Advanced Linear Continuous Control Systems
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Lecture - 01
Introduction to State Space

Today we start with Introduction to State Space; in this we will cover why there is need for state space representation.

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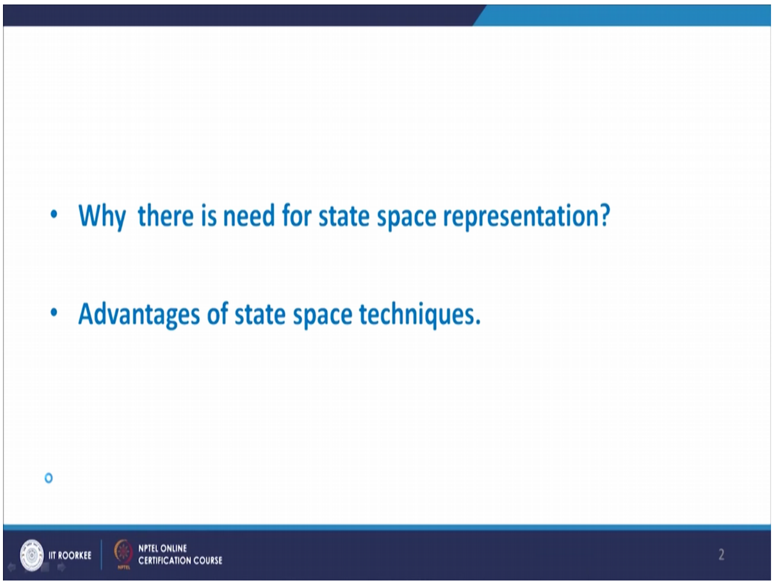

And second advantages of state space techniques, now why state space representation is essential.

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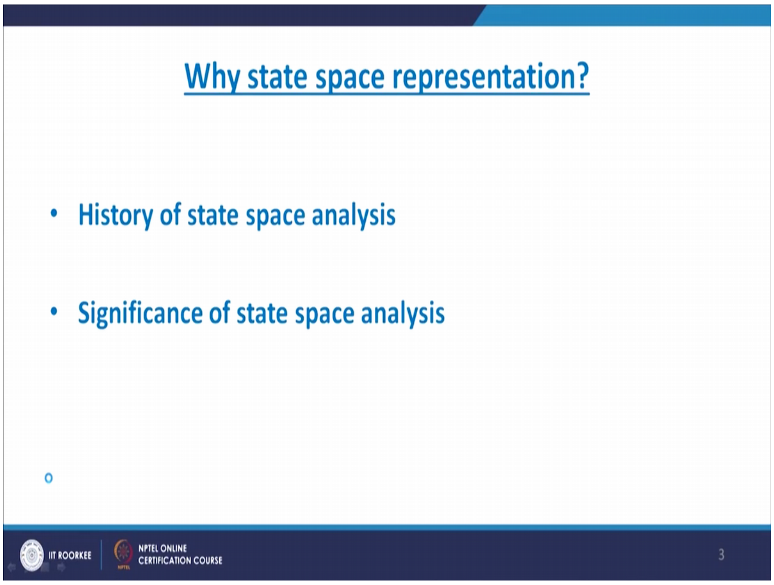

In this we will cover history of state space analysis and significance of state space analysis, there we if you see a control engineer what is the role of control engineering, role of control engineering is to develop model then checks stability and if the system is unstable then it has to design a controller and moreover he has to change the performance.

Particularly if you want to check the stability, there are various techniques are wherever the techniques are Routh Hurwitz, criteria, root locus, nyquist plot, bode plot, Nichols chart. So, all these techniques are the classical techniques then as per the control design is concerned the main controllers which are use in a classical control approach that is PID control and larly control.

So, all these techniques are base on transfer function approach and that is been develop around say between 1920 to 1950 as still the research workers going on in this directions, but because of the development after which will computer, the state space technique as been come and that is will come after 1960. And this state space technique are base on time domain approach, but you know that before 1920 that is before the development of a transfer functions the all the methodologies are base on time domain approach.

But after 1978, the all the state space approaches and classical approaches had been combined and they form new approaches that is called robust control approaches. Now about the important role of this state space in comparison to the classical approaches, in case of state space we have to know the, what are the internal parameter of this systems along with input and output. Where as in case of transfer function approach we we don't need, we miss we cannot get any information about the plant only we are knowing we are applying input and we are getting the output.

Now, I will explain this point through an example, suppose I have some medical problem what I will do. I will go to doctor and definitely, I will go to doctor which is nearby to my place that I will tell my health problem to doctor. Doctor hear my problem and what will happened he will prescribe the medicines, then I will take that particular medicines and then I will wait for 3 days. If I would not be cured then what I will do, I will go to same doctor again then he will say no you have some major problem no in that case you have to go to higher centre, or in big city we have to go.

Whenever, I will go to that big city or higher centre when I will go to that particular doctor who is the specialist what he will do he will not describe medicine, first of all what will do he will say that you test you do some testing particular urine test blood test, and whatever report will come base on this he will prescribe the medicine. So, this is nothing, but the advance control approach or we can case say state space approach where as the earlier when we applying input getting output just like a previous doctor. So, that is we can say classical approach.

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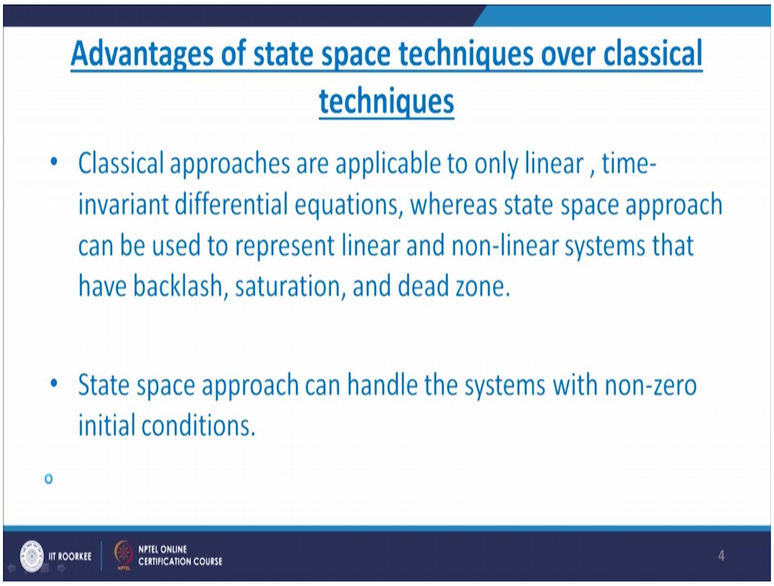

Now, about the advantages of state space techniques over a classical technique, classical approaches are applicable to only linear time invariant differential equations. Whereas state space approach can be use to represent linear and non-linear systems that have backlash saturations and dead zone, but you will find it, in case of classical approach we are using linear system only.

So, original system and we know that practical systems are non-linear, but analysis using classical approach cannot be possible for non-linear system therefore, we have to use the linear model defiantly whatever result will get that is not exact that is approximated result, but if the same time if you design your controller testing has been done on the non-linear model your analysis will be perfect results are perfect. So, all these type of analysis that is particularly linear plant is possible in classical approach where as if the plant is non-linear it is possible.

In case of advance control approach that is the first advantage of state space or the classical approach say; state space approach can handle the system with non-zero initial conditions. Classical approach is base on transfer function approach, and you know that in transfer function we have to neglect the initial conditions, but where as if you go for a state space approach we can consider the initial conditions.

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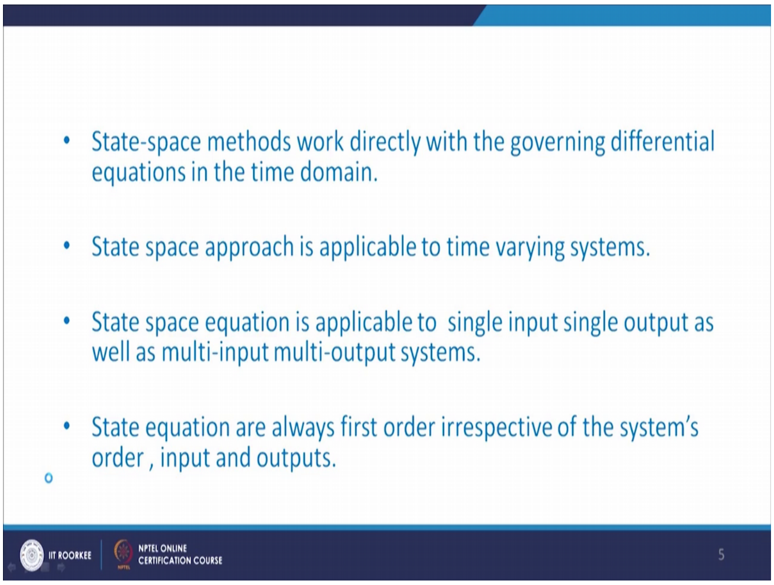

Third state space methods work directly with the governing differential equation in the time domain as I told earlier also state space approach is basically based on time domain.

And classical approach is based on S domain or frequency domain, there state space approach [vocalize-noise] is applicable to time varying system, you know you the see the air craft air craft has some fuel and it is move off what even fuel decreases that again the weight is decreases weight of the air craft is decreases therefore, whatever technique you will [vocalize-noise] developed there should be for time varying system.

So, that time varying concept can be applicable in case of state space where it is cannot be applicable in case of transfer function approach, third state space equation is applicable to single input, single output as well as multi input, multi output systems. You know in a practice most of the systems are multi input multi output time therefore; we should have techniques of stability as well as controlled design which are applicable for multi input multi output system. So, that type of approach is particular stability and control design are easily available in state space approach whereas, it is very cumbersome for in case of the classical approach.

Last point, state equation are always first order irrespective of the systems order input and output. If you see the any transfer function how define transfer function, transfer function is Laplace transferm output to Laplace transferm of input initial conditions are 0. So, here in transfer function we get different orders system third order fifth order hundred order, but if you see the state, state space equation that is always a first order equations.

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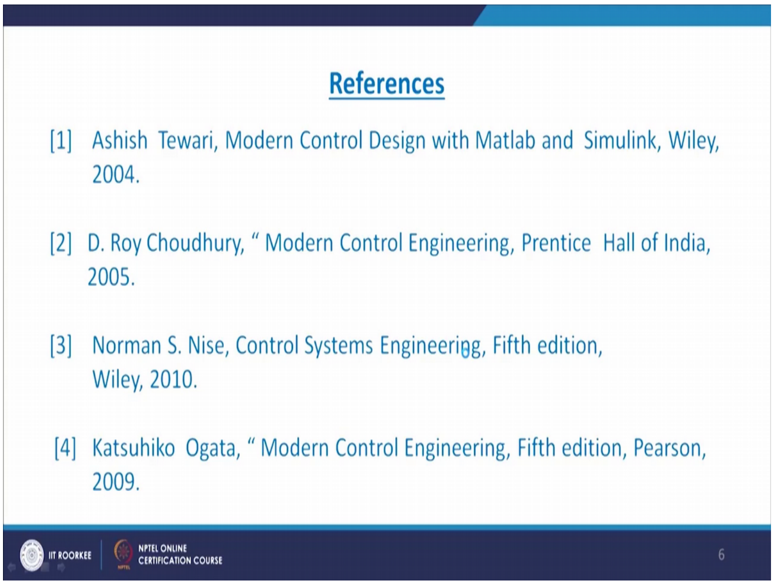

Now, whatever point have discuses you can also fine these points in following references Ashish Tewari, D Roy Choudhary, Norman S Nise and K Ogata. So, these are advantage I have explain, that is general advantage, but there are advantage which are the helpful from practical side, that is from practical point of you particularly from control design point of you so that advantage is.

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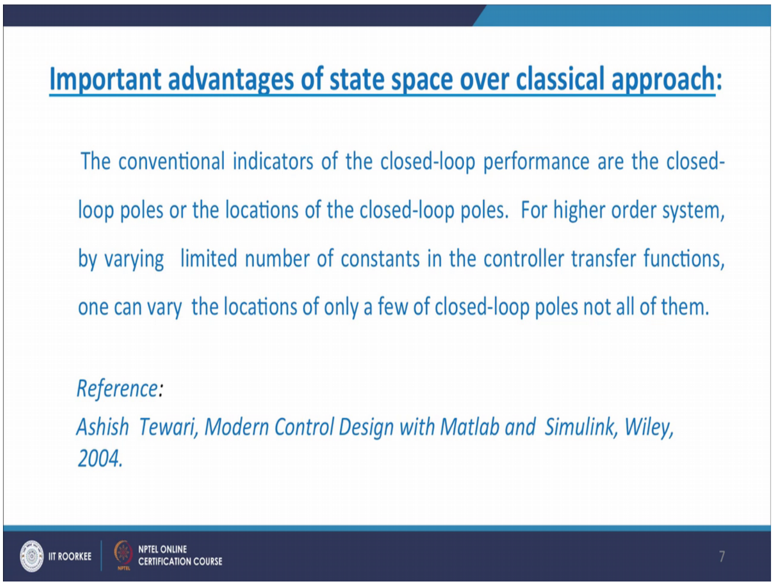

The conventional indicators of the closed loop performance are the closed loop poles or the location of the closed loop poles for high order system by varying limited number of constants in the control transfer function, 1 can vary the location of only a few of closed loop poles not all of them. See here if you want to design a controller that controller design means final result in phase on a closed loop poles. So, what my aim is that that I can plays, the poles anywhere in the expense when each poles controlled by in the performance of the system.

So, we will find that if you go for the classical approach the controller design including, we design a controller we cannot place the poles as per our requirement. So, this is the basic disadvantage of the classical approach, so we will see this point through an example.

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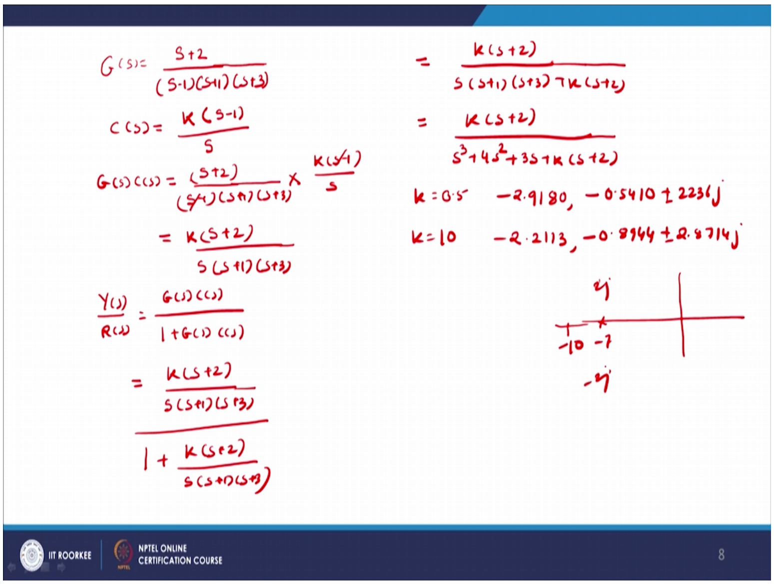

Suppose if you take G S equal to like this G S equal to say S plus 2 divide by S minus 1, S plus 1 S plus 3. Now this is a transfer function you will find at in this transfer functions there are 3 poles and 1 0, and in this case this 1 pole on the right side of the expense therefore, these plant G S is unstable; now we have to design a controller for this plant, so how you design the controller.

So, in order to design a controller what is the base time is here that this S minus 1 4 in the denominator we can cancel it. And the second thing what we can do is that even that there should not be any steady state error we can at one integrator. So, therefore, we can write a controller C of S equal to I can add one gain factor k, then this is S minus 1 this need to be canceled. So, I can write in the numerator and divide by S. So, this is control GS is a plant CS is a controller now what we want open loop transfer function with controllers GS into CS.

So, we can multiplied, so what will what will get S plus 2 S minus 1 S plus 1 S plus 3 and this is multiplied with this controller k S minus 1 divide by S. So, we will find it this S minus 1 and this S minus will cancelled and finally, we get as S plus 2 divide by S, S plus 1 S plus 3 and here is the 1 game. So, this is the open loop transfer functions now what we want a closed loop transfer functions how will you write closed loop transfer function.

So, closed loop transfer function is YS by RS equal to GS into CS divide by 1 plus GS into CS.

Now we write down this transfer function as k S plus 2 S S plus 1 S plus 3 divide by 1 k S plus 2 S S plus 1 S plus 3. Now we solve it and after solving we will get result as k S plus 2 divide by S S plus 1 S plus 3 plus k S plus 2. Now again you will simplify it what will get k S plus 2 divide by S cube 4 S square 3 S plus k S plus 2, this is a closed loop transfer function. Now in this case what will do you will take different values of k now first of all what will do we will take k as k equal to 0.5.

So, you will get the rules are minus 2.9180, other rules are minus 0.410 plus minus 2 2 3 6 j. And when we take k equals to ten we will get rules as minus 2.2113, 0.8944 plus minus 2.8714 j. So, we will find it for k equals to 0.5 we got this rules and k equals to ten we got this rules.

So, we will find, but what my aim is that I want that by 1 rule should we fix let us say my aim is that that my aim is that my rules should be around (Refer Time: 14:25) at minus 10, another rules are at somewhere in this locations say at around minus 7. So, this is my requirement to this type of requirement cannot be satisfied by means of this approach at therefore, whatever the desired performance we required that cannot be achievable by means of this approach therefore, this type of disadvantage can be covered by means of its state space approach in the state space approach we can plays poles anywhere in this, now would the second point you have a plant GS third order and what is the controller your controller is of a first order.

So, final plant should be of fourth order, but we will get a third order then can this thing practically possible plant is third order control is first order we got the. So, result we should get as a fourth order, but we are get is third order. So, they, but these type of things although we observe the theory, but practically there is some problems this type of things cannot be possible practical sense let us say I will draw now the root locus of the plant.

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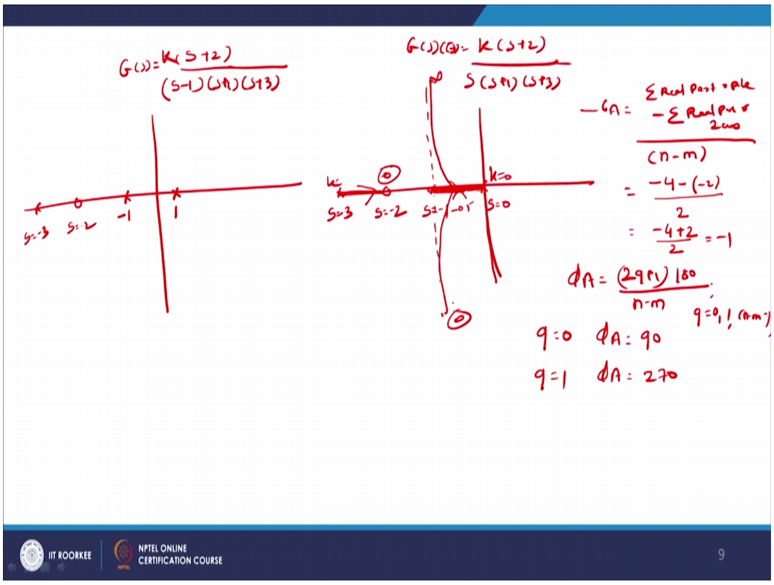

Suppose, I have taken a same plant let us see GS equal to S plus 2 divide by S minus 1 S plus 1 S plus 3 its k, now we will draw the root locus of this system. So, how will you draw the root locus, for root locus where to show all poles and zeros. Now the poles are say here S equals to here pole then here another pole 1 equals to minus 1 1 is plus S equals to minus 3 and d 0 S equals to minus 2. So, this open loop plant, but afterward what happen we have design a controller for the controller you have a plant like this GS equals to k S plus 2 divide by S, S plus 1 S plus 3 that is GS into users CS.

So, here when we show the poles and zeros, so we have 1 pole at S equals to 0 another pole at S equals to minus 1 an another pole at S equals to minus 3 And we have 0 at S equals to minus 2, so 3 poles 1 0 and this is for GS which is an unstable plant, then our aim our aim is basically for the stability of the system systems to be stable therefore, I taken this plant GS into CS. Now we have to draw locus, how will draw the root locus I think you might have studied in a classical approach root locus start from pole and terminate to zeros.

So, you will find it there are 3 poles and 1 zeros, so 1 pole will terminate to zeros then what about the other poles. So, these 2 poles will terminate to infinity along the (Refer Time: 18:15) that you have might to studied in the root locus in classical approach. So, now, here we will draw the root locus of the system. So, first of all we see the location of root locus on the real axis. So, the real axis you say, so this is the basic locations when there are exist odd number of poles the root locus will are here is a odd number, but here this is even. So, root locus will not lye here and root locus lye here, so odd number of poles.

Now, in this case what will happen root locus start from pole and it will also start away here is start, if will terminate to 0, but for these two case root locus will term t into iterating other purpose we have to calculate this centroid. So, how will you calculate centroid, centroid is that is summation of real part of pole minus summation of real part of zeros divide by number of poles minus number of zeros n is poles m is 0. So, what will get in this case what is the poles 1 at minus 1 other at minus 3. So, you will get minus 4 this minus what is the 0 at minus 2, so you will get minus 2 then how many poles 3 poles 1 0, so you will get 2.

So, you will get minus 4 plus 2 2 is minus 1 and what about angle, angle defined as 5 equals 2 q plus 1 180 divide by n minus m and in this case your q varying from 0 1 up to n minus m minus 1. So, when q equals to 0, so 5 a will get as q 0 will get n minus from 2 we will get 90 degree and when q equals to 1 will get 5 as if you add q. So, 180 into 3 dived by 2, so will get around 270 degree.

So, here we were assembled to 2 angle, so root locus start from pole it will start from here and there is a break a point. So, that we calculate by dk by ds here I am not calculating. So, approximately value will coming at minus 0.5, so here what will happen toward root locus star form here, it will move this side and another will this side and another root locus start from this pole entered to be 0. This is a location of root locus relaxes as well as in the imaginary axis.

Now what about the stability of this term you will find that you can varying k from 0 to infinite, this is infinite this is k equal 0 infinite 16 is stable. Whatever k will get, but can it possible in a real time sometimes it is happened at students are working as a project and in simulations they are getting result, but when the same thing they are implemented in a real time they are seen that result are not coming because in the simulation we can getting result, but hardware real time result are not getting.

So, if you use the classical approach this can be possibilities. So, here also we will find at in this particular example for k varying for 0 to infinity k like 0 to infinity your system is stable, but if you take this example in the real time applications this is not possible system is unstable. So, this type of verification is possible using state space approach if you take this problem is the state space you will find it system become unstable. So, whenever I will teach the modeling in the state space and analysis control design.

I will take same example and I will show that this plant with controller is unstable this is happened, because we have done a pole 0 cancellation therefore, when particularly when we are doing analysis in a classical approach we should avoid this pole 0 cancellation in the pole 0 cancellation is there this is the problem just the as we are seen the problem here problem is that your order has been this system is stable, but real time system is unstable. So, in the next part I will explain you how to model a state space approach then different types of modeling, now you have e some references.

Thank you.