Advanced Linear Continuous Control Systems
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Lecture - 12
Modelling of DC Servo Motor (Part-I)

Today we start with modelling of DC Servo Motor. So, the motor is very important instrument or we can say device, which is useful in control applications. Whenever control engineer develop any algorithm, it is to verify real time.

So, the DC motor is an very important problem. Therefore, this is need to study the modelling of this motor. There are 2 types of motors, AC motor and DC motor. So, first of all we see; what are the differences between AC motor and DC motor.

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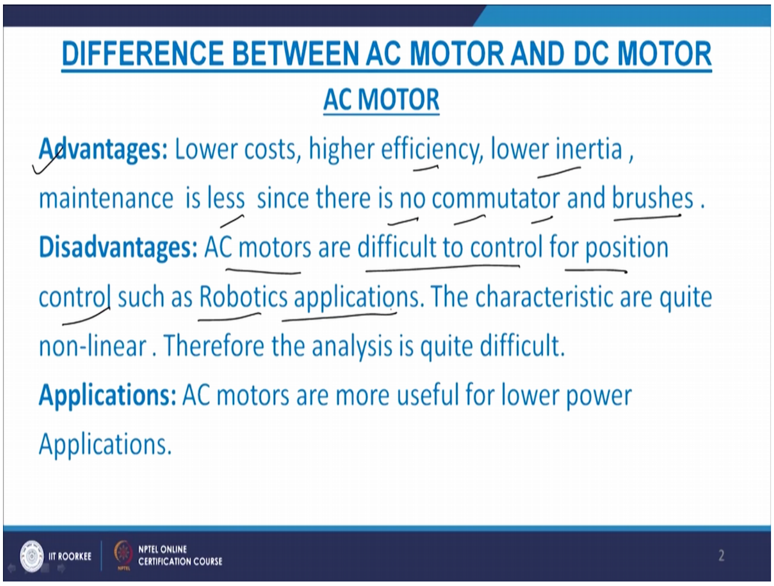

Now, if you compare the DC motor and AC motor, which is the best? So, first of all we see what are the advantages and disadvantages of AC motor. Now about the advantages, lower costs higher efficiency lower inertia maintenance is less since there is no commutator and brushes these are some advantages of AC motor, but there are some disadvantages. Because finally, what is the result we required output? So, we will find that that output which obtained by see it is difficult to control, and again the characteristics particular (Refer Time: 01:54) characteristics is somewhat non-linear.

Therefore, for analysis point of view the AC motor is not better in comparison to DC motor.

So, what are the disadvantages? AC motors are difficult to control for position control such as robotics applications. As I told you that last time, that we have to control the position of a robotic arm so, how to control it? So, here suppose this is an arm, and this arm is there and this is a point gripper, and here the gripper is fitting the some object and it place it so, what happened? If you move like this; so here it has to come this point. So, here we will find that the controlling task or movement is not so proper. So, that is a one disadvantage of this AC motor.

Second the characteristics are quite non-linear therefore, analysis is quite difficult. Now what are the applications? AC motor are useful for low power application, because of the part the things that analysis quite difficult. Therefore, if you go for higher order system it is difficult to use this AC motor. So, in that case we will see later on the DC motor is quite better, now about a DC motor.

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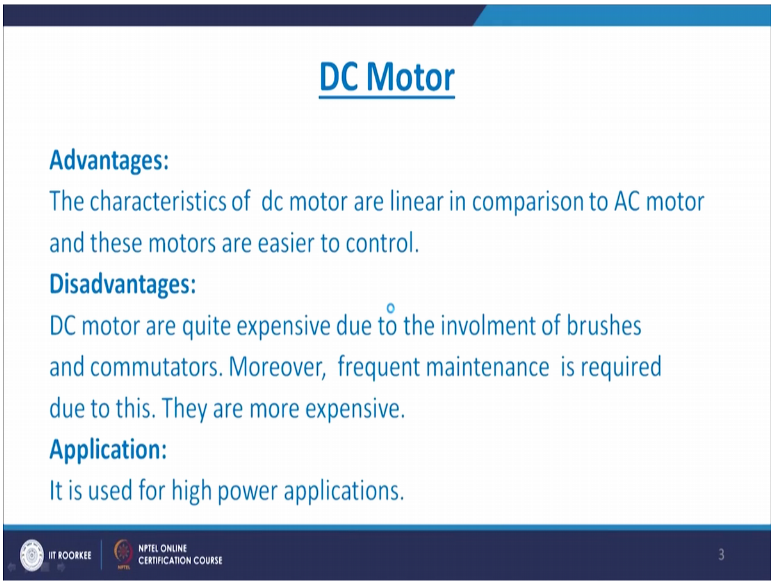

The characteristics of DC motor are linear in comparison to AC motor and these motors are easier to control, and if you see the control aspects, these motor are very easy to control, then there are some disadvantages. The DC motor are quite expensive due to involvement of brushes and commutators.

See in this DC motor this involvement of brushes and commutator, and naturally the size increases. And again we have to maintain this brushes and commutator, and therefore, it is quite expensive, but today due to development of the power electronic engineering, brushless motor is also come.

So, so, in other words we can say that now DC motor are better in comparison to in comparison to AC motors. Now application it is used for high power applications. So, as the controlling is quite easy therefore, we have we can use this motor for higher power applications.

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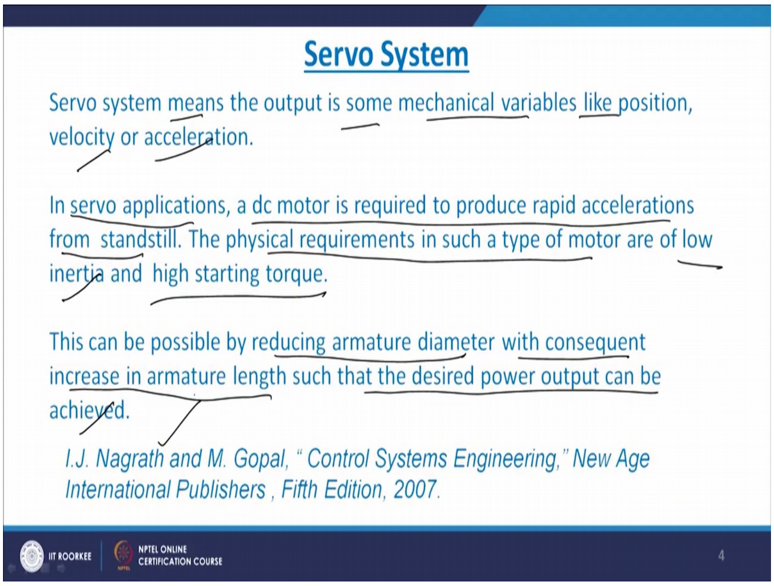

Now, about the servo system, normally those we are setting electrical machine subject, normally called as motors AC motor and DC motor. But whenever this motor we have to use in control, it is called as servo motor. What is servo? Servo means your output it is in terms of position velocity or accelerations. So, here point is servo means output is some mechanical variables like position velocity or acceleration.

Now, what is a requirement of such type of AC motor? What do we wanted? The motor once we give input motor should immediately on, that is rapid acceleration, for the standstill, so that is the requirement of AC motor. So, it is written servo application DC motor is required to prop produce rapid acceleration from standstill.

The physical requirement in such type of motor are of low inertia and high starting torque. So, this is possible rapid acceleration is possible whenever there is inertia is should be less then and then we get high starting torque.

So, this can be possible by reducing the armature diameter. So, we have to reduce the armature diameter with consequent increase in armature length, such that the desired torque output can be achieved. So, desired output can be achieved by reducing the armature diameter which consequently increase the armature length. So, you can see this reference.

So, this DC motor can be used in 2 modes, one called armature control mode and another is called field control mode. So, what is the difference between armature control and field control? In armature control mode, we have to keep filed current constant, and in case of field control mode, we have to keep armature current constant.

Now, we have to see the modelling of DC servo motor, in both cases that is in armature control mode and field control mode. So, we will see this part, ok. So, first of all now we are developing the transfer function model of a DC servo armature control motor. And afterword we will develop the state space model of an armature controlled DC servo motor. So, first of all I am drawing the circuit diagram of an armature control DC servo motor.

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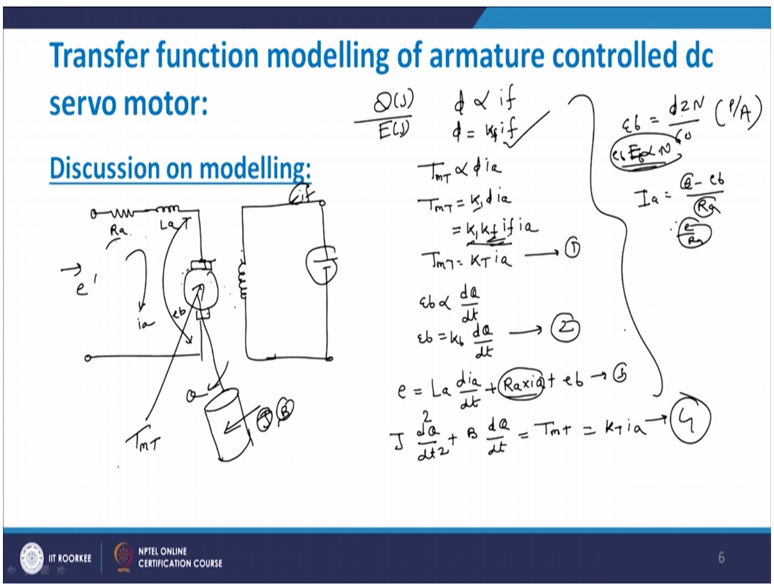

So, in a armature control DC servo, motor or we say any motor this is a involvement of armature resistance. Then we have inductor and here is brushes, these are brushes this one and this one, see it is armature resistance, this is inductance, then we required the field winding. So, here we say armature winding, but here we need a field winding now here this is a field current e is here applied voltage, now here we will get back emf and here is a armature current.

So now when this motor we rotate, it has to drive against moment if inertia and a frictions. So, here is a J B, and now the torque progress by the motor Tm, Tm T is the torque progress by the motor, and here is the theta positions,.

Now, this is an circuit diagram of an armature control DC servo motor now here a main requirement is a position theta. Now this is motor, now first of all we see; what is a difference between the generator and motor. In generator will get the emf so, how we how we are getting the emf? When the conductor cuts the flux the emf is induced, the flux, the how the flux is produce by the field winding, and in case of motor by the current carrying armature winding cuts the flux then emf is induced. So, that is the basic difference between the generator and motor. In generator we get the output voltage in case of motor we get the toque.

So, in case of motor when conductor current carrying conductor cuts the flux torque is produced, generator conductor cuts the flux only, in that original conductor there is no current is flowing the that is difference between generator and motor. Therefore, if you see in this particular diagram, circuit diagram we have applied voltage. So, applied voltage it to this motor means that it is it to the system we fall as a motor.

So, here we will find that this is field winding. So, what is the purpose of the field winding is to produce the flux, and this is a supplier to produce the field current. So, if changes the field current changes, if changes the flux changes. And this flux is responsible for this the getting the torque, then this conductor here cuts the flux we get the torque. And now here is the eb, a back emf and e is the applied voltage, and now or main purpose is to develop a transfer function model of these servo motor.

That is our main purpose is to get theta s by E of s. Now we start deriving this now what is the first step, the first step we see that this field current because of this field current the flux is produced. And if you consider linear range of magnetization curve so, we write expression as the flux is proportional to your filed current.

Flux is proportional to your filed current, and here we assume linear range of magnetization curve, therefore, you write this expression as phi equal to k 1 equal to. This is we replaced by k 1, now finally, what is the aim of the motor? Aim of the motor is to produce a torque. The toque is proportional to flux and armature current. Therefore, the torque develop by this motor is proportional to, where flux into armature current. Now this is a proportional signed, this is also remove it. So, we can write motor torque equal to k 1 flux into armature current.

So now here this is flux this flux, we have already determined. So, what we can do we can replace this flux equation of flux here. So, we will get k k 1 this actually we take this is kfs this is k 1 k 1 kf if into i. So, this flux is proportional to kf into if the motor torque is proportional to flux into armature current. So, this flux we are replace by kf into if here, kf into if and this is an armature current.

So, in this case what we are doing? We are keeping the field current constant, this current we have to keep constant, therefore, what we can do? This k 1 is also constant this k 1 constant, this kf is a constant of this, and this if we have to keep constant. Therefore, these total kn k 1 kf if, we replaced by another constant. So, Tm motor torque equals to equal to KT into ia, let us say this is equation number one. So, we have got the motor torque.

Now, then about the back emf back ms is very important in motor, because whatever the current flowing in the motor is depends upon this back emf, because if you see the back emf Eb equal to phi z n 60 p by. So, this expression you might have studied in electrical machine subject; that is flux number of zn number of armature conductor speed p poles a is the number of parallel path. And here this eb is proportional to eb is proportional to speed. So, this part you have studied in electrical machines.

Now, what is important role of eb? Important role of eb is in the initial current, if you see the any motor the initial current I equal to e minus eb, we can take small eb here eb divided by Ra.

So, if you start the motor what happen? If we finish start the motor speed is very less. So, when speed is less, what is the eb? Because eb is proportional to seev speed therefore, this eb is also less, and that is the current is e by Ra, current is in this case e by Ra, this is quite large. But as start the motor as motor gather speed eb increases the armature current decreases.

So, this eb in this case it is just like a starter, it control the armature current. So now, here we have to write sown the expression for this eb. This eb is proportional to d theta by dt, because theta is positioned d theta by dt is a speed. Therefore, you removed here a proportionality signed we will get eb equal to Kb, d theta by dt; this is equation number 2. So, we have got equation number one, for motor torque we have got expression for back emf, now what are the things which are involved in this motor, that is register inductor. So, this part we have not covered. So, that part also we have to taken into the expression of the modelling, therefore, what we do? We apply a conventional Kirchhoff's law to this part.

So, what especially we will get e applied voltage equal to La di a by dt, plus Ra into ia plus eb, this is equation number 3, this is nothing but your Ra into ia; that is armature resistance into the armature current La di by dt plus eb, eb is the back emf. Now, this part is completed now come into this part. So, J moment of inertia and Bs friction, as we apply a torque is coming in the existence electrical torque, but it is some motions, just like if I walking here walking. So, I have apply my force so, it is opposite by my inertia my body weight, similarly when motors are rotating it has been opposite by the moment of inertia J and also the friction the friction is with discuss friction last time we have seen this.

So, here therefore, here also we can write the expression of the torque developed by the motor that is a that is a mechanical torque. So, here we can write the expression, like this torque equations J d square theta by dt square plus B d theta by dt equal to T MT, and this order we derived that is equal to KT into ia, now this equation number 4.

So, these are equation number 1, 2, 3, 4 and this all this equations covered the model of this servo motor. Now, but our main purpose is to get the model in terms of s, that is theta is theta is a position theta s by Es, e e e offers is the applied voltage.

Now, what is the next step? So, it means that we have to apply the Laplace transform to all this equation 1, 2, 3, 4 and doing some mathematical adjustment we have to get theta s by s. So, whatever things know I am explained now, again you see it in various steps. So, first step that is torque torque equations.

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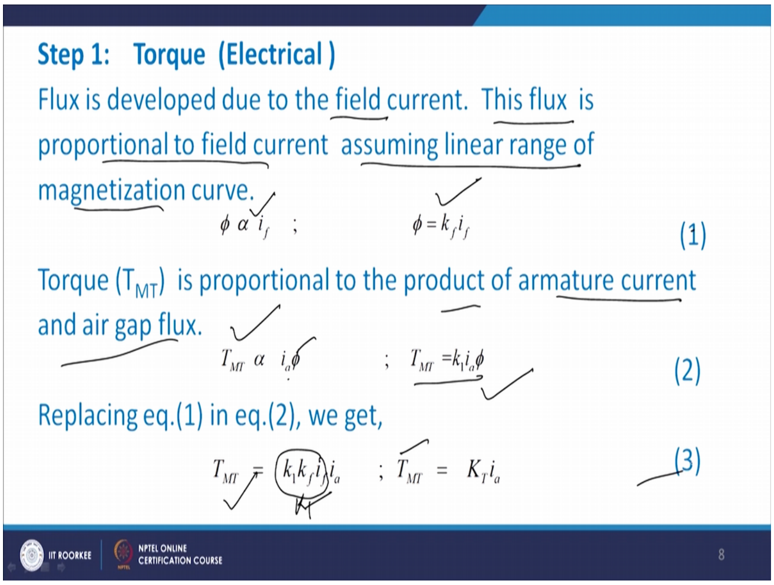

The flux is developed due to the field current, these flux is proportional to field current, assuming linear range of magnetization curve. So, flux is proportional to if so, flux equals to kf into if. So, we have seen this expression earlier so, this is next.

This torque is proportional to product of armature current and air gap flux therefore, this motor torque equal to armature current into flux so, this is expression. Then replacing equation one this is equation 1 in equation 2, this is 2 so, we got TMT. And here this part is replced by a constant KT, therefore, motor torque equal to KT into ia.

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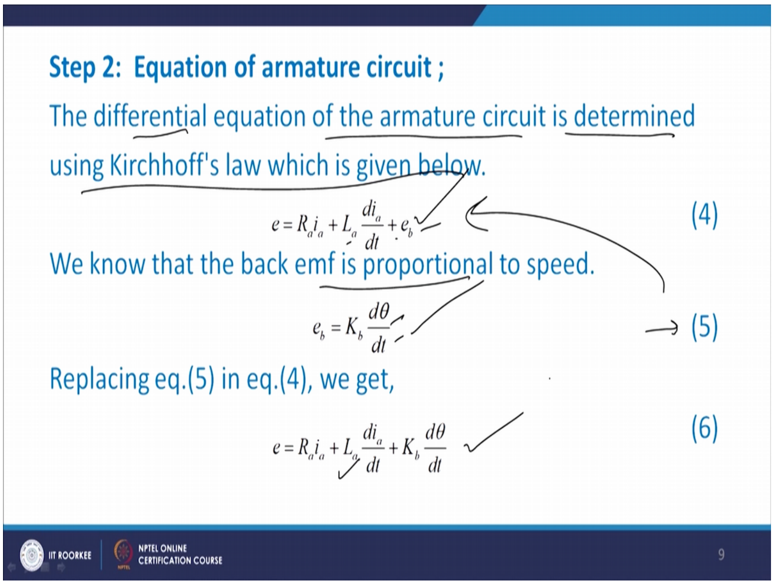

Then equation of armature circuit, differential equation of the armature circuit is determined using Kirchhoff's law which is given below. So, e Ra ia La di by dt plus eb.

Then third we know that back emf is proportional to speed. So, eb equal to Kb d theta by dt, then replacing equation 5 this 5 in equation 4, this is 4. So, we will get expression as e Ra ia La di by bt plus Kb into d theta by dt, step 3 torque mechanical effect.

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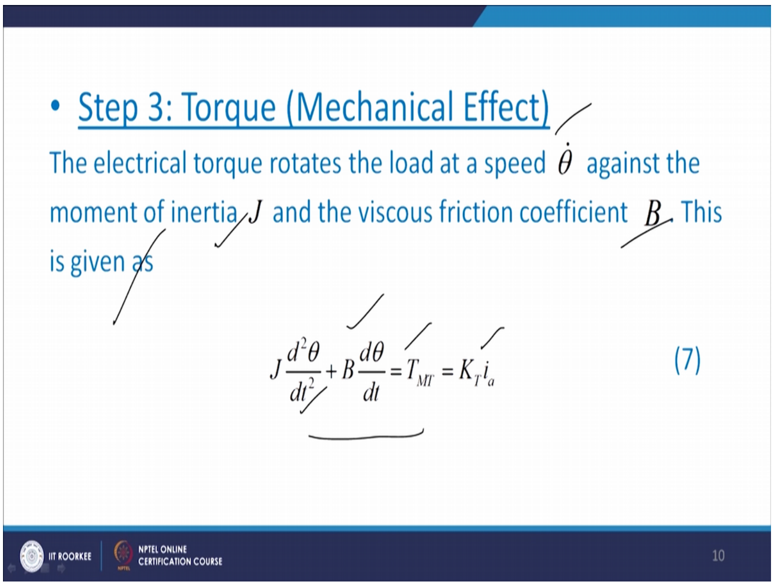

The electrical torque rotates the load as speed theta dot against the moment of inertia J, and the viscous friction coefficient B. So, this is give as like this and J d square theta by dt square, B d theta by dt TMT KT into ia so, this is equation.

So, whatever we have explained earlier, all these so, this is represented here, all this equations. Now we have to develop the model in transfer function form. So, definitely we have time domain so, we have to apply Laplace transform. So, here this all this equation I have written again.

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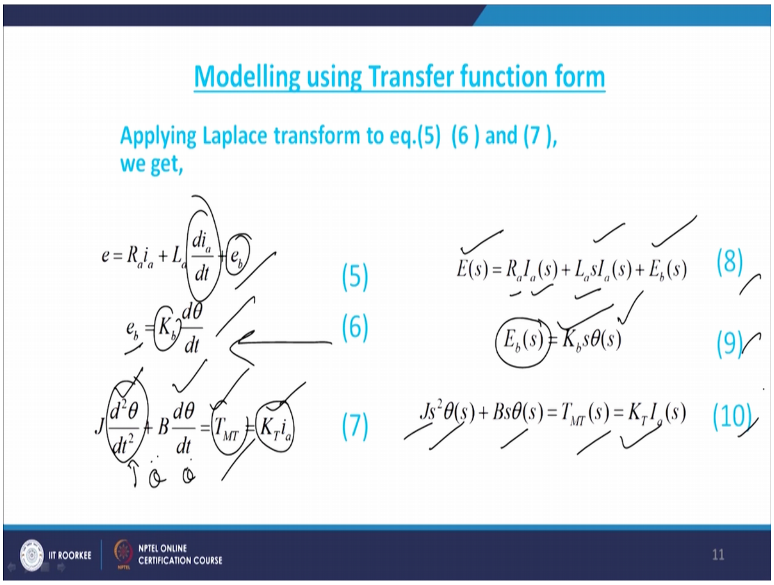

So, that you we can easily write down the expression we has domain. So, here e so, we write Laplace of e E of s Ra in to ia of s the La, this sia s and this e eb equals eb of s. Then about this equation number 6, this eb is represent by eb of s and this constant Kb, and this d theta by dt theta dot is represented by s theta of s. Then motor torque J d square theta dt square, B d theta by dt, motor torque is KT into I a.

So, see here J see here J d square theta dt square that is it is theta we can say dot double dot; that is represented by s square theta s, then d theta by dt theta dot is represented by s theta s and this TMT of s, and here this is KT into ia of s 8 9 and 10, this equations we have got.

So, what we are doing here? See equation number 8, from this equation 8 we are taking I a of s outside so, we have got this.

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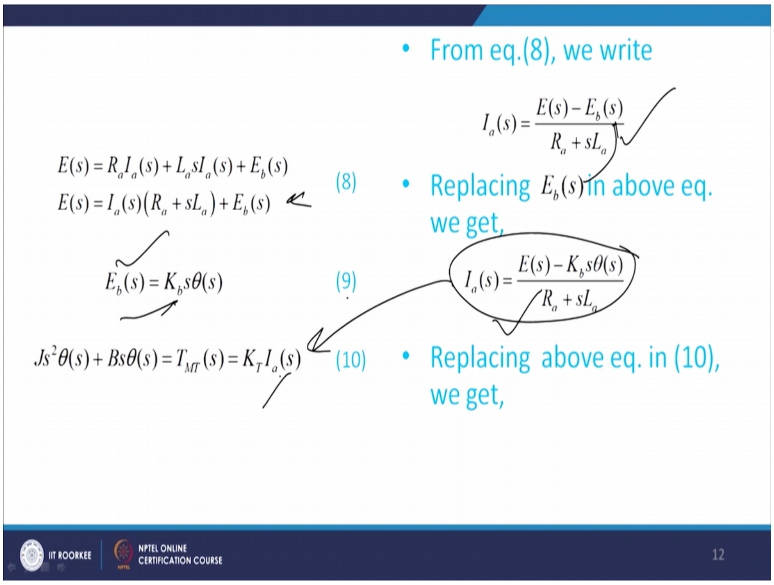

And again we have determined eb of s this is eb of s, this is equation number 8 and now here equation 9 corresponding to this eb of s. And now what we do replacing eb of s here? So, we have got ia of s like this and again replacing the equation of I a of a a of s in equation number 10 this.

So, you will get these equations. So, we will J square theta s plus Bs theta s TMT KT into es this part. This part is basically your ia of s, this is I a of s.

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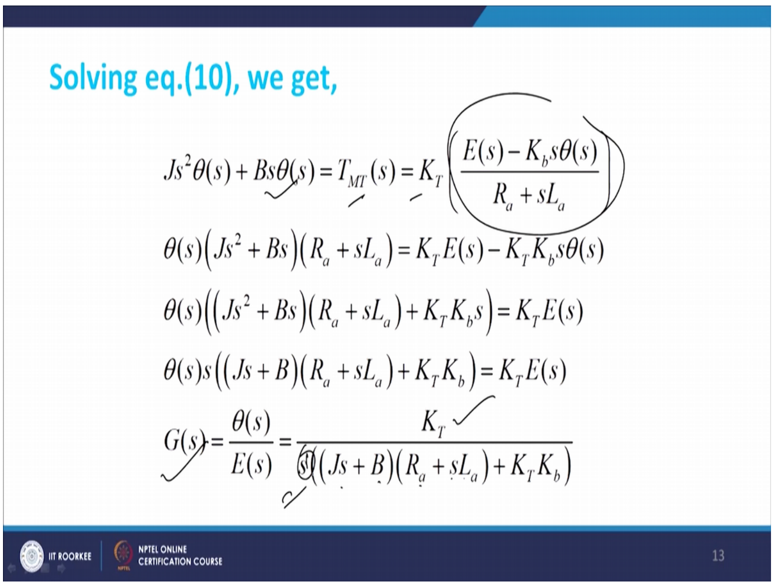

And now we have solving cross multiplying taking some elements common, and finally, we have reached the expression for Gs equal to theta s by Es equal to KT. Js plus B Ra sLa KT into Kb.

So, this is a a transfer function model of a DC servo motor which is in armature control motor. And again we will find that, it will has one pole at origin this is a type one system. And see here open one open with plants here for a position control is not stable. This is a one important point we can observed. So, all the stability issues regarding this we will see later on, but just now the this modelling issues has come so, this is very important point to be noted.

Sometimes instead of position, we required a speed control. So, in that case how to how will you get the model? So, we have got the model like this, this is the third order model see here s s s. But if you go for the speed control, can be get a say third order model or can we get the 4th order model, now this need to be verified.

Therefore, now we are doing here the model of a DC servo motor, armature control that is basically speed control model speed model.

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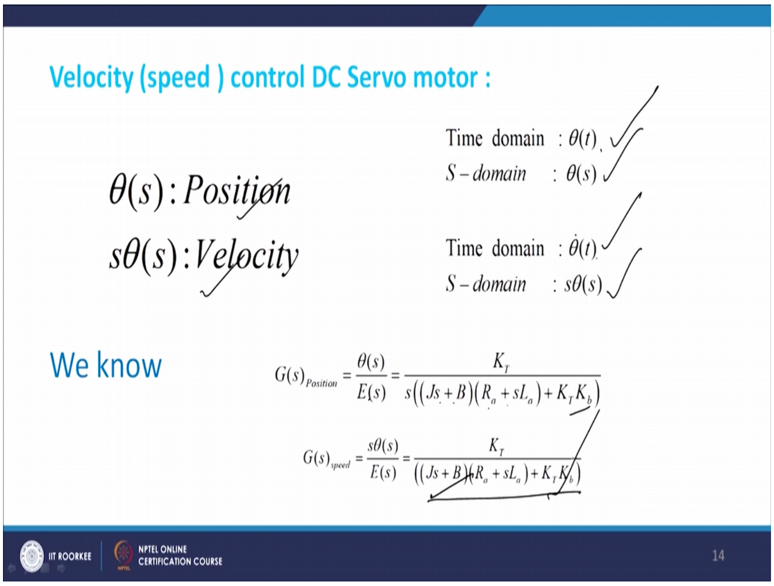

So, theta s is position so, s theta of it is velocity. So, time domain theta T s domain theta theta of s, then time domain you get the theta of d s domain equals to s theta of s. This is for the position and this is for velocity.

So, Gs position equal to theta s by s equals to Ks KT s Js plus B Ra sLa KT Kb that is same expression this expression. And when we are moving to the speed so, what will happen to the speed? Here s theta s by Es; that means, your s is cancelled, and you will find that the model of a DC servo motor that is that is a speed control model is second order whereas, the position is third order.

And we will find that this model this is second order system, and you know that when the all the coefficient positive this system is always stable. So, all these are positive if positive this model is a is stable model that is open loop stable, whereas, this open loop stable model. And this is third order that is armature control model is particularly position control model is third order whereas, speed control model is second order model. Now you just see the block diagram of a position control DC servo motor.

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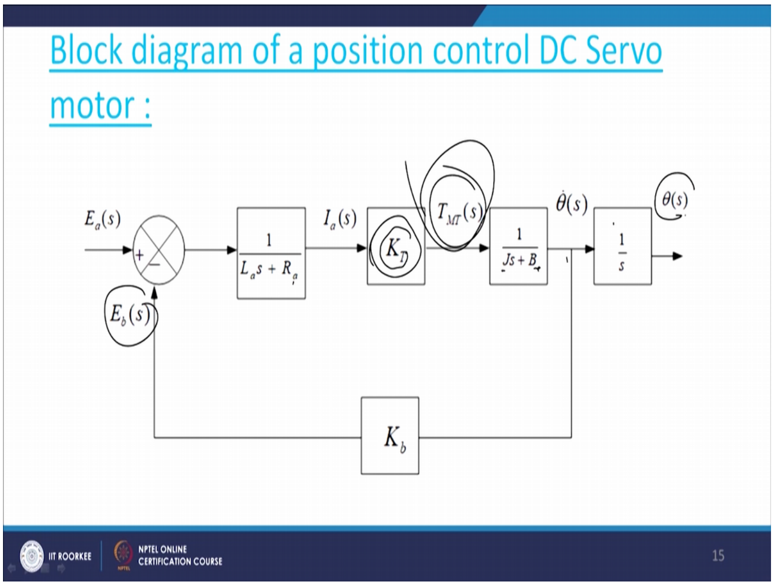

We will find that this is 1 by La s by Ra; that means, you if you see the original figure s we apply input, input it has to go through us Ra and La. And because of this Ra and La, the current is flowing and this current flowing and because of the current flowing a torque is produced and torque is opposed by this J and B.

Therefore, first of all we have write down you have written the expression for La s Ra, then this is constant (Refer Time: 26:17). We have got torque and that torque is opposed by a 1 by Js plus B, and this is a back emf. And if you want a speed a position we have got theta s 1 by s. So, this eo ea of s input opposed by back emf, and because of that current is flowing we drive through this las by Ra.

Then is a motor torque constant, so that means, a motor torque is produced, and that has to drive the motor against 1 upon 1 by 1 by Js plus B, this is a velocity and this is we can say a positions.

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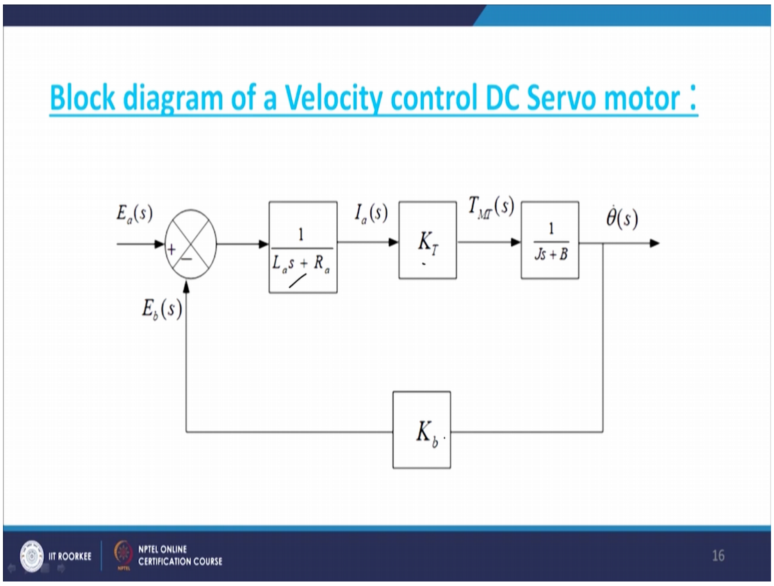

Now, block diagram of a velocity control DC servo motor same thing, what we have done here, here theta dot s same 1 by ls Ra KT Js plus B and this Kb.

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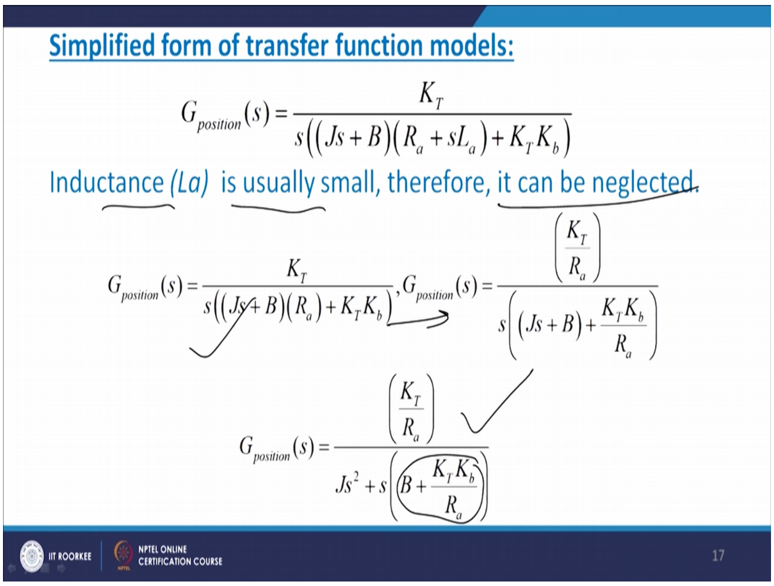

Now, the simplified form of transfer function model. As I have seen that the model that is position control model is third order. In control literature we will see that there are various modern ordination techniques are available. They can reduce the model higher order to lower order model. So, there are some techniques are available, but can we do that some model itself just looking after read can it reduce to second order or first order, without using any techniques. You just see the model here in this model we have G position is s Js plus B Ra sLa KT Kb.

But you know, if you take the armature inductance inductors of armature it is very small so, that we can neglect it. So, here we are neglected this La. So, what we are written here? Inductors La is usually small. Therefore, it can be neglected. So, here after neglecting La you will find we get this expression. Again we have some modified this one, and finally, the position control model we have got like this. And here this B KT by Ka k is represented by another constant.

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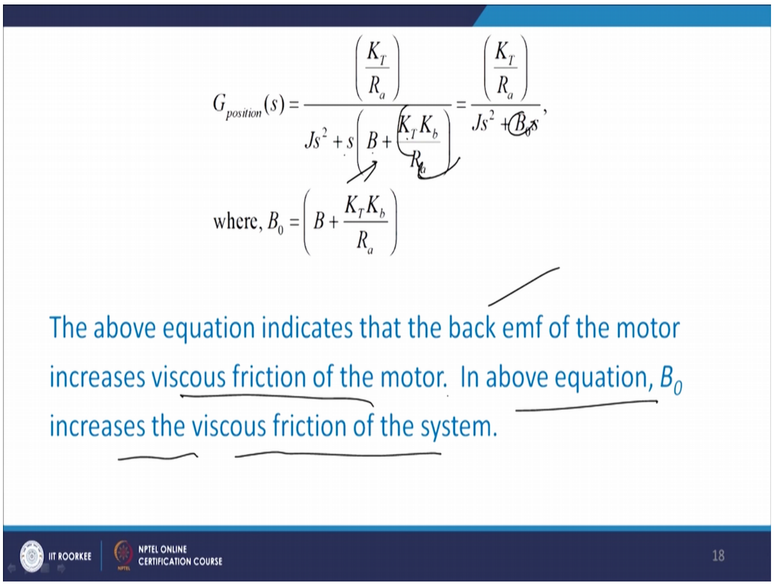

So, here G position s equal to KT by Ra Js square s B KT Kb Ra that equal to KT Ra Js square B 0 s, where B 0 equal to B KT Kb Ra; that means, it is B 0 is like this.

So, B is the friction which opposes a motion so, what we will happened? Here KT Kb these are responsible for the back emf so, what happened? Because of this back emf the viscous friction is also increases. So, above equation indicates that the back emf of the motor increases viscous friction of the motor. So, in above equation B 0 increases the viscous friction of the system.

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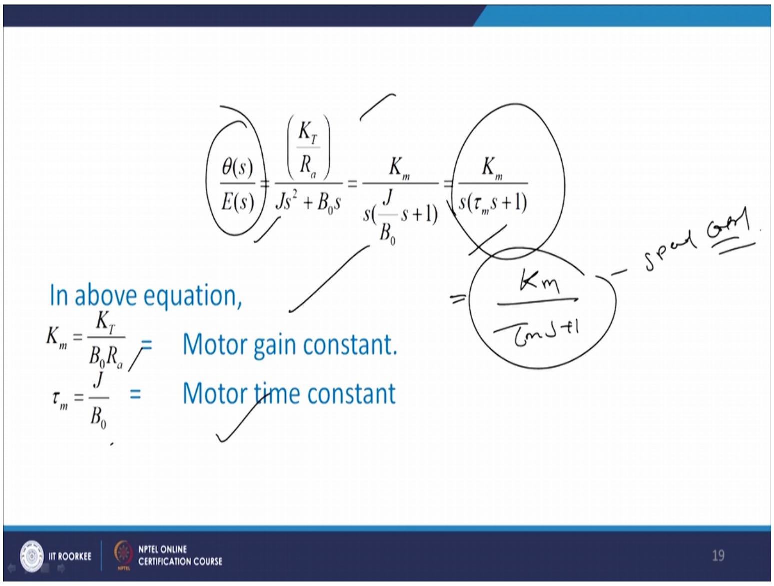

And again it has been simplified and we have got the Km equals to KT B 0 by Ra motor gain constant and tau m J B 0 motor time constant. So, that is the and simplified model of an armature control DC servo motor particularly DC is a your position control model. And if you on the speed control model so, we will get we can write down as Km tau m s plus 1.

So, this is model is a speed control. And this speed control model it reduce to first order so, see a this is a motor, and this particular motor model, we are used to only first order particular speed, whereas, in case of position control it become a second order model. And therefore, a suppose in motor what is the requirement, requirement is to a speed suppose we want a speed of 1500 rpm.

And suppose we are not getting exacts speed so, what we have doing? We are we design a controller so, at that time this model which we determined. Now, this is helpful in the designing of the controller.

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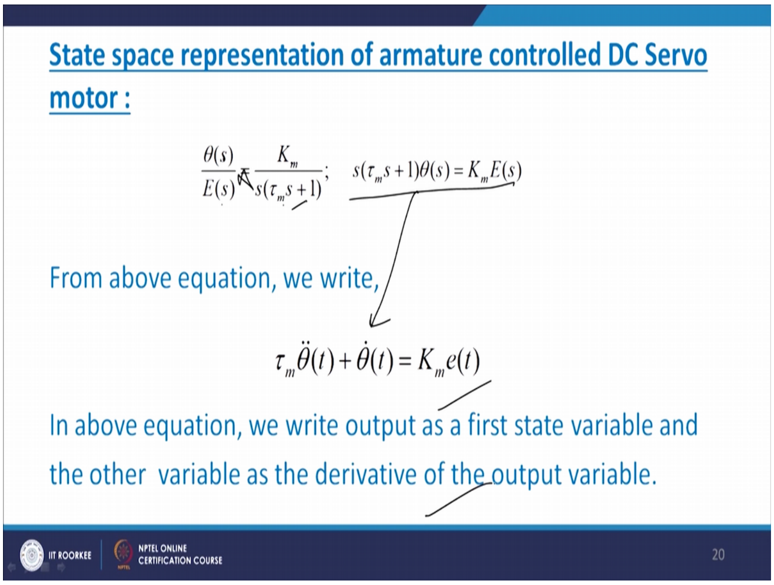

Now, this is the model which is concern with the transfer function. But now we are running the advance control, for the advance control; we are in the state space modelling. So, in the state space model, we have seen that based on the system we have to assume the state variables.

So, here now the state space representation of an armature control DC servo motor, that is same system which we are determine earlier. So, we have to determine it is state space model so, what we have done? Theta is by s equal to Km s tau m s plus 1 so now, we have done cross multiplication.

So, this is cross multiplication, after that after solving this we have got tau m theta dot t plus theta dot t equal to Km into e of t. In above equation we have write output as a first state variable, and other variable as the derivative of the output variable.

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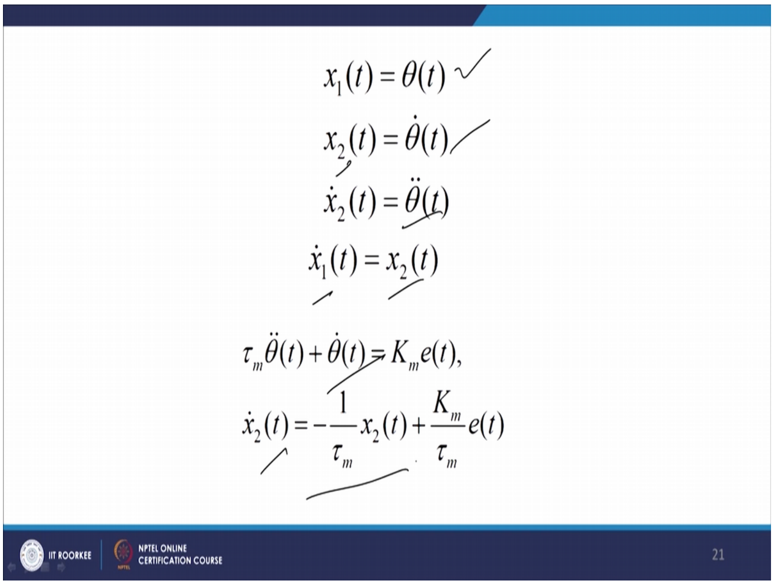

So, here we have taken x 1 of t equal to theta of t, x 2 of T equal to theta dot t, then x 2 dot of t equals to theta dot t, then x 2 dot of t equals to theta double dot t. So, because why because here we theta dot t.

Therefore, x 1 of t equal to x 2 of T, and then after solving this that is, this equation this particular equations v comma cross the value of x 2 dot t is like this.

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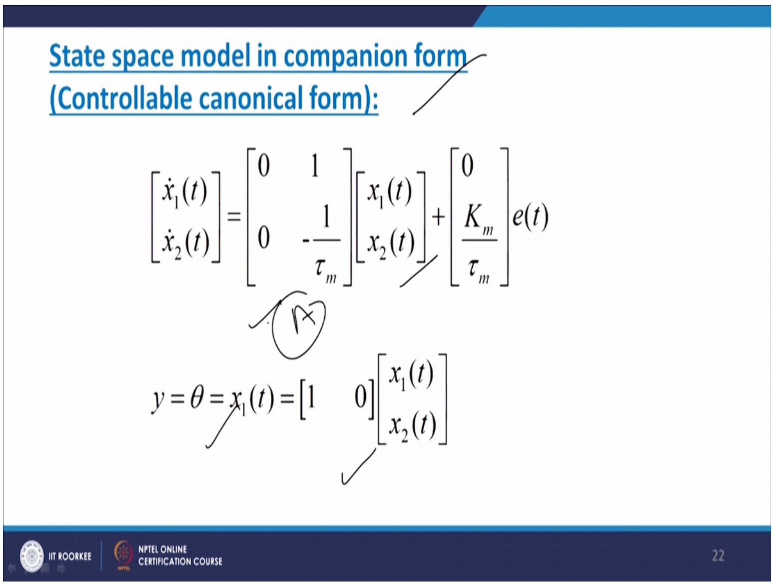

And finally, this model has been presented in the comparison form or controllable canonic form. So, we have got this particular this; that means, we have started with the transfer function model, that is a classical approach and the from classical approach we have develop and state space model, now this is a one type of approach.

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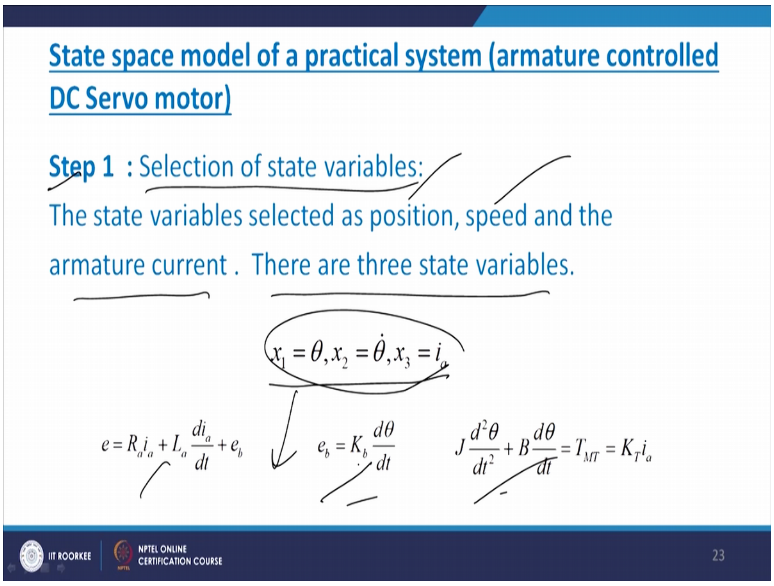

But in sometimes we can design a model or we can develop the model, based on the variable which is involved in our system. As we have to seen last the earlier classes, that we have to considered the variable as the ni storage element or very important variables. So, here we will now develop the model of a DC servo motor in a state space considered the variables. And we will see is there any difference so, first of all we wills we will consider state variables. So, in this case, that is a step one selection of state variables. The state variables selected as position, speed and the armature current so, there are 3 state variables.

Because we have got expression into from d theta by dt d square theta by dt square, and how much of current is also very important here. Therefore, we have considered this 3 state variables. And we have order determined the equation for armature circuit back emf, and a torque equation of a motor we refer to the moment of inertia and a friction so, these are 3 equations. And now these variables we have to adjusted in these equations.

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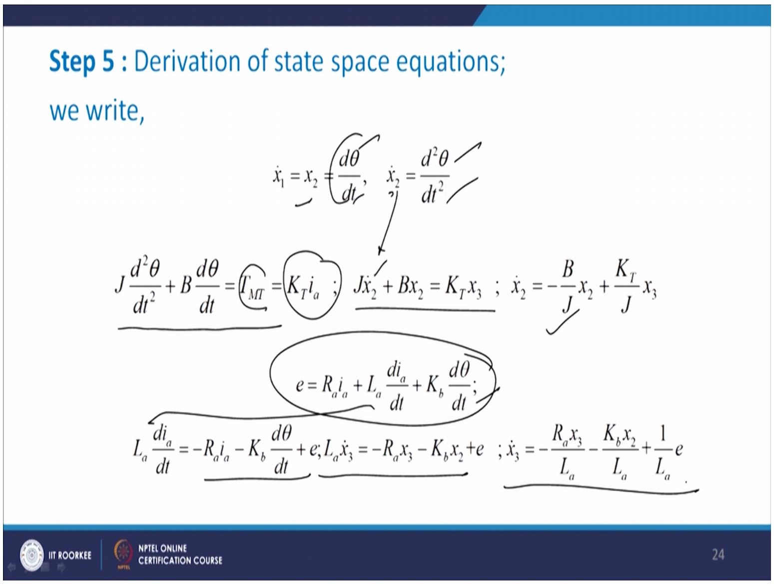

Now derivation of state space equation so, we write x 1 dot equal to x 2, d theta by dt then x 2 dot equal to d square theta by dt square.

Now, this equation J d square theta by dt square B d theta by dt equal to T MT T means motor torque constant. That is equal to KT into ia so, here J d square theta by dt square represented by x 2 dot see here this. Then d theta by dt this is dt theta by dt equal to x 2 we replaced by x 2. And here KT into ia into x 3, and now after solving this equations we have got this x 2 dot equal to minus B by J x 2 plus KT by J into x 3.

Now, this is an equation for an armature circuit. Sorry, e Ra into ia La di by dt plus Kb d theta by dt, and now again we have taken this part here, and other part of right sides right side we have got this. And again replacing the all the state variables, we have got the expression for x 3 dot like this.

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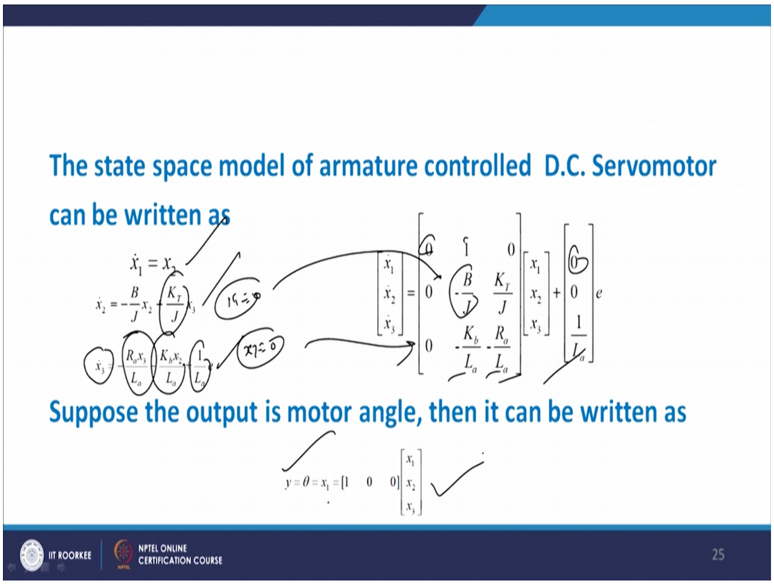

Now, the state space model of a armature control DC servo motor can be written like this. So, same x 1 dot x 2 dot x dot; that means, what are we have determined here? X 1 dot x 2 dot x 3 dot and x 1 dot x 2 dot x 3 dot, we have written like this, and from this x 1 dot equal to x 2.

So, here 0 1 0 or the input is 0 then x 2 dot equals to minus B by J is to see here. X 1 term is 0 here so, for it is 0, and here KT by J x 0, then x 3 dot this is x 3 dot equal to. So, it is a term of x 1 just see here, J x 3 x 2 again here term of x 1 is 0. Therefore, we are replace by 0, and here term of x 2 we have written as minus Kb by La, and this term of x 3 we replaced by Ra by La and here particularly one La represented by like this.

Suppose the output is motor angle. Therefore, it can be written as y equal to theta, theta is x 1 and this is expression.

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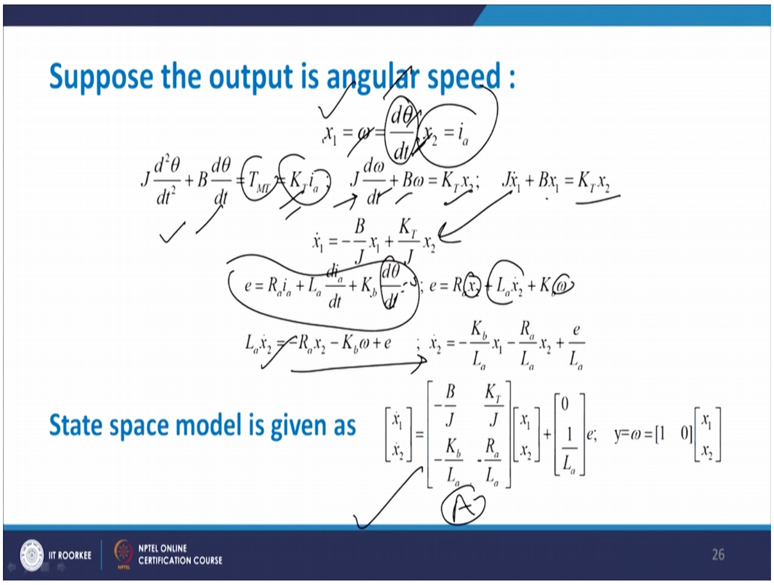

Now, here we want output should be this is like a speed just like position. We have taken now it is speed, now we have variables we have to change. So, here now my variable is x 1 equal to omega equal to d theta by dt. That is the first variable d 3, there is no need for position variable here, we because we required speed.

If your position we may have taken x 1 equals theta, which was seen last time, but here our purposive speed. Therefore, what we have done we have taken d theta by dt equal to x 1, and second variable because we have to control armature also armature current. Therefore, x 2 equal to ia therefore, we have writing the expression J d theta dt square B d theta by dt motor torque KT into ia, and now here d theta by dt equal to omega.

So, it has been replace like this J d omega by dt, B into omega KT into ia into x 2, and here J x 1 dot B x 1 KT into x 2. And now this equation is solved. Then equation of an a armature circuit e Ra, what is ia? Ia into x 2, La di by dt x 2 dot and Kb into d theta by dt equals to a equal to omega. So, we have written omega and again we have solve it we have got this equation and form this we have got this. The x 2 dot equal to minus Kb by La x 1 minus Ra by La x 2 plus e by La so, this is the expression.

So, state space model is given as like this. You just see here we have got 2 state space model, one state space model we have determined from the transfer function approach and one state space model we have determined by this by this actual approach that is actual state space approach.

Then can we get the same result, or can we get the same roots of the matrix, that is matrix A, this is matrix A, and earlier also we determined this matrix so, this is matrix A we have determined.

So, can we get the same Eigen value or same performance so, this is a very important issue. So, in future when we learn the stability and performance, we will again come with this modelling part or again we take this model and we take the performance. But here for modelling point of view we have got models. So, that means, the state space model we can determined form the given transfer function, or we can determined from the state space models.

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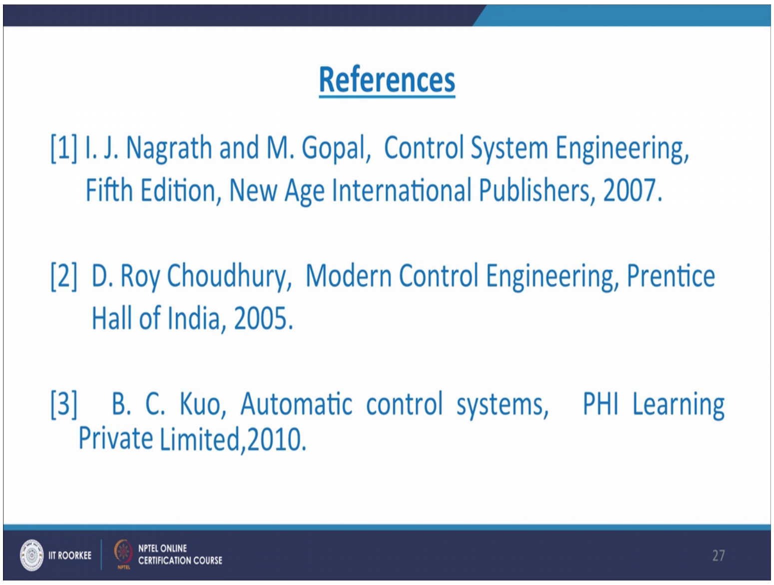

Now, these are some references.

Thank you.