 1 a 1 equal to the trace of the given matrix what is the trace trace is here 0 0 minus 6 and minus 6 minus of this you get 6. So a 1 equal to 6 now to get P 2 how to get P 2, P 2 equal to P 1 into a plus a 1 into I so we write

P 1 equal to 1 0 0, 0 1 0, 0 0 1 that is P 1 A is S is 0 1 0 0 0 1 minus 6 minus 11 minus 6 and what is a 1 a 1 we have got 6.

So, we can write at write down as 6 0 0, 0 6 0, and 0 0 6. Now we have to multiply this and then add it, so after multiplication and addition we will get result as 6 1 0 0 6 1 minus 6 minus 11 0. Now we have got P 1 P 2 now we want a 2.

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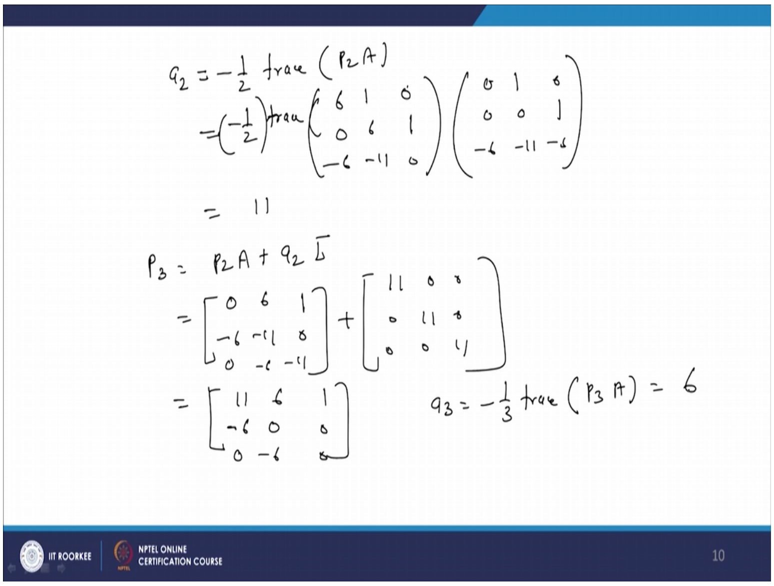

So a 2 equal to A 2 equals to minus 1 by 2 trace of P 2 into A. So, if you multiply it we will get minus 1 by 2 trace of 6 1 0, 0 6 1, minus 6 minus 11 0 multiplied by a is 0 1 0 0 0 1 minus 6 minus 11 minus 6 and now you have to multiply this then the taking the trace of this one and divide by a minus 1 by 2 and after doing this one you can get result as 11 a 2 will get at 11.

So, we have got P 1, we have got P 2, we have got a 1, we have got a 2 now they need to calculate the P 3 and a 3. Now to get P 3 this P 3 equal to P 3 equal to P 2 into A plus a 2 into I. Now what is the value of P 2 this P 2 equal to 0 6 1 minus 6 minus 11 0 0 minus 6 minus 11 P 2 into A then this a 2 into I a 2 is 11 I is a identity matrix.

So, we can write down as 11 0 0, 0 11 0, 0 0 11 and after addition of these two matrices we will get it as 11 6 1 minus 6 0 0 0 minus 6 0 ok. We can write down as a 3 equal to minus 1 by 3 trace of P 3 into A. This is that gives you 6, so in here we are determined P 1, P 2, P 3, a 1, a 2, a 3 and now this had to replace in this equation and after replacing it we have we will get the same transfer function model which we have got earlier that is we have we will get a transfer function G s as 1 upon s cube plus 6 s square plus 11 s plus 6. So in this way we have calculated the transfer function from the given state space model.

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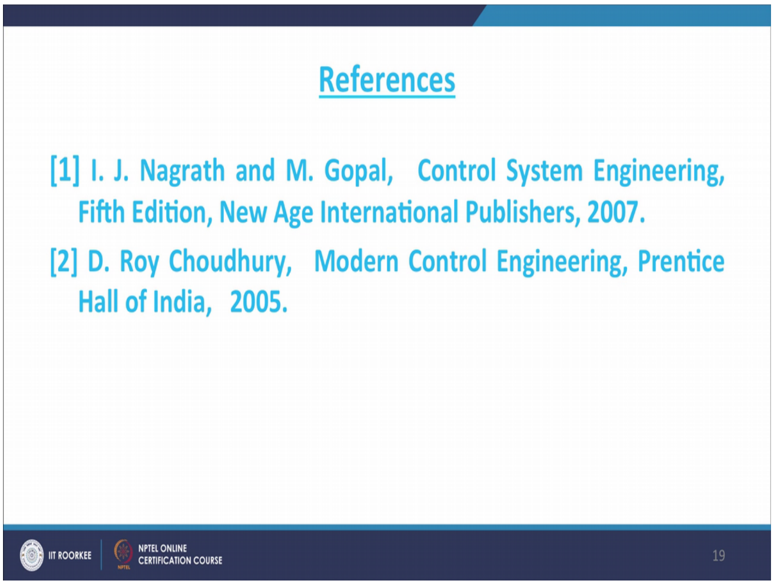

Now, these are some references.

Thank you.