Phase-Locked Loops
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Lecture – 15
Frequency Domain Insight in Frequency Acquisition for Type-II PLLs

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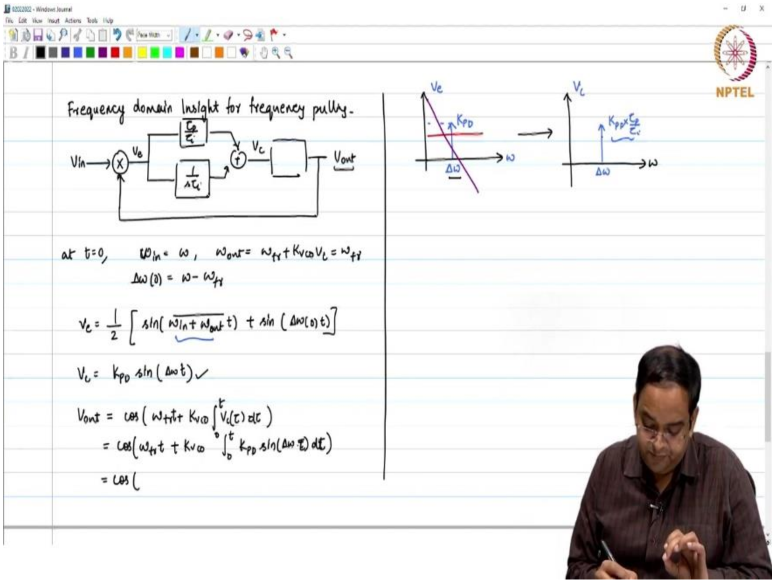

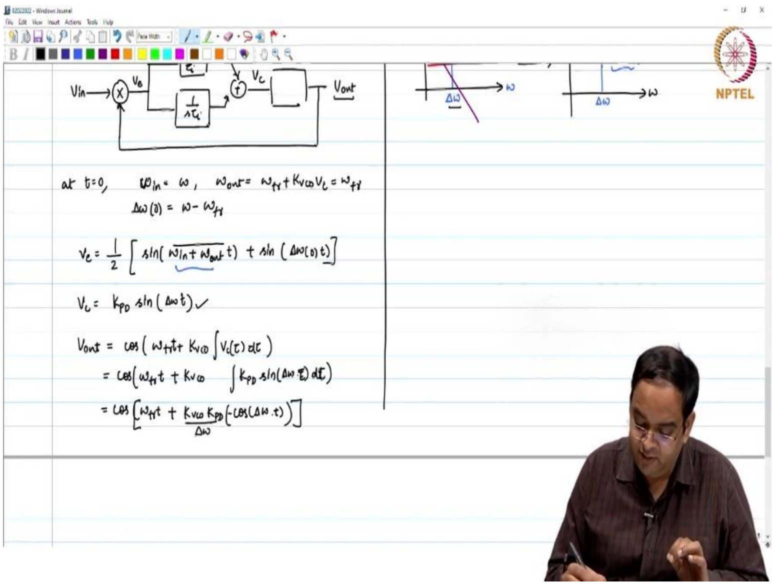

Welcome to this session. We would like to discuss the frequency domain insight behind frequency pulling. So, just enjoy these two frequency terms here. So, we have this mixer based phase error detector, then we have the loop filter with proportional and integral paths, and the VCO. So, we have already seen the time domain analysis. Now, we will look at the frequency domain for frequency pulling.

So here, we have,

$$\text{At } t = 0,\ \omega_{in} = \omega,\ \omega_{out} = \omega_{fr} + K_{VCO}V_c = \omega_{fr}$$

So, the frequency error at the beginning is given by,

$$\Delta\omega(0) = \omega - \omega_{fr}$$

Now, you have this frequency error, and we know that V_{in} and V_{out} are sinusoids. The error voltage is given by,

$$V_e = \frac{1}{2}\left[\sin\left((\omega_{in} + \omega_{out})t\right) + \sin(\Delta\omega(0)t)\right]$$

So, for this particular frequency error, let us look at the frequency spectrum of V_e. When you start your system at the beginning, you look at V_e spectrum, V_e versus ω, and it has a component at $\Delta\omega$. I am neglecting $\sin\left((\omega_{in} + \omega_{out})t\right)$ component because I know that this is going to be filtered by the loop filter. So, we have $\frac{1}{2}\sin(\Delta\omega(0)t)$ component. The loop filter which you are seeing here has two paths, namely, proportional and integral. The integral path will actually filter this out more or less most often if the frequency error is large. So, the proportional path is going to amplify this because the gain is finite for the proportional path.

So, the proportional path has gain like this and the integral path has gain like this. So, integral path filters out and proportional path gain remains. So, what do you get at the output of the loop filter which is the V_c node? You get the same frequency component, may be amplified with some value and this is $\Delta\omega$. So, that is what you have here. So, if you look at the magnitude, this is K_{PD} and here you have $K_{PD} \times \frac{\tau_p}{\tau_i}$.

Now, when you have a frequency component at $\Delta\omega$, what happens to the control voltage? We will see it here. So, the control voltage is given by,

$$V_c = K_{PD}\sin(\Delta\omega\ t)$$

The output voltage is given by,

$$V_{out} = \cos\left(\omega_{fr}\ t + K_{VCO}\int V_c(\tau)\ d\tau\right)$$

Let me just use another variable instead of t, let us say, τ. This is just for integration purpose, so, this is what you have. If you look at this, the control voltage which you see here changes the VCO frequency and modulates the output signal. Now, how does it modulate the output signal? You need to do a little math here. So, we have,

$$V_{out} = \cos\left(\omega_{fr}\, t + K_{VCO} \int K_{PD}\ \sin(\Delta\omega.\tau)\ d\tau\right)$$

We assume that during the integration, $\Delta\omega$ remains constant because we are considering only one frequency. Hence, we get,

$$V_{out} = \cos\left(\omega_{fr}\, t + \frac{K_{VCO}\, K_{PD}}{\Delta\omega}\ (-\cos(\Delta\omega.t))\right)$$

You can forget the limits here for now. So, I am just doing indefinite integration, you can do definite integration also. The motive here is to find out how this control voltage is going to modulate V_{out}. So, actually there will be limits, you can do with that also, here I am just going to do it without the limits. So, there is no problem with that if you do with the limits or without the limits. The objective here is to find out how this will change the output voltage. So, you are having two components, one is $\omega_{fr}\, t$ and the other component is $\frac{K_{VCO}\, K_{PD}}{\Delta\omega}\ (-\cos(\Delta\omega.t))$.

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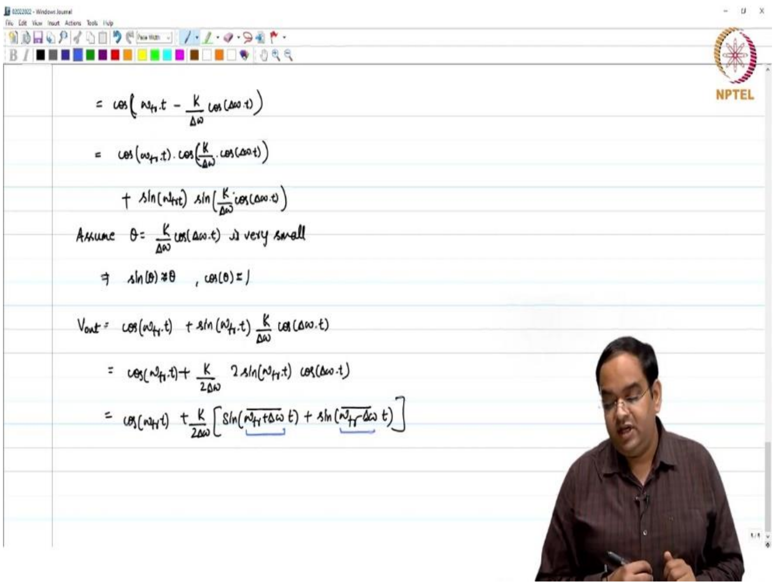

So, we get,

$$V_{out} = \cos\left(\omega_{fr}\, t - \frac{K}{\Delta\omega}\ \cos(\Delta\omega.t)\right)$$

Since, $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$, so, we get,

$$V_{out} = \cos(\omega_{fr}\, t)\cos\left(\frac{K}{\Delta\omega}\cos(\Delta\omega . t)\right) + \sin(\omega_{fr}\, t)\sin\left(\frac{K}{\Delta\omega}\cos(\Delta\omega . t)\right)$$

Now, we assume here that $\theta = \frac{K}{\Delta\omega}\cos(\Delta\omega . t)$ is very small. If we make this assumption, then we know that $\sin(\theta) \approx \theta$, and $\cos(\theta) \approx 1$. So, we get,

$$V_{out} = \cos(\omega_{fr}\, t) + \sin(\omega_{fr}\, t)\frac{K}{\Delta\omega}\cos(\Delta\omega . t)$$

We are doing this for the first time but we will do these things multiple times in PLL, so that is why I am going through all the steps. So, you have,

$$V_{out} = \cos(\omega_{fr}\, t) + \frac{K}{2\,\Delta\omega} 2\sin(\omega_{fr}\, t)\,\cos(\Delta\omega . t)$$

Now, apply the identity, $2\sin(A)\cos(B) = \sin(A + B) + \sin(A - B)$. So, we get,

$$V_{out} = \cos(\omega_{fr}\, t) + \frac{K}{2\,\Delta\omega}\left[\sin\left((\omega_{fr} + \Delta\omega)t\right) + \sin\left((\omega_{fr} - \Delta\omega)t\right)\right]$$

So, when we modulate the control voltage of the VCO with frequency $\Delta\omega$, then at the output of the oscillator, V_{out}, we get two more frequencies, one is $\sin\left((\omega_{fr} + \Delta\omega)t\right)$, and the other is $\sin\left((\omega_{fr} - \Delta\omega)t\right)$.

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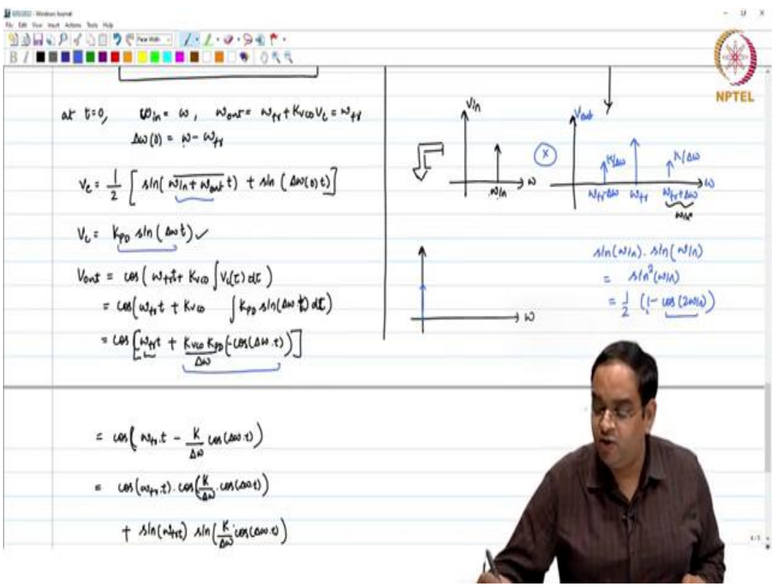

So, this implies that I have more frequencies, and one of the frequencies is ω_{fr} for the VCO as it was, and the other frequencies are $\omega_{fr} + \Delta\omega$ and $\omega_{fr} - \Delta\omega$ with magnitude $\frac{K}{2\,\Delta\omega}$. So, this is V_{out}. So, when we modulated the control voltage with a sine wave, the output of the oscillator gives rise to two more frequencies, $\omega_{fr} + \Delta\omega$ and $\omega_{fr} - \Delta\omega$.

So, then it comes back at V_{in}. V_{in} is still sitting at ω_{in}, you did not change V_{in}, so V_{in} is still at ω_{in}. What is ω_{in}? You see from here that we have,

$$\omega_{in} = \omega_{fr} + \Delta\omega$$

This frequency is actually equal to ω_{in} only. So, when you multiply these two signals, one signal is having these three frequency components, the other signal is having only one frequency component, then this multiplication will give rise to more frequency components, and what are those more frequency components? Let us just see here.

I just do not want that you take this downward arrow also as some frequency, so let me just avoid this and maybe use a little nicer arrow. So, you have this ω here, and $\omega_{in} = \omega_{fr} + \Delta\omega$. When these two frequencies combine, you are going to get the DC component. If you are wondering what I am doing, I am multiplying $\sin(\omega_{in}t)$ with $\sin(\omega_{in}t)$. If I do this, then this multiplication is $\sin^2(\omega_{in}t) = \frac{1}{2}(1 - \cos(2\omega_{in}t))$. So, you get two components, one component is at higher frequency and the other component is at DC. So, this multiplication leads to a component at DC at V_e.

You start with a non-zero DC component at V_e, to begin with, because there was no change, V_c was not there, then you come back in the loop and you see how the output frequency gets modulated, you produce a DC component. In addition to this, you will have components at $\Delta\omega$, $2\,\Delta\omega$ and so on, but the important part is that you are having a DC component. And what is there at the output of V_e? There is a loop filter which has a proportional path and an integral path. The integral path has infinite gain at DC and you produce a DC component. So, if you produce a DC component at the input of the loop filter, the loop filter has infinite gain at DC, it will just amplify it, your control voltage will increase by a lot of margin and it is going to help you in compensating for the frequency error.

So, this is another way of looking at the frequency acquisition. Previously, we looked at it completely in time domain. This time we looked at how these frequencies multiply to give you the DC component and that DC component gets amplified. So, this actually completes our frequency acquisition for the PLL and a similar procedure can be followed for different kinds of PLLs. We have only taken a simple example and you can use it for different PLLs. Thank you.