

**Advanced Linear Continuous Control Systems**  
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**Lecture – 13**  
**Modelling of DC Servo Motor (Part-II)**

Now, we again start with modeling of DC Servo motor part 2, in previous part we have seen the modeling of an armature controlled DC Servo motor. And in that case we have keep a field current constant, but as you know motor there are two current in im. So, we have keep field current constant, but suppose if you keep armature current constant the in that case can we get any different model or can we get the same model. Therefore, here our main purpose is to develop the model of a servo motor when your armature current is constant.

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**Field controlled DC Servo motor**

**Circuit Diagram**

$$\frac{\Theta(s)}{E(s)} \frac{Q(s)}{E(s)} = \frac{Q(s)}{E(s)}$$

$\phi \propto i_f$   
 $\phi = k_f i_f$   
 $T_m \propto \phi i_a$   
 $T_m = k_t \phi i_a$   
 $= (k_t k_f) i_f i_a$   
 $T_m = k_t' i_f$

$L_f \frac{di_f}{dt} + R_f i_f = e_f \quad \text{--- (1)}$

$J \frac{d^2 \theta}{dt^2} + B \frac{d\theta}{dt} = T_m = k_t' i_f \quad \text{--- (2)}$

That means when the armature current is constant, what is the theta S bar by E of S that is need to be determined. So, first of all we draw a circuit diagram of a field controlled DC servo motor. So, these field resistance is Lf there is Rf, then here is the applied voltage ef and here is the armature part.

And here current flowing is i that need to be constant. And here we got torque TMT, what torque this as I told ef is for a development of the flux, Rf is a field resistance, Lf is field inductance. Then this torque is produce it has to drive or the motor has to against

the moment of inertia and a friction therefore, here  $J$  and  $B$ ,  $J$  is the moment of inertia and  $B$  is the friction, here current flowing here  $i_f$ . So, this is the a circuit diagram of a field controlled DC servo motor.

Now, we have to determine  $\theta$  that is  $\theta$  here  $\theta$  a  $S$  by  $E S$ . So, how to get it? Here, we have to develop  $\theta$   $S$  by actually  $e_f$  of  $S$ , that is  $e_f$ . So, how we do it? So, first of all we see here that the flux is produce because of this  $e_f$  that is, the field current is produce and because of this field current will get the flux. Therefore, here we write this flux is proportional to field current therefore, this flux equal to  $K_f i_f$ . So, this proportionality constant prepares by like this. Then torque develop torque develop is proportional to your field current flux into armature current, torque develop is proportional to flux into armature current; last time you have seen.

So, again we can replace this by a constant  $K_1$  flux  $\phi$  into armature current. Now, in this expression what we do? We replace this  $\phi$  with this  $K_f i_f$ . So, will write we write  $K_1$  then  $K_f i_f$  into  $i_a$ . So now, we want the motor torque with respect to some parameters. So, in this case armature current is constant. So, this armature current is constant this  $K_1$  is constant this  $K_f$  is constant therefore, this motor torque we can write down as  $K_t$  into  $i_f$ .

Now, the respect to torque the expression we obtain, now how to handle other part of it. So, other part just we observing we are getting the part respect to  $R_f$  and  $L_f$ . So, this part we can solve or this we have to solve, and here there is no need because here armature current is constant. If you see in a previous part that is the armature controlled a DC servo motor, we have solve the armature circuit  $R_a L_a$  like this. But here basically here we have this  $R_f$  and  $L_f$  because your field is varying, but here and in this case  $i$  is constant. Therefore, we have to apply Kirchoff's law to this particular part.

So, we can write down this as  $L_f \frac{di_f}{dt}$  plus field resistance  $R_f$  this is field resistance  $R_f$  into  $i_f$  equal to  $e_f$ ,  $e_f$  is the applied voltage to this field part. Here also we applied, but this we are keeping constant. Now, we have developed the circuit for this portion then we have got the expression for motor torque. And here will find one difference there is no back emf.

If you see last time in a armature controlled there is a back emf has come, but here there is no back emf, because here we are keeping armature current constant therefore, there is

no back emf. Let me just see there is we can consider here there is no controlling part then, let us see what will happen afterward. So, here we have got expression like this now we see the expression for torque that is the mechanical torque. So, here we can write down  $J \frac{d^2 \theta}{dt^2} + B \frac{d \theta}{dt} = \text{motor torque } K_t i_f$  into if that is this is the 3 bodies.

So, we got the expression for this and expression for this and this another expression, but we have adjusted like this. So, we have got these 2 equations. So, everything every part we have covered here and now we have to develop a model that is,  $\theta$  S by ef in a in case of your armature controlled we had it when  $\theta$  S by ES because in this case we have applied input to the armature side, but here input is to the fields type that is the important issues because that is varying.

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### Field controlled DC Servo motor

**Step 1: Motor torque:**



$$T_{MT} = K_1 K_f i_a i_f = K_T' i_f \quad (1)$$

In field controlled motor, armature current is constant

In above equation,  $K_T'$  is a constant

**Step 2 : Field control circuit:**

$$e_f = L_f \frac{di_f}{dt} + i_f R_f \quad (2)$$



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So, whatever I explain now again just see it. So, here motor torque  $K_1 K_f i_a i_f$  into is. So,  $K_t$  dash into if. So, this means this we have got then about a field control circuit  $e_f = L_f \frac{di_f}{dt} + i_f R_f$  this part and third part is torque equation get into i f.

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**Step 3 : Torque equation:**

$$T_{MT} = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = K_T' i_f \quad (3)$$

Taking Laplace transform of eq.(2) and eq.(3), we obtain,

$$E_f(s) = (R_f + sL_f) i_f(s) \quad (4)$$

$$T_{MT}(s) = Js^2\theta(s) + Bs\theta(s) = K_T' i_f(s)$$

$$i_f(s) = \frac{Js^2\theta(s) + Bs\theta(s)}{K_T'}$$

Replacing  $i_f(s)$  in (4), we get,

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That means, this expression we are shown here. So, we have got equation in the time domain. So, we want in s domain therefore, basically what we do we apply Laplace transform. So, we are applying Laplace transform of equation in 2 and 3 that is this is equation number 2 and this is equation number 3 we got like this. And now, from this expression this if of s is written like this and now this expression of if of s in 4 replacing if of s in 4 this 4. So, we got this result.

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Hence,

$$E_f(s) = (R_f + sL_f) \frac{Js^2\theta(s) + Bs\theta(s)}{K_T'}$$

$$E_f(s) = (R_f + sL_f) \frac{(Js^2 + Bs)\theta(s)}{K_T'}$$

Thus,

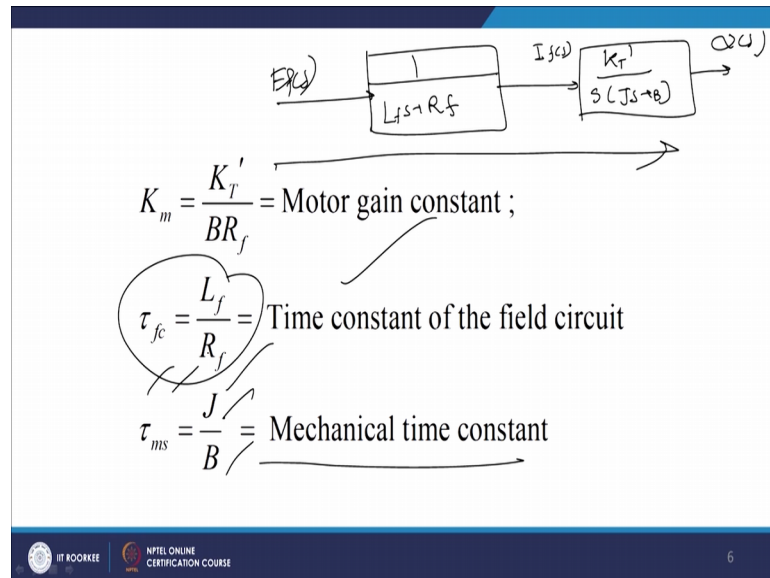
$$\frac{\theta(s)}{E_f(s)} = \frac{K_T'}{(R_f + sL_f)(Js^2 + Bs)} = \frac{K_T'}{s(R_f + sL_f)(Js + B)}$$

$$\Rightarrow \frac{\theta(s)}{E_f(s)} = \frac{\left(\frac{K_T'}{BB_f}\right)}{s\left(1 + s\left(\frac{L_f}{R_f}\right)\right)\left(1 + \left(\frac{Js}{B}\right)\right)} = \frac{K_m}{s(1 + s\tau_{fd})(1 + s\tau_{ms})}$$

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Now, what to do next? Now, we have to simplify it. So, here in order simply function the theta of s is taken common here. And afterward this theta S digital like this and ef these and adjusted the variable like this. And from this we have got this expression then again this B and Rf this adjusted like this. And we have got this one could expression that is Lf by Rf is replaced by this tau fc and J by B is by tau ms. So, what is mean by this.

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So,  $K_m = K_T' / B R_f$ . So, here this point this  $K_T'$  by  $K_m$  is replaced by constant  $K_m$  there is called motor gain constant, then here  $\tau_{fc}$  is replaced by  $L_f / R_f$  there is called time constant of the field circuit. And again we see that in a time constant. What is time constant time required for system to the 63 percent of the value?

So, as the time cost is more your system become little bit slow. If your time constant is less your system is fast you will find that the time constant in case of this field controlled is depend upon  $L_f$  and  $R_f$ . So, normally the field binding resistance or inductance is field binding inductance is generally more. Therefore, you will find that the time constant of the field circuit is quite more then about the mechanical time constant  $J$  by  $B$ .

So, will find that this system is working in an open loop because there is no (Refer Time: 10:48) came there is no controlled. So, therefore, if you draw the block diagram of these the block diagram of these you will get like this, that is here,  $1 / (L_f s + R_f)$  to if of  $s K_T' / (s J s + B)$  into theta of s ef of S.


So, if behavior of this is as good as open loop whereas, in case of armature control we have got the closed loop again we will find that about the time constant. Now we just see the difference between armature controlled DC servo motor and field controlled DC Servo motor the first point.

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**Difference between armature controlled DC Servo motor and Field controlled DC Servo motor**

(i) The transfer function model of position controlled servo motor can be reduced to second order in case of armature controlled dc servo motor whereas the model of Field controlled dc servomotor cannot reduced to second order.

$$G_{field\_controlled}(s) = \frac{K_m}{s(1+s\tau_c)(1+s\tau_{ms})}; G_{arm\_controlled}(s) = \frac{K_m}{s(\tau_m s + 1)}$$


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The transfer function model a position controlled servo motor can be reduced to second order in case of armature controlled DC servo motor whereas, the model of field controlled DC motor cannot reduce to second order. Why it is so? Because we see that in case of armature controlled DC servo motor these armature inductance and that is very, very small therefore, that can be neglected. And therefore, because of that reduction we have got the model as a second order model, but whereas, in case of field controlled motor this is not the possibility.

We cannot reduce the feeder inductance to 0 therefore, we can say that the model of an field controlled motor particular position controlled model is always be a third order. Whereas, in case of armature controlled servo motor, we can considered the model and the second order by reducing the it is inductance.

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$G(s) = \frac{k}{(s+a)(s+b)} - T(s)\omega(s)$

(ii) The maintaining a constant armature current ( $i_a$ ) is more difficult than the maintaining field current ( $i_f$ ) because of the back emf of the armature circuit.

(iii) The stability of the field circuit is poorer to that of an armature controlled dc servo motor.

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Next the maintaining a constant armature current, second point the maintaining a constant armature current  $i_a$  is more difficult than maintaining the field current if because of the back emf of the armature current. Although we got some advantage that we can use the model in case of armature, but as far as the armature current is concerned, it is difficult to control because here back emf. As we change the as speed changes the armature controlled changes. So, controlling of this back emf also very challenging therefore, these a one problem is that we it is difficult to keep a constant armature current whereas, it is easier to keep a constant field current. So, this is the, if issue or the difference between the armature controlled and the field controlled the third point.

The stability of the field circuit is poorer to that of an armature controlled DC servo motor. As stability is very important issue in control engineering. So, the stability of state space these things we will see in the next part, but here it is say that stability of the field circuit is poorer why it is so? We find that system is very simple they even no grow loop, but it is say that stability of the field circuit is poorer to that have armature controlled DC servo motor, why it is so? This we can easily see from the while itself, you see that the armature controlled motor it can be used to second order whereas, field control model is always a third order.

And you know that if the all the coefficients are positive is a third order and the second order, what about the effect of the stability? If you succeed if I am taking one example, 1

upon  $S$  plus  $a$   $S$  plus  $b$  this is  $S$  this is  $GS$  this is  $a$  in open loop plot, open loop plot. And now what about a closed loop we can write down close loop as  $1 + G$  of  $S$ . So, we will find that if you draw the root locus of this let us say root locus, let us see it is minus  $a$  and this is at minus  $b$ .

So, for second order system will find that let us say root locus means that locus of root or locus of close loop caustic position when one of the variable or gain  $k$ , varying from some  $0$  to maximum value or  $0$  to infinity that part you might have studied in the classical controlled. So, here now I am drawing you the root locus for this was what one were root locus form here this is a pole and there are no  $0$ s it to start from here, one is going like this other is going like this like this.

So, for second order system the root locus like this that is  $k$  equal to  $0$  and here  $k$  equals to infinity, here  $k$  equals to infinity. So, one root locus start from here it will move like this other root locus like this. So, this is the an imaginary axis, and we see that if the roots are lying on the right side of the  $s$  plane system is always unstable left hand side system is stable. So, you will find that for a second order system the root locus is like this. I can again showing here and this one is here. So, one is root locus starting from here these a break a point other is moving like this, other is moving like this is same point you can see the how to did my root locus from any of the book.

But for this point expression of root locus is important. So, here you will find at for second order system as we change  $k$   $0$  to infinity here  $k$  equal to  $0$  to infinity, the root locus always lies on the left hand side. Therefore, you know you just see here your armature controlled motor ok, is always is first order or even a second we can say if you say the speed controlled motor is always a in terms of first order or in case of the we can say the position control it becomes second order.

Therefore, if you go for the third order you will find that the root locus is moving towards the right side of this; that means, if I am taking the third order plan. Let us say  $K$   $S$  plus  $1$   $S$  plus  $2$   $S$  plus  $3$ . So, I am showing the roots  $S$  equal to minus  $1$  another root  $S$  equals  $2$  minus  $2$  and here  $S$  equal to minus  $3$ ,  $S$  equal to minus  $2$  minus  $1$  these are  $3$  roots. So, in this case what should happen the one root locus in this case if you see the root locus start from terminated into  $0$ . So, here the root locus like this and another is root locus like the real part, and here one root locus from here it will move like this



another root like this. So, will find at add some value of gain  $k$ ,  $k$  equals some value let us say here  $k$  equal to infinity and some value of gain here system become unstable.

But in second order system is always stable. So, here as you see that the field system field controlled circuit is always a third order. Whereas, in case of armature controlled it is a second order, that is why as the order increases stability reduces. Therefore, here we have say that stability of the field circuit is poorer to that of an armature controlled DC servo motor, then time cost is higher in the field controlled DC servo motor in comparison to armature controlled DC servo motor. You find that the time constant in case of field a field controlled DC motor servo motor depends upon  $L_f$  therefore, as  $L_f$  is normally high. Therefore, time cost is higher that part we can say this system is to be slow.

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(iv) The time constant is higher in the field controlled DC Servo motor in comparison to armature controlled DC Servo motor.

(v) Field controlled motor is useful for small size motors and armature controlled motor is useful for large size motors.

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The other point the field controlled motor is useful for small size motors and armature controlled motor is useful for large size motors. It is because you know in case of field controlled motor it is difficult to control the armature current and therefore, this is not. So, therefore, this type of design field controlled design cannot be useful for high power application whereas, in case of armature controlled field current can be easily keep constant. Therefore, we can say that armature controlled servo motor can be useful for high power applications. Now, these are some references

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**References**

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Thank you.