Advanced Linear Continuous Control Systems
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Lecture – 10
State Space Representation:
Numerical Examples on State Space Modelling (Part-II)

Today, we start with again Numerical Example on State Space Modeling. Till date, we have seen examples on lower order systems. But we know that practical systems are of higher order. For example, a power system model power system model you can say it is 100 order, 500 order. So, in that case we can it is possible for us to the modeling of the system. Therefore today, I am taking some examples on higher order system; along with this some other example which are important that I am also taking.

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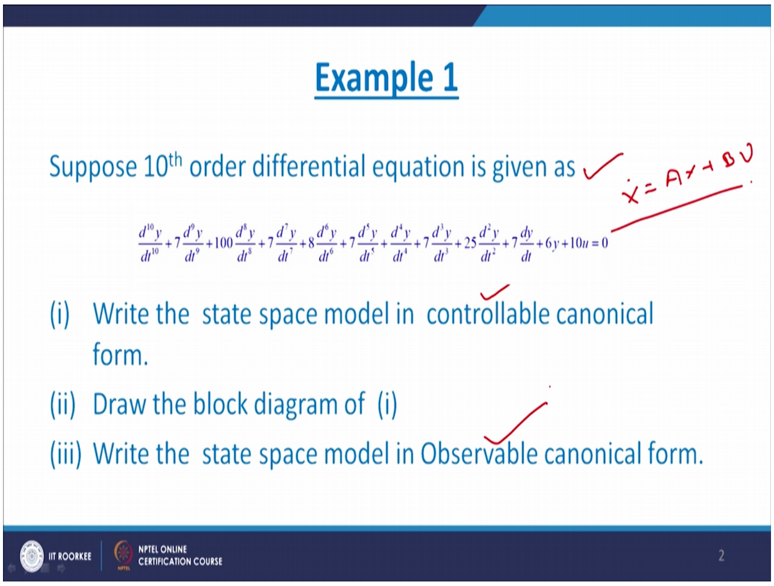

Start with example 1. Suppose, a 10th order differential equation is given as so, this is 10th order differential equations. Now, here our main purpose is to determine the model of this differential equation in state space form; state space form X dot equal to Ax plus BU. This form, this we want. Second, we have to draw the block diagram for this and third, we required write the state space model in observable canonical form; that is we required in controllable canonical form, also in observable canonical form.

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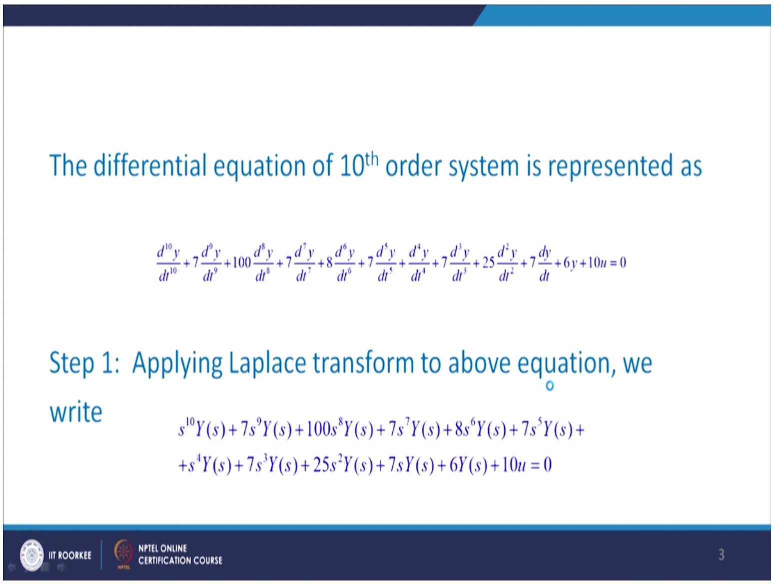

So, differential equation of 10th order system is represented as, this is 10th order system. So, you have seen last time that whenever any differential equation is there, we have to apply Laplace transform. So, here for this 10th order system we will apply Laplace transform. So, applying Laplace transform to above equation, we get this. (Refer Time: 02:17) as the 10th order as such to 10th s into 0 10 u equal to 0. Now the simplified form of equation is represented as. So, what we have done? We have taken Y s common; this Y s; this we have taken common and we have got these equations.

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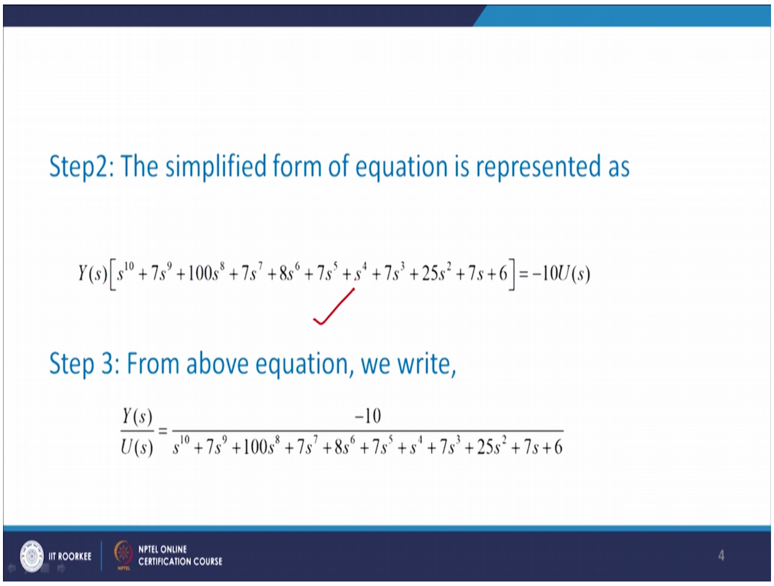

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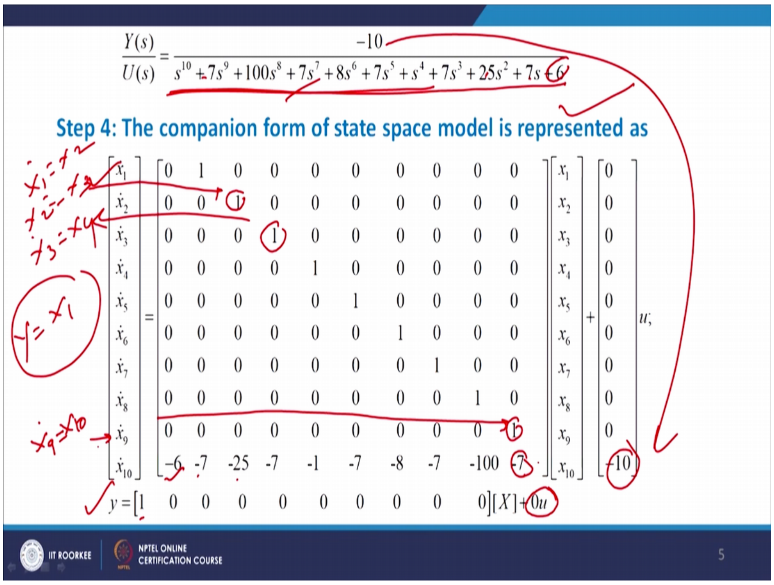

Now, from above equation Y s by U s; this is Y s by U s, we have got this equation. See here, 10th order equation; this minus 10 has come here this side. The companion form of state space model is represent as; now we have to represent this equation in companion form or controllable canonical form. This controllable canonical form, space variable form, companion form; they have same meaning. In many of the textbook they have written companion form; in some of the textbook it is written as controllable canonical form.

So now, we have to write already we have seen this part. So, now, we are writing down the state space model in companion form for this model. So, we are in x 1 dot, this is 0 1. So, what we are done here x 1 dot equal to x 2; that is why it is written as 0 1. Then here, x 2 dot equal to x 3. So, we have got 0 0 1. So, here this element one corresponds to x 3; then x 3 dot equal to x 4. So, you see here 1 is corresponding to x 4. Similarly, if you go for this x x 9 dot; this x 9 dot equal to x 10.

So, you will find at 0 0 0 0 1 and this is 1 has corresponding to this x 10. And about this denominator, we have to write down in a reverse order. Here, this is 6 within minus 6 7 minus 7 minus 25 minus 25 and lastly this element is minus 7 and this b matrix 0 0 0 0 minus 10. So, this minus 10 will appeared here and what are the output? So, output here Y equals to X 1 because the basic philosophy of companion form or controllable canonical form is based on output as a first variable. So, here X 1 equals to Y. Therefore, you defined it Y we written has 1 0 0 0 0 and 0 0. As I told last time, when there is when the order of the denominator is more than the order of the numerator, we get 0. But whenever both are same, then some elements will come here d is 0.

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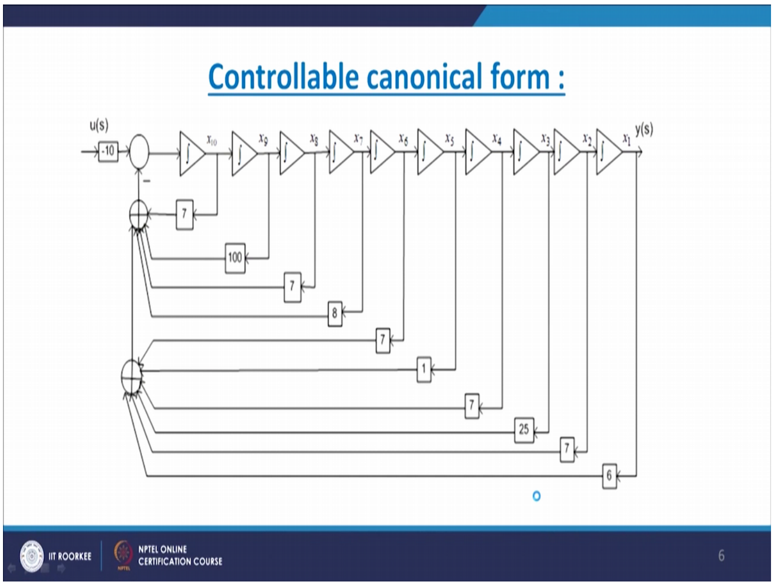

Now, just see here. This is a block diagram of a controllable canonical form that is companion form. So, as the order of the system is 10. Therefore, how many integrators are required? So, we require 10 integrators; so, here integrator 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; 10 integrators and see this element 7, 100, 7, 8, 7, 1, 7, 25, 7, 6. So, you will find at this element are nothing but the coefficient of the denominator. So, (Refer Time: 05:57) of your represent like this.

So, this is corresponding to x 10 dot. So, this is nothing but x 10 dot. So, x 10 dot equals to 7 x 10; 100 x 10 like this and what about is Y equal to X. So, here x 1 we have shown and about U; U is x 10 dot equal to minus 10 into U. So, it has been shown like this so, Y s by U S. This is nothing but these particular equations.

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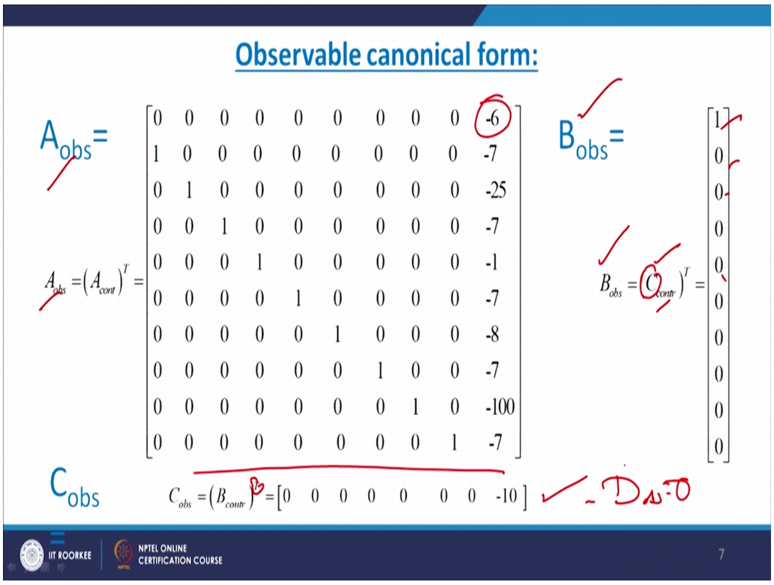

Now, about the observable canonical form; last time I have explained. Observable canonical form is also useful in state space analysis particularly in checking in checking observatory, also when we need to design observer, this type of form is useful. Therefore, here we have written the observable canonical form for this particular system; but will be write the observe form, it is better. First of all, we write in a companion form and then you convert it because it is very easy.

So, see here A observable. A observable equal to transpose of A controllable matrix. You see here this is your A control matrix and here this we have to make transpose of it. So, transpose of it means that this minus 6 which is here, it will go this side; you see here this minus 6. So, you will find at 0 0 0 minus 6 and up to 0 0 minus 7. Here, we have simply make the transpose of A controller matrix. Now, would B observable. B observe means transpose of controllable matrix means matrix which is obtained in controllable canonical form that is C matrix.

So, transpose of C matrix which is the controller which is in a companion form or controllable canonical form is represent 1 0 0 0. So, you will see that this is 1 0 0 0 0 by transpose of it and we make transpose we will get 1 0 0. Now, about the C observable matrix; So, C observable matrix equals to B; a controllable transpose of B controllable canonical matrix. So, B controllable canonical matrix; this is transpose. So, what is the elements of B? B involved 0 0 0 0 0 0 minus 10 and we make transpose, you will get 0 0 0 0 minus 10 and here, D is d observable s equal to 0.

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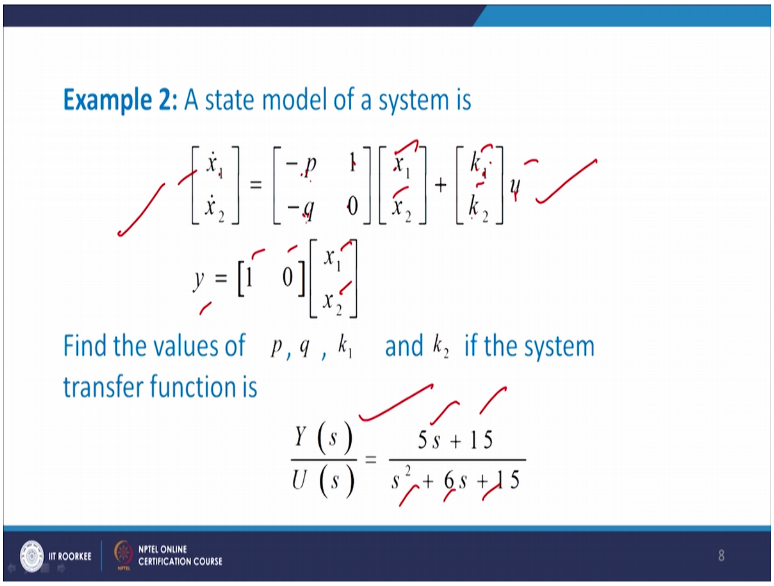

Now, let us take the another example till date what we have done? We but we have done that we have taken a particular state space model and from this we have got some transfer functions. Now here, we have given the transfer function model and state space model is also given; but there are some unknowns. So, we will see that in this example 2. Here x 1 dot x 2 dot minus p 1 minus q 0 x 1 x 2 k 1 k 2 u and y equals to 1 0 x 1 x 2.

So, in this example a nodes are minus p q k 1 and k 2 and a transfer function model Y s by us is given as 5 s plus 15 s square plus 6 s plus 15. So, using these two part, we have to determine p q and k 1 and k 2. So, how will you do it? The first step; first step is we will solve this equations this is x 1 dot equal to minus p 1 x 1 plus k 1 into u and what is the x 2 dot x 2 dot equal to minus q x 1 plus 0 into x 2 plus k 2 into u; just see here this we have written.

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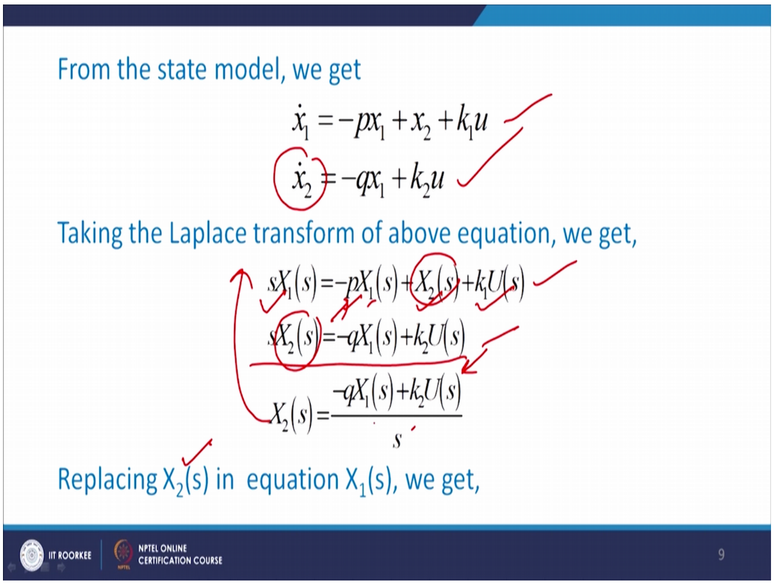

As this is in, this equation in time domain; so, when time domain is there, if you want in s domain we have to take its inverse Laplace transforms. Oh sorry. We have to take its Laplace transform. So, Laplace transform is s X 1 of s p X 1 of s X 2 of s k 1 into U of s; see here k 1 U of s. Now, the here Laplace of this is x x 2 of s and here one important thing that we have neglected the initial conditions; so, here s X 2 of s minus q X 1 of s plus k 2 U of s. Now, see here this equation. So, this is equations. So, we have to solve this equation. So, with the best option is here, we will find, we replace the value of X 2 of s in this equation.

Therefore, this equation is represent as X two of s equal to minus q X 1 of s plus k 2 U of s divided by s. So, replacing X 2 of s in this equation, we get s X 1 of s minus p s X 1 of s you see here is p X 1 of s. Here, we are replace this. So, you will find you will get this plus k 1 in into U of s; so, k 1 into U of s.

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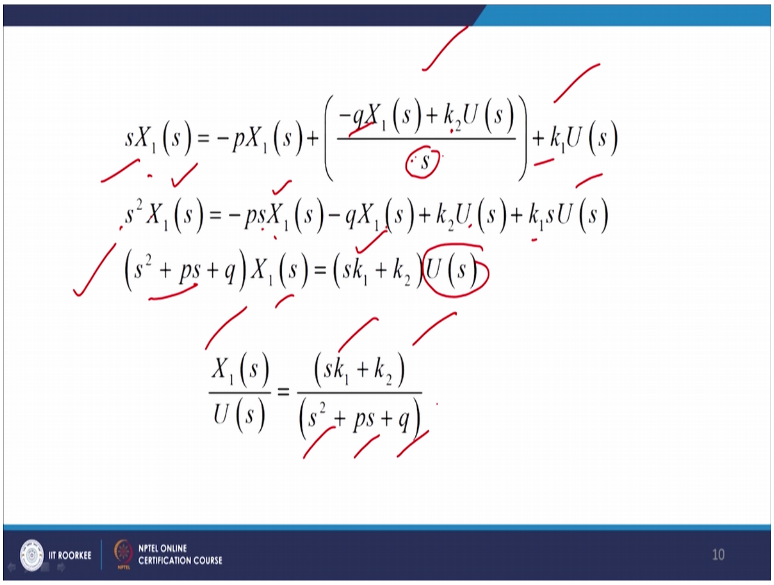

Now, further we have solved. So, how you solved? This s is multiplied both side; this side as well as this side; When you multiply as this side, will get s square X 1 of s p s X 1 of s minus q X 1 of X 1 of s k 2 U of s and k 1 s U of s. Now, what we have done here? We have taken the terms ready to X 1 of s 1 side. So, you see a X 1 of s X 1 of s X 1 of s. So, term related to this X 1 of s, we have taken this side and here terms with respect to U of s, we have taken this side plus here.

X 1 s by U of s will get s k 1 plus k 2 s square p s plus q. So, this is the equation, we obtained and you have given this equation. So, we will find at there is similarity between these two equations. What is the similarity? This is second order that is denominator is the second order; numerator is also first order and here denominator is also is second order and numerator is first order. So, we can compare both these equations. So, here we have written and here Y s by U s and here y t equals to x 1 of t because here you will find at y equals to X 1 plus X 2.

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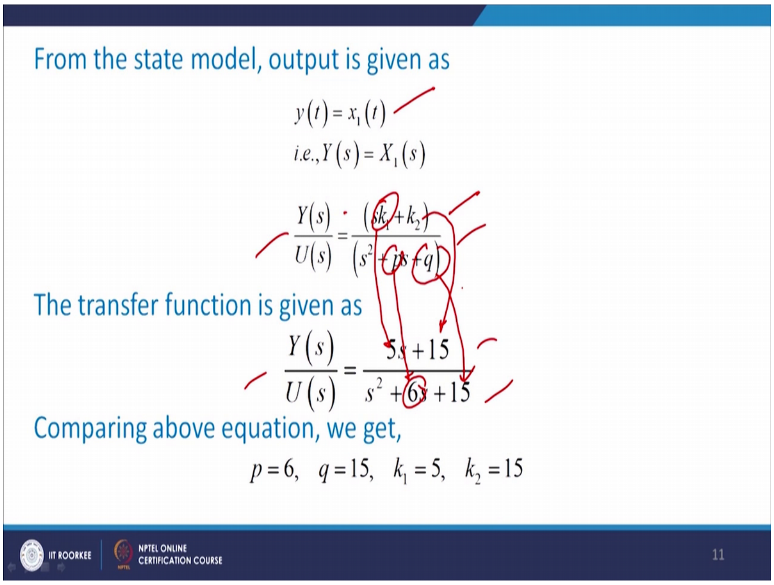

So, here that is why we have written here Y of s equal to X 1 of s and this is Y s by us like this and now we have compare. So, after comparison will find at this is p; p s this is 6 s. So, p corresponds to 6; Now, here q; this q corresponding to 15 and this k 1 corresponding to 5 and k 2 corresponding to 15. So, in this way, we have determined the unknown in the state space model. Now we start with the, another example.

Up till now, we have seen the different form comparing form that is controllable Canonical form; then in a Diagonal form, Jordan form. So, they are different type of form can also possible. Therefore, here I am taking one example. So, by if you solve this example you are getting some different form.

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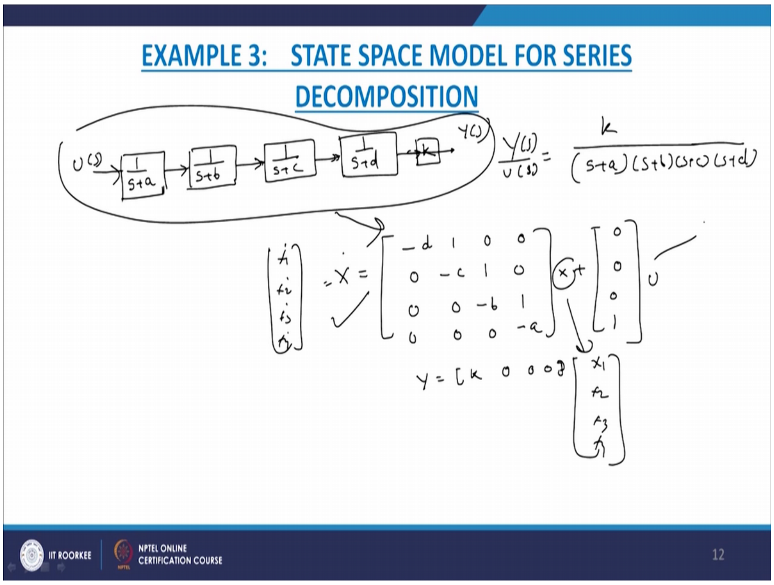

So, just see this example. Now here I am drawing one block that block is 1 upon s plus a. Now we will take another block 1 upon s plus b. Now, I am taking another block s plus c; I will take another block 1 upon s plus d. Now here, arrow I am showing. So, here we have we applied U s and output is Y of s Y of s and here, I am considering another block 1 k. So, here my aim is that the Y s by U s, in this case we should get as as k upon s plus a s plus b s plus c s plus d; this is Y s by U s.

And here, we have to prove that x dot x dot equal to minus d 1 0 0 0 minus 6 minus c 1 0 0 0 minus b 1 0 0 0 minus a. This x plus 0 0 0 1 and output y equal to k 0 0 0; x 1 x 2 x 3 x 4; this same this x is also like this often. This x dot means here x 1 dot x 2 dot x 3 dot x 4 dot. So, your main purpose is that this is a plant or a system and this system; we have to represent it in this form. So, this system can be represented in a companion form, canonical form we have see last time. Now this is a another type of form.

So, this is nothing but some series connection; if the series connection is there, we get s dot equation like this. Now, how to prove it? So, we can prove by a way that is see here s plus a 1 upon s plus b s plus c s plus d. So, s is there 4 s; that means, that there are 4 states; it means that there are 4 integrators. So, here we have to assign states.

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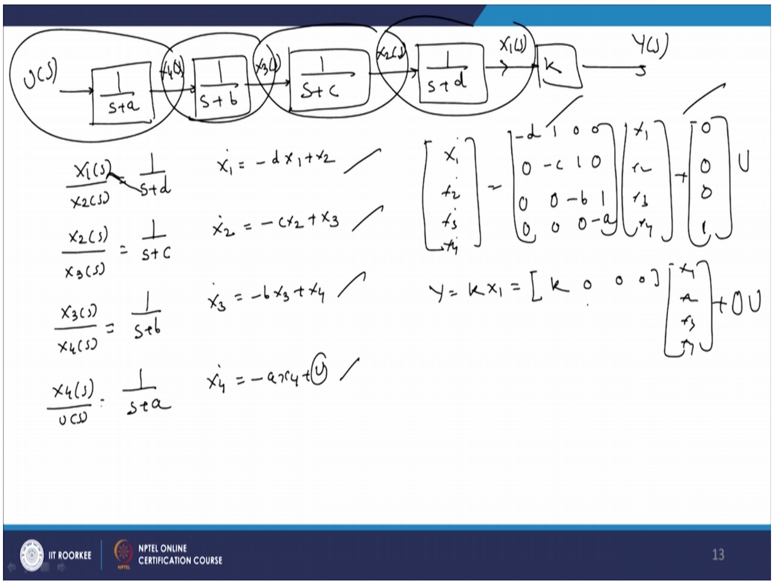

So, first of all, we assign the states to this plant. So, how do you do it? Let us see. 1 upon s plus a 1 upon s plus b 1 upon s plus c 1 upon s plus d; this is k; this is Y s. This is input U of s; I am showing arrows have been shown a b c d k Y of s. So, this is Y of s. Now, we have to show the steps. So, normally we have seen, we have started all state with X 1 X 1 X 2 X 3 X 4 like this. So here, we will show state as X 1 of s. Now here integrator means is x 1 dot of s. So, we can write this as X 2 of s; then here, we write X 3 of s and here we write X 4 of s. So, we have written X 1 of s, X 2 of s, X 3 of s and X 4 of s.

So, now we have to solve it. So, how to solve it? So, we will find that here, we write this block this repetition. So, this can be represented as X 1 of s divide by X 2 of s that is equal to 1 upon s plus d. Now you cross multiply it; then take its inverse. So, what will get? We will get X 1 dot equal to minus d into X 1 plus X 2 here; This s X 1 of s X 1 dot. Now d into X 1 so, will go right side, it become minus d X 1 plus X 2, know about this block, this block.

So, this block is X 2 of s divide by X 3 of s. They are equal to 1 upon s plus c. So, again we solve it that is again we apply inverse Laplace. So, what we will get? We will get X 2 of x 2 dot equal to minus c X 2 plus X 3 along coming to this block. So, this block is X 3 of s divide by X 4 of s that is equal to 1 upon s plus b. Now again, the cross multiply taking inverse. So, what we will get? We will get X 3 X 3 dot equal to minus b X 3 plus

X 4 and now regarding this block that is X 4 s by U of s. So, X 4 s by u of s, we can write as 1 upon s plus a.

Now again, we cross multiply it. What we will get? Will get X 4 dot equal to minus a x 4 plus U. So, here we got 4 equations X 1 dot X 2 dot X 3 dot and X 4 dot. Now, we have to represent in a state space model. So, how you represent it? We have X 1 dot X 2 dot X 3 dot X 4 dot that is equal to the right X 1 X 2 X 3 and X 4 and this is terms for U. Now, here what is X 1 dot? X 1 dot equal to minus d X 1 plus X 2; so minus d. So, we are right here minus d and what is a term or X 2 is 1; the rest of the elements are zeros and corresponding to U element is also 0. Then what about X 2 dot? X 2 dot equal to minus c X 2 plus X 3. So, we write as 0 minus c 1 0.

Now, what about X 3 dot? X 3 dot equal to minus b x 3 x 4. So, here term of terms of x 1 and x 2 are 0. So, here I am writing down 0 0 minus b 1 and lastly x 4 dot equal to minus Ax 4 plus U ; that means, here term of x 1 x 2 and x 3 are 0. So, when 0 0 0 minus a; but here U is there. So, U is 1. But let us see that about other x 2 dot x 3 dot the terms of u is 0. So, here only in x 4 dot term someone has come 0. So, u so, what is my what is the requirement? This minus dc b a 0 0 1; so, this we are automatically prove and will find that. And the important feature is that corresponding to each element a b c d, we have we are get the elements of 1 1 1; this is the arrangement.

Similarly, if you increase the blocks so, we can get it say d minus c. So, around this will get 1 if f 1 like this will get it. Now, about the output so, output Y equal to k into x 1. So, we can write down as k 0 0 0 x 1 x 2 x 3 x 4 and definitely there is 0 into U. So, this is a b c matrix. Now we start with another example on a MIMO system, multi input multi output system. Most of the process control from, are of MIMO type. Therefore, if your system a in a transfer function form or we have a differential equation which is in a having a multi input multi output in that case also we can determine this step is a model of the system and will find one unique feature in this case.

Unique feature is that if you take any model in a in a transfer function, we obtain the order hundred order; but here in MIMO system particularly when represent state space form or driving order is always first order; first order model only. There is no miss even 1000 order system is there in a state space order is always first order.

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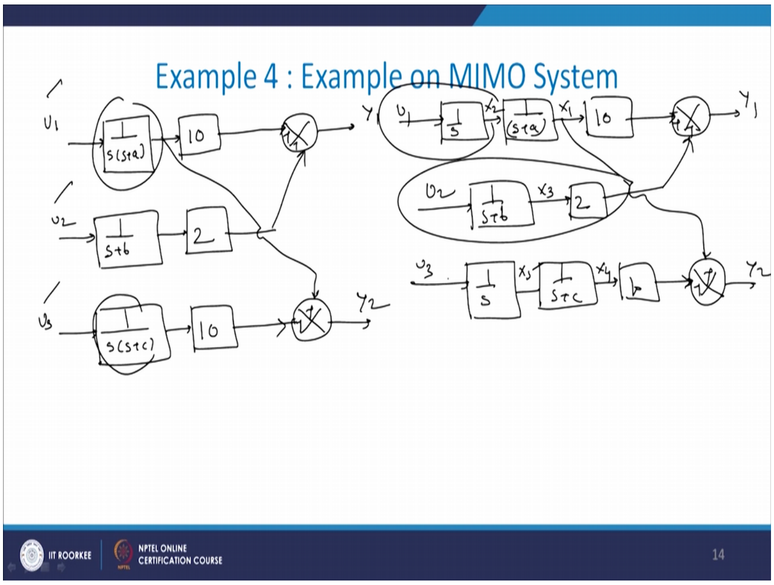

Now, here we solve an example where here we show that for a MIMO system also, we are getting this state model in a first order form. So, we start doing this example, see now here I am taking one block U 1. This is one input; 1 upon s s plus a. Now here I using 10; then here is a block I am showing. Here is Y 1. Now, another input is coming that is U 2 and here that transfer function is s plus b. Then here, is 2. Now it is connected here.

Now this U 2 is connected to Y 1. Then you take another input say U 3 and here the transfer function block is s upon s plus c and here again obtained and it is showing to this cement block. This positive, positive; here is also positive, positive in from here this path is coming from here, here and is connected here and here is another connection and this is output Y 2.

So, here output is Y 1 Y 2 input as U 1, U 2, U 3; blocks are 1 upon s s plus a 1 upon s plus b 1 upon s s plus c again as we shown as 10 to 10. Now we have to solve this problem definitely when 1 integrator is there is 1 state; in a 2 indicators are there are 2 states. So, will find at if you separate this equation just now we have seen last example. So, it is possible that we can simplify this part and easily we can get the state space model. So, how we will do it? So, here I am re diagram the block diagram again. Here, I am showing 1 upon s 1 upon s plus a, then 10 and here is this summing block and this is Y 1 here is thing is same no change that is 1 upon s plus b, these 2 and here it is connected like this.

So, here U 2 U 1; So, here one change away around this 1 upon s s plus s, it has been separated as 1 upon s plus a U to s 1 upon s plus b 2 and lastly, this is U 3. It has been also be separated as 1 upon s 1 upon s plus c gain of 10 and here is a block summing block. So, you see positive values and the air connections. So, from where it has come. So, it we you see that from this; it has come from this side, this side. So, here input U 1 is connected to output Y 2.

Now we have separated. Now we have to assume the state. So, what happened? Here I have taken 1 state x 1 and now here we can take state as x 1 dot. So, that x x 1 dot equals to x 2; then here of for x 1 x 1 here is also an integrator. So, we can take state as x 3; then coming here 10, we can take the state x 4 and here we can take state as x 5. So, here all the states has been defined. Now what you do? We have to take x 1 as by x 2 of s x 3 is by U to of s x 4 by x 5 of 5 of s; then x 5 by user. So, all the equation we have to write or individual transformation we have to write.

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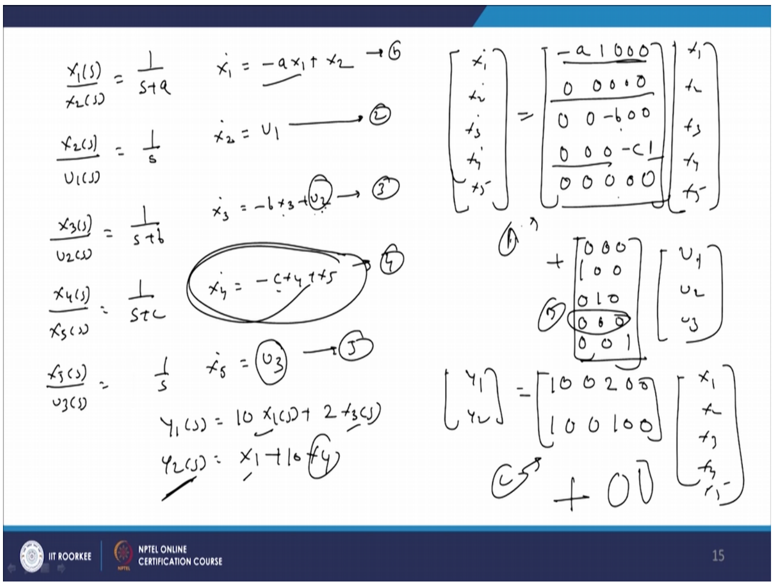

So, now we write this you see your X 1 X 2. So, we write this as X 1 of s by X 2 of s equal to 1 upon s plus a. So, equation we write we cross multiply it will get X 1 dot equal to minus a X 1 plus X 2. Now we have seen for this block. Now coming to another block that is this, this part; this part is X 2 of s divide by U 1 of s equal to 1 by s. So, you cross multiply it, will get X 2 dot equal to U 1. So, you can say this equation number 1; I can take this equation number 2. So, this portion has been completed now coming to this portion.

So, here X 3 by U 2. So, X 3 as by U 2 of s. So, we write this as X 3 of s divide by U 2 of s equal to 1 upon s plus b again for cross multiplying and take inverse Laplace, will get X 3 dot equal to b X 3 plus U 2. So, this equation number 3. So, this portion has been finished now coming to this. So, here two blocks that is X 4 with respect to X 5 and X 5 with respect to U 3. So, we have write down the equation for this blocks. So, we can write down as X 4 s by X 5 wave of s equal to 1 upon s plus c; you see here U 1 by s plus c. So, you have to like this and now we solve it.

So, we will get X 4 dot equal to minus c X 4 plus X 5. This is equation number 4 and lastly X 5 by U 3. So, write this as X 5 of s by U 3 of s; then equal to 1 by s. So, after solving will get X 5 dot equal to U 3, this equation number 5. So, we have got 3 5 equation number 1 2 3 4 5. So, now, we have to solve this; that means, we have to represent in a state space form. So, here we write this as equation as X 1 dot X 2 dot X 3 dot X 4 dot X 5 dot that is equal to we have model X 1, X 2, X 3, X 4, X 5 plus.

Now, here how many inputs are there you see here; they are three inputs U 1 U 2 U 3 and outputs are Y 1 Y 2. So, here we can say the inputs are U 1, U 2, U 3 and we have to represent output as Y 1 Y 2. So, this U 1, U 2, U 3 corresponding to input Y 1 and Y 2 corresponds to output. So, here this here we write down as X 1 X 2 X 3 X 4 X 5; just check this, see here x dot this is a matrix; this is b matrix corresponding input matrix; this is c is the output matrix.

Now we have to write down the elements in elements in a b and c just see here x 1 dot equals to minus a 1 x 1 plus x 2. So, here we get the elements of only x 1 and x 2. So, we write minus a 1 and there are 5 variables. So, with 5 elements that is 3 additional elements 0 0 0; Then, corresponding to x 2 dot x 2 dot equal to U 1. So, there are no terms for x for there are no terms corresponding to x 2 dot. So, we are write down as 0 0 0 0 0.

But here U 1 is 1. So, here we can write down as 1 0 0; but whereas, if you see the first equation I have written this one; but now we can write is 0 0 0. There are no elements of X 1 dot corresponding to U 1 U 2 U 3 that is why written 0 0 0 and for this we have written 1 0 0. Now, now come to X 3 dot. The elements corresponding to X 3 dot is minus b X 3 U 2; that means, we get only terms for X 3 the rest of the elements are 0. So, we can write down as 0 0 minus b 0 0 and about and what about the elements for this in U matrix that U 1 U 2 e three. So, we will get elements as U 2 is 0 1 0. Now corresponding to X 4 dot in a X 4 dot there are no elements of U.

So, we can directly write down here 0 0 0 and now about this c X 4 X 5 terms for X 1, X 2, X3 are 0. So, we can write down as 0 0 0 and this minus c 1 and lastly we come to X 5 dot equal to U 3. So, U 3 is this U 3 is there; that means, corresponding with this 0 0 1 and here will not get the any elements for X 5. So, all the elements are 0's and corresponds the now coming to the output; output Y 1 and Y 2. So, you going to see that this Y 1; So, Y 1 equal to 10 X 1 plus X 3; then about Y 2 see here corresponding to X 1 and X 4. So, we are write down the equation for Y 1 and Y 2.

So, equation for Y 1 of s equal to 10 into X 1 of s plus 2 into X 3 of s; you just see here 10 into X 1 2 into X 3 and now corresponding to Y 2 say the input is coming from this side. So, here X 1 plus 10 into X 4. So, we can write down as Y 2 of s equal to X 1 plus 10 X 4 and here in a state space form or output equation, we can write down as this is X 1 is 10 that element of X 2 is 0; X 3 is there we can write down 2 0 0. And corresponding to this Y 2 of s, the elements we can write down as X 1; this 1 0 0 and X 4 term is there that is 10 and X 5 term is 0 and here this is no d is involved so, 0 into U.

So, you can again cross check it. So, x 1 dot equal to a X 1 plus X 2. So, we return a 1 1; then x 2 dot equal to n rest of the limits as 0; then x 3 dot equal to minus b x 3 into u into U 2. So, you will find that elements of b rest of the element 0 u t are depend as 0 0 1 0. Now coming to x 4 dot the elements of U are totally 0's whereas, the other elements that is x 4 x 2 x 3, 0. So, we have seen 0 and is c 1 and about the x 5 dot equals 2 we have say U 3. So, let us say x 5 dot we can say I can check it this is U 3. So, we can write down as elements as 0 0 1 and about the other elements are 0 0.

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So, this is a state space model for MIMO systems. We can see there are some references Ogata I. J. Nagrath; then B. Sarkar.

Thank you.