

**Advanced Linear Continuous Control Systems**  
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**Lecture - 14**  
**Determination of Transfer Function from State Space Model (Part-I)**

Today we will study, how to Determine Transfer Function from State Space; that is our main purpose is Determination of Transfer Function model from State Space Model.

So, first of all in this case, we will see introduction, conventional approach and some numerical examples. Till date we have seen that when transfer function is there or differential equation is there, we have converted into a state space model and we have seen there are lots of advantages of state space over transfer function. But today, we are discussing the conversion of state space to transfer function when the state space has so much advantage. We have why we are studying this conversion that is form a state space to transfer function.

This is because in most of the industry. Even nowadays, instead of advanced controller a conviction controller has been used just like a PID controller and when we were designing a PID controller; that means, we require a model and if the model it is a transfer function form, we have lot of techniques so that we can easily design a PID controller.

Therefore, sometimes we need a transfer function model that is when we have a state space model and we have to design a controller PID controller. It is better first of all we convert the given state space model into transfer functions; that is first point.

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Introduction

- (i) Suppose for a plant, if we have developed a model, which is based on state space. But, the main purpose is to design a controller (PID type controller). The most of the PID controller design techniques are based on transfer function approach. In that case, there is need to convert state space model into transfer function form.
- (ii) In control theory like model order reduction, the most of the conventional techniques are based on transfer function form. In that case, also there is need to convert state space into transfer function.

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Suppose for a plant, if we have developed a model, which is based on state space; model which is based on state space. But, the main purpose is to design a controller PID type of control that is here. Main purpose is to design a classical PID type of controller.

The most of the PID controllers design techniques are based on transfer function approach. Mean the researchers have developed a various tuning techniques that is tuning of PID control parameters. They are based on transfer function approach. Therefore, in that case there is need to convert state space model into transfer function form.

Now next point: In control theory there is 1 area which is called model order reduction and there also we are reducing higher order model to lower order. But, if you are suppose if the system is given in a state space form. Then, in that case to get the model is quite difficult or the lot of technique which are available that is based on transfer function approach.

Therefore, in that case what we will do? We have a state space model; state space model we have to convert into transfer function, then we reduce that model. And finally, that reduction reduce model, we convert into a state space form. So therefore, these are some advantages of having a transfer function even though we have state space model.

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**Conventional approach**

- The procedure for determining transfer function from state space model is presented below in the following steps:
- Step 1: Consider the state space model as
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \quad (1)$$
- Step 2: Applying Laplace transform to above equation (Assuming initial conditions are zero), we get,
$$sX(s) = AX(s) + BU(s) \quad (2)$$
$$Y(s) = CX(s) + DU(s)$$

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Now, how to get the transfer function model from when the model which is given to you is in state space form. So, let us see here, we have model  $\dot{x}$  equal to  $Ax$  plus  $Bu$  and  $y$  equal to  $Cx$  plus  $Du$ . So, consider the state space model like this. Now what we want? We want the model in transfer function form. Transfer function means that  $s$  domain. Therefore, what we are doing? We are applying Laplace transform to this equation 1.

So, when we apply Laplace transform to this these to this equation 1, we are neglecting initial conditions. So, here initial conditions are neglected. Therefore, we got model as  $X$  of  $s$  equal to  $A$  into  $x$  of  $s$  and this is  $B$  into  $U$  of  $s$ . Similarly,  $Y$ .  $Y$  of  $s$   $C$   $x$  of  $s$  plus  $DU$  of  $s$ ; that is for 1 to 2, we have obtained a model. A model means we have obtained a transfer function for model or transfer function equations we obtain. Now, we have to solve these questions. How do you solve these equations?

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Step 3: From eq.(2), we write,  $Y(s) = C X(s) + D U(s)$

$$\begin{aligned} [sI - A] X(s) &= B U(s) \\ X(s) &= [sI - A]^{-1} B U(s) \end{aligned} \quad (3)$$

We know,  $Y(s) = C X(s) + D U(s)$

Replacing  $X(s)$  in above equation, we get,

$$Y(s) = C [sI - A]^{-1} B U(s) + D U(s) \quad (4)$$

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So, here what we do? We have taken  $sI$  minus  $A$  1 side and  $B$  into  $U$  of  $s$ ; then here what we have done? We have den we have done  $X$  of  $s$  here and  $sI$  minus  $A$ . We have move on the right hand side. So, it become  $sI$  minus  $A$  inverse into  $B$  into  $U$  of  $s$ . So,  $x$  of  $s$  is equal to  $sI$  minus  $A$  inverse  $B$  of  $B$  into  $U$  of  $s$  and what is  $Y$ ?  $Y$  of  $s$  is  $C$  into  $X$  of  $s$  into  $D$  into  $U$  of  $s$ . Just now here shown.

So, what we do we replace the value of  $x$  of  $s$  in this equation. So, we replace  $x$  of  $s$  in this equation, what we will get? We will get a  $Y$  of  $s$  equal to  $C$   $sI$  inverse  $A$   $B$   $U$  of  $s$  plus  $D$   $U$  of  $s$ . Now  $U$  of  $s$  and  $U$  of  $s$ , we take in common.

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Step 4: Solving eq.(4), we get,

$$\frac{Y(s)}{U(s)} = G(s) = C[sI - A]^{-1}(B + D) \quad (5)$$

Step 5: The above equation can be written as

$$\frac{Y(s)}{U(s)} = G(s) = C \frac{Adj[sI - A]}{\det[sI - A]} B + D \quad (6)$$

The slide also contains handwritten notes in a bubble:  $\dot{X} = AX + BU$  and  $Y = CX + DU$ . Arrows point from these equations to the terms in equation (5).

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And then, after solving what we have got? We have got Y of s divided by U of s equal to G s. Now, this is our transfer function model equal to C sI A inverse B plus D. So, this is G s that is the G s we are obtained for  $\dot{x}$  equal to AX plus BU. Y equal to CX plus DU. This is state space model and this is equivalent transfer function and you will find that in this particular transfer function, we have A B C D both are involved.

Now, how to find out the transfer function? So, you will find that here, we need to calculate the inverse and when the need to calculate inverse; that means, that we need to calculate its adjoining as well as determinant. Therefore, this equation sI minus A inverse we have written in this form. What is the form adjoining of sI minus A divided by determinant of sI minus A C B and D.

So, here the main purpose is to calculate adjoin of sI minus A as well as determinant of sI minus A.

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**Step 6: Determination of  $\text{adj}[sI-A]$ .**

(i) The  $\text{adj}[sI-A]$  is defined as the transpose of cofactor of  $[sI-A]$ .

$$\text{Adj}[sI-A] = (\text{Cofactor}[sI-A])^T$$

(ii) Cofactor of  $[sI-A]$  is related to principle minors of a matrix. Suppose the minor of element  $(a_{ij})$  is denoted by  $|M_{ij}|$ . The  $|M_{ij}|$  denotes the  $i$ th and  $j$ th column of a determinant  $[A]$ . The cofactor of the element  $(a_{ij})$  is given as minor  $(-1)^{i+j} |M_{ij}|$ .

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Now, how to calculate, how to calculate adjoint of  $sI$  minus  $A$ ? So, first of all we define what is mean by adjoint of  $sI$  minus  $A$ . So, adjoint of  $sI$  minus  $A$  is defined as the transpose of co-factor of  $sI$  minus  $A$ . That means, you have to calculate the cofactor of this  $sI$  minus  $A$  and then, we have to make it transpose. Therefore, adjoint of  $sI$  minus  $A$  equal to co-factor of  $sI$  minus  $A$  transpose.

Now, how to calculate the cofactors? The cofactors of  $sI$  minus  $A$  is related to minus of a matrix. So, for every elements in the matrix there are some minus. So, that minus have to be calculated and once we get a minus, we will get cofactor and once we get cofactors, we will get adjoint. So, here a steps have we written. A cofactor of  $sI$  minus  $A$  is related to principle minors of a matrix. It is related to principle minors of a matrix.

Suppose, the minor of element  $a_{ij}$  is denoted by  $M_{ij}$ ;  $ij$ ,  $i$  vary from 1 to  $n$ ,  $j$  varying from 1 to  $n$ . So, the  $M_{ij}$  denote the  $i$ -th and  $j$ -th column of determinant  $A$ . The cofactor of the element  $a_{ij}$  is given as minor  $(-1)^{i+j} M_{ij}$ .

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- (iii) Determinant of  $[sI-A]$
- Step 7: Replacing values of  $[sI-A]^{-1}$  in eq.(5), we get the transfer function model of the system.

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And then, we have to calculate the determinant of  $sI$  minus  $A$  and finally, last step replacing the values of  $sI$  minus  $A$  inverse in equation 5. If you see your there is equation 5, we will get we get the transfer function model of the system. That is the main purpose is to calculate the adjoint and also the determinant.

Now, how to calculate the adjoint and determinant of a matrix?

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### How to find inverse of third order system?

Consider the third order system as  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  (row 1) (col 1)

$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$  (row 2) (col 2)

#### Step 1: Finding the minors of matrix $[A]$ :

Minor of $(a_{11}) : k_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$	Minor of $(a_{12}) : k_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$	Minor of $(a_{13}) : k_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$
Minor of $(a_{21}) : k_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$	Minor of $(a_{22}) : k_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$	Minor of $(a_{23}) : k_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$
Minor of $(a_{31}) : k_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$	Minor of $(a_{32}) : k_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$	Minor of $(a_{33}) : k_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

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So, this is the matrix. For simplicity we have taken 3 cross 3 matrix. So, a 11 a 12 a 13 a 21 a 22 a 23 a 31 a 32 a 33. So, let us say these are some elements and this matrix say

and this matrix is square matrix. Now, how to find out the minors of matrix A? So, in order to get the principle minors of matrix A first of all I will take this a 11 that is minor of a 11. We have written as  $k_{11}$  that equals to just remove this row and these columns. So, what we will get a 22 a 23 a 32 a 33. So, a 22 a 23 a 32 and a 33 this is.

Similarly, to calculate the principle minors of a 12, remove this particular row and this particular column. So, what is remaining is a 21; we have a 21 a 23 a 31 and here a 33 that is this is basically your, a 21 and this is basically your, a 31 and here a 23 and a 33.

Now, the principle minors of this a 13, the principle minors of a 13, remove this particular row this column. So, we will get a 21 a 22, then this is a 31 and here a 32, then, about the principle minors of this particular element. The principle minors of this element, it is a we have to remove that particular row and columns. So, here this row and this column has been removed. So, what we will get we will get element of a 12 a 13, then if you will get the element of a 32 and this a 33.

Similarly, the principle minors of a 22. So, a 22 means particularly this row and this column. So, last element is you have a 11; then a 31; then we have a 13 and then a 33. Then a principle minors of this particular element a 23; a 23 means we have take remove this row and this column what is we have a 11 a 12; then a 31 this one and a 32.

Now, next we have look at the principle minor of remove element that is particularly a 31. So, a 31 means we have to remove this row and this column. So, what element we will get we will get the value element of a 12 a 13; then we will get the elements of a 22 and here a 23. Then, the principle minor of a 32; so, we remove this row and columns.

So, what we will get a 11; we will get a 21; then a 23 and lastly the principle minors of a 33 particularly this you remove this row and this column corresponding to this a 11 a 12; then a 21; then a 22.



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Cofactor of matrix =  $(-1)^{i+j} M_{ij}$   
 where, i is row and j is column.

$$\text{Cofactor of } (A) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

$$\text{Adjoint of } (A) = \text{transpose of (Co-factor of } (A)) = \begin{bmatrix} m_{11} & m_{21} & m_{31} \\ m_{12} & m_{22} & m_{32} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}$$

Now, the cofactors of matrix: Cofactors of a matrix minus 1 raised to 1 plus j into M ij. So, whatever elements is get you see the which rows and which columns; particularly when you see this particular element. This lies in your first row and first column. So, here I equals to 1 j equals to 1; so 1 plus 1 2. So, minus 1 minus 1 raised to 2 is 1. Therefore, is element corresponds this is positive. Similarly, if you take this element or you can say this a 12. So, just see a 12; where it lie? It lying in the first row; this is row 1 and what is the column? Column is 2; first column sorry first row second column; first row second column; so 1 plus 2 plus 1, 3. So, minus of minus of 3 equals to minus 1.

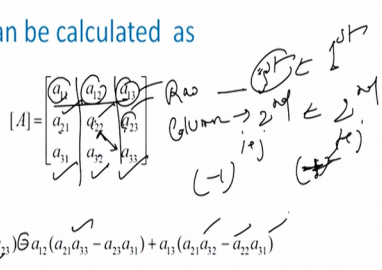
So, you will find that the element is negative. So, minus k 12. So, if you check all the elements in this particular matrices, you will find that this, this, this and this that is it is A. You can say these particular elements, they are they are they have negative signs all have positive signs. Therefore, what we want? We want the adjoint of a the adjoint of a nothing but the transpose of cofactor of A; that means, once we get cofactor of A. By this arrangement, what you do? We have to make a transpose of the elements to transpose means you have taken A m 11 here; m 21; then here m 31; see here m 31; then here m 12; then here m 22; then here m 32 and now here m 13 m 23 and m 33 that is here m 11 m 12 m 13 m 21 m 22 m 23 m 31 m 32 m 33.

And now these elements you have to make transfer; that means,  $a_{11} a_{22} a_{33}$  and lastly  $a_{13} a_{21} a_{32}$ . So, in this way we have got the adjoint of A. Now the calculation of the determinant, how to calculate determinant of the matrix?

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**Calculation of determinant of matrix [A] :**

The determinant of [A] can be calculated as



$$|A| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

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So, for determinant, what we do? Let us say  $a_{11} a_{12} a_{13} a_{21} a_{22} a_{23} a_{31} a_{32} a_{33}$ . Now, how to calculate the determinant? So, to get the determinant, what I have taken? I have taken  $a_{11}$  that equals to this cross multiplication; that is that is similarly the adjoint here the procedure is some more same that is for this  $a_{11}$ , we remove this row and columns; so here  $a_{22}$  multiplied by  $a_{33}$  minus cross multiplication of  $a_{23}$  into  $a_{32}$  just here. Now, to get the  $a_{12}$  I have to use or negative sign I have used because this lying in the first row; row is first row first row and what is column? Column is second row. That is first row, this is first and here is second row; so  $1 + 2, 3$ .

So, this is odd that is minus 1 of  $i + j - 1$  that is you can say minus 1 of  $i + j$ . So,  $i$  is 1;  $j$  is 2. So, minus 1 raised to 3, we will get minus 1. Therefore, we get minus. Minus of  $a_{12}$ ; then  $a_{21}$  multiplied by  $a_{33}$  minus minus  $a_{31}$  into  $a_{23}$  this and lastly  $a_{13}$  plus  $a_{31}$  that is equals to remove this row and columns, we will get  $a_{21}$  multiplied by  $a_{33}$   $a_{32}$  minus  $a_{22}$  multiplied by  $a_{31}$ .

In this way, we have to calculate the determinants of A. So, here we have calculated adjoint and determinant of a matrix. So, this is particularly 3 cross 3 matrix. Similarly, we can try for other order of matrices and we will find that as the order increases the

competition will increase. But nowadays because of software we are not facing such problems, but for theoretical point of view or from calculation point of view it is better to solve the manually for lower order system high order it is cumbersome.

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**Numerical Example 1 :**  
**Determination of transfer function model from state space model using conventional approach :**

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

**Step 1:**  $[sI - A] = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} = \begin{bmatrix} s & 1 \\ -1 & -2 \end{bmatrix} = s^2 + 2s + 1$

$G(s) = C [sI - A]^{-1} B + D$

$$\begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}$$

Now, I am taking here 1 example. Example 1: Determination of transfer function model from state space model using conventional approach. Now, here I am saying a conventional because afterward I will go for some other approach which is somewhat better than this. But, however, the approach is now I am presenting is a conventional approach. So, whatever we are discussed that is the conventional approach.

Now,  $\dot{x}$  equals to  $01$  minus  $1$  minus  $2$   $0$   $1$   $u$ ,  $y$   $1$   $0$ . So, here we know  $A$ ; this is  $B$  and this is  $C$ :  $A$   $B$  and  $C$ . So, now, we have to follow the steps. So, we want a transfer function; that means what we want? We want  $G$  of  $s$ . For this plant, but  $G$  of  $s$  means  $C$   $sI$   $A$  inverse  $B$  plus  $D$ .

This we required. As I have discussed earlier, we want now  $sI$   $A$  inverse first because  $C$  is given, what is  $C$ ?  $C$   $1$   $0$  and here  $b$  is also given. But we need  $sI$   $A$  inverse. So, how to get it? So, first of all I have to written  $sI$  minus  $A$  that is that is nothing but  $sI$  means here  $s$   $0$   $0$   $s$  minus  $A$ ; what is  $A$ ?  $A$  is  $0$   $1$  minus  $1$  minus  $2$

So, just you see here if  $s$  minus 0; then here 0 minus 1 minus 1; then 0 this is minus 0 minus of minus 11 and here  $s$  minus of minus 2 that is plus 2; then  $s$  plus 2. So, now, we have got  $sI$  minus  $A$ .

Now what we want? We want its inverse to get the universe first of all we determined adjoint and for getting adjoint we need cofactor: so cofactors of  $sI$  minus  $A$ . So, to get the cofactors of  $sI$  minus  $A$  you will find.

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• Cofactors of  $(sI-A) = \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}$

•  $\text{Adj}(sI-A) = \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}$

•  $\text{Det}(sI-A) = s^2 + 2s + 1$

•  $G(s) = C [sI-A]^{-1} B = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s^2 + 2s + 1} = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix}}{s^2 + 2s + 1} = \frac{1}{s^2 + 2s + 1}$

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So, here see a step  $s$ . What is the cofactor of  $s$ ? Cofactor of a  $s$  is  $s$  plus 2. Cofactor of this element is 1. So, you will find that here  $s$  plus 2, but now here it is written minus 1; it is minus 1 because here this is in the first row and second column. So, what are elements will get that has to assign a negative sign. So whatever elements will get here, we have to assign a negative sign. So, here we are connected a attach negative sign. Similarly, the cofactor of this one is 1 and here the cofactor of this is  $s$ . Now what we do we have take the adjoint adjoint means what we do? We have taken  $s$  plus 2 1 minus 1 into  $s$ ; this we got adjoint and then we required a determinant.

So, how to calculate a determinant? The determinant, we calculate as here this is write as this  $s$ ; this  $s$   $s$  plus 2, then here we have plus 1 that is you will find here  $s$  square plus 2  $s$  plus 1; that means, we will get.

So, we have got  $s^2 + 2s + 1$  that is the we can say determinant that is again, I will proceed this. How do I get this? That is we have  $s - 1$  here  $1s + 2$  and how do you get it? Here  $s^2 + 2s + 1$ . So, here basically it is  $s - 1$ . So,  $s - 1$  here  $1s + 2$ ; we get  $s^2 + 2s + 1$  ok.

So, I think you understood see here the  $sI - A$  we got  $s - 1$   $s^2 + 2s + 1$ . So, here  $s^2 + 2s + 1$ . Now here we have got this adjoint of  $s^2 + 2s + 1$  a cofactor determine we have got. Now we want a  $G(s)$ . What is  $G(s)$ ?  $G(s)$  equal to  $C(sI - A)^{-1}B + D$ . What is  $C$  which is  $C$ ?  $C$  is our  $1 \ 0$ .  $C$  have to  $1 \ 0$ , then  $sI - A$  inverse.  $sI - A$  inverse we already calculated it. So, here  $s^2 + 2s + 1$  minus  $1s$  and what is the element of  $B$ ;  $B$  is we have  $0 \ 1$ ,  $B \ 0 \ 1$ . Therefore, what we got? We have got after solving this one we got  $1$  upon  $s^2 + 2s + 1$ . So, this is a one example we have solved.

Now, we see we solve some other example and moreover what we will do? We will take 2 different state space model and we see that for 2 different state space model, what type of results will get? So, we start with an example. So, we will take example.

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**Numerical Examples**

$$\dot{X} = AX + BU \quad Y = CX$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$Y = [1 \ 0 \ 0] X$$

$$[sI - A] = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 6 & 11 & s+6 \end{bmatrix}$$

$$\text{Cofactor } [sI - A] = \begin{bmatrix} s^2 + 11s + 11 & -6 & -6 \\ -s - 6 & s^2 + 6s & 11s + 6 \\ 1 & -s & s^2 \end{bmatrix}$$

$$= \begin{bmatrix} s^2 + 11s + 11 & -6 & -6 \\ s + 6 & s^2 + 6s & -11s + 6 \\ 1 & s & s^2 \end{bmatrix}$$

$$\text{Adj } [sI - A] = [Cofactor]^\top = \begin{bmatrix} s^2 + 11s + 11 & -6 & 1 \\ -6 & s^2 + 6s & s \\ -6s & -11s + 6 & s^2 \end{bmatrix}$$

$$\det [sI - A] = s(s^2 + 11s + 11) + 1(6) + 0 = s^3 + 6s^2 + 11s + 6$$

$$G(s) = C [sI - A]^{-1} B + D$$

Now, here  $\dot{x}$  equal to  $Ax + BU$ . This is and  $Y$  equal to  $CX$  and here the model here I am taking as  $0 \ 1 \ 0 \ 0 \ 0 \ 1$  minus  $6$  minus  $11$  minus  $6$ . Here we write  $x_1 \ x_2 \ x_3$  plus  $0 \ 0 \ 1$  into  $U$  and what is the output  $Y$ ?  $1 \ 0 \ 0 \ X$  this is the example. This  $abc$  has been shown  $d$

Now, what we want? We want the stress transfer function model. So, what is the first step? We have to determine  $sI - A$ . So, what is first step  $sI - A$ . This is  $sI - A$ . So, how do you get  $sI - A$ ? The  $Ss - I - A$ ; that means, from how, how you write?.

This is  $s - 1$  0; then here 0 0 and here this is 1 in minus  $a$  it becomes  $s - 1$  and here we will write  $6 - 11s + 6$  that is  $sI - A$ , we have got it.

Now, next step. Next step. We what we want? We want the cofactor of this  $sI - A$ . So, how we will write cofactors? So cofactor of cofactor of  $sI - A$ ; so this is  $S$ . Now this is  $s$  the  $G$   $s - 1$  0 0  $s - 1$   $6 - 11s + 6$ . Now, we cofactors of  $sI - A$ ; so here particularly for this element, we have to remove this row this column. So, what we will get? So, we will get it as  $s$  square plus  $6$  x this  $1$  and now here plus  $11$ ;  $s$  square plus  $6$  x plus  $11$ .

Now, about this element you see minus  $1$ . So, here you have to remove this row and this column. So, multiplication is  $s$  plus  $6$  introduce and minus  $6$ . So, and minus  $6$  and minus of minus  $6$  is plus  $6$ . So,  $s$  square plus  $6$   $s$  plus  $11$  and here is  $6$  and particularly this one, this particular case for this element we multiplies  $11$  and minus  $6$  is so minus that is minus  $6$  is here and then, here for this row we can write the element like this minus  $s$  minus  $6$  then  $s$   $s$  square plus  $6$   $s$  plus  $11$   $s$  plus  $6$  and lastly, we write  $1 - s$  into a square.

Now, after word after word what we want at this to get adjoint the we see that the elements this signs of these elements to be change because all these four elements will lies in this minus  $1$   $i$  plus  $j$  form. Therefore, the adjoint of elements is nothing but the factors transpose along with these particular elements.

Therefore, we write this particular element  $s$  square plus  $6$   $s$  plus  $11$ , then here this sign minus  $s$  minus  $6$ , we write as  $s$  plus  $6$  plus  $1$  this one; then here  $6$  because of this minus  $1$  raised to  $1$  plus  $j$  this element become minus  $6$  and here is minus  $6$ ; then this is  $s$  square plus  $6$   $s$ ; then here we get minus  $11$   $s$  plus  $6$  minus  $6$  and here write  $1$ , this is  $s$  and this is square.

So, we have got cofactor of  $sI - A$ . Now we require adjoint? To get the adjoint, we have to make transpose of this. So, we write adjoint of  $sI - A$ : so adjoint of  $sI - A$

A. So, what we will take? We take transpose of this that is equal to co factor of sI minus A. This is transpose. So, we write down this as s square plus 6 s plus 11; then here s plus 6 that is 1; then here minus 6 s square plus 6 s s; then minus 6 s minus 11 s plus 6 minus 6 s square.

So, now we have got adjoint of sI minus A. Now, next step is we have to calculate the determinant of sI minus A. So, how to calculate determinant? So, we write determinant of sI minus A. So, determinant of sI minus means how we will write? This is s; this one s square plus 6 s plus 11 and now what this we have taken. Now about this element this element, this 0: that means what we will get? We will get it as s square plus 6 s plus 11 is plus 1 and this 1 into and about this, this is 0.

So, all the element concerned with this become 0. So, we will get it as s cube plus 6 s square plus 11 s plus 6. Now, we have got adjoint; we have got determinant and now our next step is to calculate model. So, your model is G of s equal to C sI minus A inverse B plus D this is a model.

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Handwritten derivation for the transfer function  $G(s)$ :

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s^2 + 6s + 11)(s + 6)} \begin{bmatrix} s^2 + 6s + 11 & s + 6 & 1 \\ -6 & s^2 + 6s & s \\ -6 & -11s - 6 & 0 \end{bmatrix}$$

$$G(s) = C [sI - A]^{-1} B + D = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{s^2 + 6s + 11s + 6}$$

Now, we replace the values. So, here we write our element C. What is the value of C is? C is your 1 0 0. Now what is the value of B? B is your check it 0 0 1, you can check this here. So, here 0 0 1 is b and this is C. So, we have written here C and B and what is sI minus A inverse? So, sI minus A inverse. So, sI minus A inverse, we have got as s square

plus  $6s$  plus  $11s$  plus  $6$  minus  $6s$  square plus  $6s$  plus  $s$  minus  $6s$  minus  $11s$  minus  $6$  into  $s$  square that is  $s$  square.

Now, the next step is we have to multiply this  $C$  here  $sI - A$  inverse  $B$ . So, here this  $C$  is  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  and here this  $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$  you have to multiplied with this  $sI$  inverse. So, if you multiply let us say this is  $A$ ; this part this is  $A$  into  $B$ . Here, we can say  $A A^{-1}$ . Let us say  $A^{-1}$  and now  $A^{-1}$  and now here we have to multiply. So, what we will get if you multiply this  $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$  is  $0$  with this one. So, what happen? The all the elements concern with this second row and third row will come  $0$ . So, what we will get? We will get as  $s$  square plus  $6s$  plus  $11s$  plus  $6$  into  $1$  that is  $C (sI - A)^{-1}$  we have got this and now it is to be multiplied by  $B$ . So, your  $B$  is  $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ .

So finally, after multiplications, what we will get? We will get as  $1$  up  $1$  and here  $sI - A$  inverse is there a adjoint; but it should be square multiplied by determinant and what is the value of determinate is  $s^3 - 11s^2 + 6s + 6$ . So, here we have got  $1$  and this is like this  $s^3 + 6s^2 + 11s + 6$  that is  $G(s)$  we have got like this.

That means, you have a original model  $x \dot{=} Ax + BU$ ; first of all we are determined  $sI - A$ , then you determined cofactor of  $sI - A$ . Then we adjoint adjoint how we have got? We are taking the transpose of cofactor of  $sI - A$  and while taking the cofactors what we are taking?

Whatever the elements in particular  $i$ th and  $j$ th row if they come in they are coming odd, we are assigned negative sign just see here minus  $6$  plus here minus  $11$  is minus  $6$  and here minus  $s$  to be checks and then determinant we have calculate; after that to this  $sI - A$  inverse we have multiplied by  $C$  and  $B$ .

So,  $C$  is  $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ ;  $B$  is like this and after multiplication of all these  $C$  and  $B$ , we have got  $G(s)$  equals to  $1$  upon  $s^3 + 6s^2 + 11s + 6$ . So, this is the example which shows that the we can convert the given state space model into transfer function. Now we will take another example.



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$\dot{x} = Ax + Bu \quad y = Cx$   
 $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} \frac{1}{2} & -1 & \frac{1}{2} \end{bmatrix}$   
 $[sI - A] = \begin{bmatrix} s+1 & 0 & 0 \\ 0 & s+2 & 0 \\ 0 & 0 & s+3 \end{bmatrix}$   
 Cofactor  $[sI - A] = \begin{bmatrix} (s+2)(s+3) & 0 & 0 \\ 0 & (s+1)(s+3) & 0 \\ 0 & 0 & (s+1)(s+2) \end{bmatrix}$   
 $G(s) = \begin{bmatrix} \frac{1}{2} & -1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} (s+2)(s+3) & 0 & 0 \\ 0 & (s+1)(s+3) & 0 \\ 0 & 0 & (s+1)(s+2) \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Now, another example  $\dot{x}$  equal to  $Ax + Bu$ ;  $Y$  equal to  $CX$ . Now here a model I am taking as  $A$  equal to minus 0 0 0 minus 2 0 0 0 minus 3 and  $B$  here we are taking as 1 1 1 and  $C$  we are taking as 1 by 1 half by 2 minus 1 1 by 2.

$A$   $B$   $C$  you are taken;  $A$  is minus 1 0 0 minus 2 0 0 0 minus 3.  $B$  1 1 1 and  $C$  we have taken as 1 by 2 minus 1 1 minus 1 1 by 2 and now we want the transfer function model that is we want  $G$  of  $s$ . So, now, if we will follow the steps which I have discussed earlier that is first step is we write  $sI$  minus  $A$ ;  $sI$  minus  $A$ .

So, what is  $sI$  minus  $A$ ?  $sI$  minus  $A$  equal to  $s$  plus 1 0 0 0  $s$  plus 2 0 0 0  $s$  plus 3. We will find that the most of the elements in this case are 0 and this particular model which we have already seen last time that is within diagonal form. And therefore, it is simply  $sI$  minus  $A$ . So, we are taking  $s$  plus 1  $s$  plus 2  $s$  plus 3. Now we have to determine the adjoint of  $sI$  minus  $A$ ; so for adjoining be required to cofactor of  $sI$  minus  $A$ ; so cofactor. So, let us say with respect to this element  $s$  plus 1, the minors are this  $s$  plus 2  $s$  plus 3. So, we are multiplied this  $s$  plus 2 into  $s$  plus 3.

Now, with respect to the 0 elements, the minors are 0 0 0  $s$  plus 3 this is 0 and here 0; then here with respect to  $s$  plus 2 respect to 0 and with respect to are 0. The elements are this is 0 and respect to  $s$  plus 2 here multiplied this you have to remove this row and this column. So, there is a multiplication of  $s$  plus 1 and  $s$  plus 3.

So, we write  $s + 1$   $s + 3$  and here other element 0 and finally, about third 0 0 here  $s + 1$  into  $s + 2$  that is with respect to this, the minors are this row is row this row and this column we have to take remove and here element like this; SO  $s + 2$  into  $s + 1$  into  $s + 2$ .

Now, we have got cofactors and after that we require the adjoint; for adjoining what we do we had to make the transpose of this one and before transpose, we have to assign negative sign. So, here this negative: so all zeroes. So, there is no need to attach any or correct any negative sign. Therefore, what we want we have take to the transpose of this one. So, but in transpose we will find that here as this is a diagonal form whatever the cofactors of  $sI - A$ , we are getting the same as adjoint of  $sI - A$ ; that means, here your adjoint of  $sI - A$ ; here  $A$  transpose of cofactor of  $sI - A$  will get the same thing like this.

Now, the next step we have to determine it is means we have to determine the  $G_s$ . So, how to determine  $G_s$ ? For getting the  $G_s$ , we have to multiply  $C$ ; then  $s$  then we have to determine you have to say multiplied by  $A$  inverse.

So, here  $G_s$  equal to  $C$ ;  $C$  is your  $1 \times 2$  minus  $1 \times 1$  by  $2$  and here the write this as  $s + 2$   $s + 3$   $0 \ 0 \ 0$   $s + 1$   $s + 3$   $0$ , then here  $0 \ 0$   $s + 1$   $s + 2$  and now we have to mult and this is the adjoint what is the determinant and determinant is nothing but  $s + 1$   $s + 2$  and  $s + 3$  and see  $sI$  inverse. This is  $sI$  inverse;  $B$  is  $1 \ 1 \ 1$ .

Now, first of all what we do; we multiply this with respect to this matrix.

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$$\begin{aligned} & \left( \frac{\frac{1}{2}(s+2)(s+3) - (s+1)(s+3) + \frac{1}{2}(s+1)(s+2)}{(s+1)(s+2)(s+3)} \right) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\ &= \frac{\frac{1}{2}(s+2)(s+3) - (s+1)(s+3) + \frac{1}{2}(s^2+2s+2)}{(s+1)(s+2)(s+3)} \\ &= \frac{\left(\frac{s^2}{2} + \frac{5s}{2} + \frac{3}{2}\right) - (s^2+4s+3) + \left(\frac{s^2}{2} + s + 1\right)}{(s+1)(s+2)(s+3)} \\ &= \frac{-s^2+11s+6}{(s+1)(s+2)(s+3)} = \frac{1}{s^2+11s+6} \end{aligned}$$

So, you will get it as (Refer Time: 37:20) 1 by 2 S plus 2 S plus 3, then other element will get it as minus S plus 1 S plus 3 and here we will get it as 1 by 2 S plus 1 to S plus 2 divided by S plus 1 S plus 2 S plus 3; that means, how we got it? Here we have multiplied this. So, 1 by 2 multiplied by this rest is 0. Therefore, we have got S plus 2 S plus 3; then about this we are multiplied by this one. So, regarding this first element this 1 0 0; so remaining minus of S plus 1 S plus 3, so, here minus of S plus 3.

And here, you will find that this 1 by 2 this all the sorry 1 by 2 minus 1 1 by 2 is multiplied by this elements. So, here 1 1 1 by 2 multiplied by 0 0 minus 1 multiplied by 0 0 and particularly, but 1 by 2 will multiplied by S plus 1 S plus 2 will get it like this and this is S plus 1 plus 2 S plus 3. Now this particular product it is multiplied with B. So, b is your 1 1 1

So, now if you multiply actually; so what we will get? We will get 1 by 2 S plus 2 S plus 3 minus S plus 1 into S plus 3 plus 1 by 2 this is S plus 1 S plus 2 if it is S square plus 3 S plus 2 and divided by S plus 1 S plus 2 S plus 3.

Now, if you solve it. So, what we will get? S plus 2 plus S plus 3, so we will get a S square plus 2 plus 3 5 S plus 6 so divided by 2. So, what you will get S square by 2 plus 5 by 2 S plus 6 by 2 minus here you will get it as S plus four S plus 3 that is S square 3 plus 1 4 4 S plus 3 plus S square by 2; then 3 by 2 S plus 1 and divided by S plus 1 S plus 2 S plus 3.

Now, you just here this is  $S^2 + 2$ ; this is  $S^2 + 2$  by  $2$  become  $S^2$  and needs to cancel with minus  $S^2$ , then just see here this is  $3$  by  $2$   $S$ , then here  $5$  by  $2$   $S$ . So,  $3$  by  $2$   $S$  plus  $5$  by  $2$  is equals to  $4$   $S$  plus  $4$   $S$  is subtracting with minus  $4$   $S$  become  $0$  and lastly this is  $3$  by  $2$ , but and this is minus  $3$  by  $2$  will cancel, become  $0$ .

And finally, we got it at  $1$  upon  $S$  plus  $1$   $S$  plus  $2$  this is  $S$  plus  $1$   $S$  plus  $2$   $S$  plus  $3$  that is same thing and you will write it down this as  $S^3$  plus  $S^2$  plus  $11$   $S$  plus  $6$ ; that means, if you see it earlier  $S^3$  plus  $6$   $S^2$  plus  $11$   $S$  plus  $6$ . So, we have got the same thing. So,  $S^3$  plus  $6$   $S^2$  plus  $11$   $S$  plus  $6$ .

So, here we observe that we have taken 2 different state space model; but we have got the same transfer functions. It means that for a same if you want similar transfer function. We have got different state space model, but we get different state space model will get only single transfer function; that means, the state space model are not unique; but whereas, the transfer function models are always unique; that means, the state space models are not unique; whereas, transfer function models are always unique.

Now, we now we have to see what are the drawbacks of this particular approach. We cannot say that drawback as because we having a different state space model, we are getting the transfer function. So, the; that means, a variety of information about the models, we cannot get it because we are getting the same transfer function model. Now, the other thing is that here we will find that this need to calculate the inverse; but if you seeing thing say 25 years back where there were no softwares. So, that time the calculation of inverse is quite cumbersome that periodically adjoining and determinant all these things.

So, as the order increases, it is quite difficult and you know that if you take some example power system models, we know we know high order model. So, in that case it is difficult to apply this particular method. So, that is therefore, it is written here.

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**Drawbacks of the conventional approach :**

(i) As the order of the system increases, the mathematical computation burden will also increase.

(ii) Earlier, the digital computer fails to determine the  $[sI-A]^{-1}$  because the Laplace operator 's' is a scalar. However, nowadays due to advanced software, it is possible to find inverse of  $[sI-A]$ .

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As the order of the system increases the mathematical computation burden will also increase; this is the one drawback and second is that let us say if you want to determine the transfer function from the given state space model by using the MATLAB software maybe MATLAB software.

So, in that case, we have take this  $S$ ;  $S$  is a variable. So,  $S$  is scalar. So, earlier time it is it was difficult to calculate the  $sI - A$  inverse by means of a MATLAB. Nowadays it is possible and the second point is that earlier, the digital computer fails to determine the  $sI - A$  inverse because the Laplace operator  $s$  is a scalar the Laplace operator  $s$  is scalar.

However, nowadays, due to advanced software, it is possible to find the inverse of  $sI - A$ . Today, there is no as such problem to calculate the  $sI - A$  inverse because of the MATLAB has been advanced and we can easily calculate, but if you see the earlier times this procedure even it is quite difficult to calculate by means of any software.

Now, next point is that we sometimes we have to do some analysis of the system. So, in the analysis of the system there is no need there is no need to use the actual values; then you need to use the variables. So, in that case to calculate the  $A - I$  inverse is also very cumbersome even today also. So, that is why we want some method we can get easily get the transfer functions.

Therefore, in the next part we will go into study the how to calculate the  $G$  s or transfer function model from the given state space model without computing the inverse.

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Now, these are some references.

Thank you.