

**Introduction to Semiconductor Devices**  
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**Lecture – 3.6**  
**Quasi Fermi Level in Non-Equilibrium Conditions**

This document is intended to accompany the lecture videos of the course “Introduction to Semiconductor Devices” offered by Dr. Naresh Emani on the NPTEL platform. It has been our effort to remove ambiguities and make the document readable. However, there may be some inadvertent errors. The reader is advised to refer to the original lecture video if he/she needs any clarification.

Hello everyone, welcome to Introduction to Semiconductor Devices. This is lecture number 9. So, in the last lecture, we were talking about non-equilibrium situations in semiconductors. We introduce the concepts of minority carrier diffusion length and minority carrier lifetime and we said that these 2 parameters play a very crucial role in the response of semiconductor devices. **(Refer Slide Time: 00:36)**

The slide contains the following content:

- Equations:**

$$n_0 = n_i \exp\left(\frac{E_F - E_i}{KT}\right) \quad p_0 = n_i \exp\left(\frac{E_i - E_F}{KT}\right)$$

$$N_A = 10^{18} \text{ cm}^{-3} \quad n_i = 10^{10} \quad p_0 = 10^5 \quad n_1 = 10^{10}$$

$$E_F - E_i = KT \ln\left(\frac{n_0}{n_i}\right) = (0.0259 \text{ V}) \ln\left(\frac{10^{18}}{10^{10}}\right) = 60 \text{ meV} \times 8 = 0.5 \text{ eV}$$
- Band Diagram:** Shows energy levels  $E_C$ ,  $E_i$ , and  $E_V$ . The Fermi level  $E_F$  is shown as a dashed line below  $E_i$ . The energy difference between  $E_F$  and  $E_i$  is labeled as  $0.5 \text{ eV}$ .
- Text:**
  - At equilibrium  $E_F$  captures carrier concentrations fully.
  - Non-equilibrium?  $n = n_0 + \delta n$ ,  $p = p_0 + \delta p$ ,  $n p \neq n_i^2$
  - Define Quasi Fermi level (QFL):
 
$$n = n_1 \exp\left(\frac{E_{Fn} - E_i}{KT}\right)$$

$$p = p_1 \exp\left(\frac{E_i - E_{Fp}}{KT}\right)$$
  - $n_0 \rightarrow n$ ,  $E_F \rightarrow E_{Fn}$   
 $p_0 \rightarrow p$ ,  $E_F \rightarrow E_{Fp}$
  - $E_{Fn}$  &  $E_{Fp}$  are called Quasi Fermi levels

So, today, we will introduce a concept of Quasi Fermi levels. Quasi Fermi levels are very useful tools in representing non-equilibrium situations on a band diagrams. So, as we go along, we will understand what Quasi Fermi levels mean. Just to refresh your memory, we have seen these expressions before. So, basically  $n_0$  will be given by  $n_i$  times exponential the distance from the intrinsic energy level.

$$n_0 = n_i e^{\left(\frac{E_F - E_i}{KT}\right)}$$

And similarly,  $p_0$  will be again the same thing.

$$p_0 = n_i e^{\left(\frac{E_i - E_F}{KT}\right)}$$

And let us say, you have a semiconductor which is doped with  $10^{15}$  dopants, you know, let us say arsenic or phosphorous dopants, so they have donors, we have an n type semiconductor. So, what would be the carrier concentration? Well, we have seen it multiple times. So,  $n_0 = 10^{15}$  and  $p_0 = 10^5$ . In the class, I am using it as a  $10^5$ . I am using  $n_i = 10^{10}$ . That is the number  $1.5 \times 10^{10}$ .

So, and whenever we have equilibrium situation, the excess minority carriers are going to be 0. So, where will be the Fermi level in this case? So, let us say I will do it one more time in case somebody who is joining fresh. So,  $(E_F - E_i)$  by rearranging this equation, I can write.

$$E_F - E_i = KT \ln\left(\frac{n_0}{n_i}\right)$$

$$E_F - E_i = KT \times 2.30 \times \log\left(\frac{10^{15}}{10^{10}}\right)$$

So, this would turn out to be, this particular quantity here,  $KT \times 2.3 = 59$  meV. So, in the calculations that I do today or in the subsequent classes, basically, I will take this  $KT \times 2.3 = 60$  meV approximately. This will simplify my calculations. I do not want to do exactly but it will be close enough, you know, 1 milli electron volts difference.

But you know, it lead to some small error, but you can verify it whenever you need exact calculations or in the home-works and assignments or the exams, please use exact numbers. I do not want to use a calculator while teaching so I choose 60 milli electron volts and approximately that way. So, the Fermi levels, the distance of Fermi levels from  $E_i$  would be  $60 \text{ meV} \times 5$ ,  $\log\left(\frac{10^{15}}{10^{10}}\right) = 5$ .

$$E_F - E_i = 60 \text{ meV} \times 5 = 0.3 \text{ eV}$$

So, you could represent that on Fermi. We have done this multiple times in the past.  $E_F$  is going to be at 0.3 eV distance from the  $E_i$  as we increase  $E_F$  changes approximate. So, the message I wanted to convey is at equilibrium  $E_F$  captures carrier concentrations fully which I mean for example, you know, I have used  $n_0$  expression. You could also use  $p_0$  expression and calculate the distance from  $E_i$ .

If you use  $p_0$  expression  $10^5$ , so, you just use the  $p_0$  expression and calculate  $E_F$  again. It will give you the same answer. The distance of Fermi level will be 0.3 eV above  $E_i$ . It will get the same answer plus but please make sure that you verify that. So, whenever you have an equilibrium situation, all you need is simply  $E_F$ . If you know  $E_F$ , you know all the carrier concentration. So, you do not even need anything more.

But what happens when I have a non-equilibrium situation? But what do I do? Whenever you have non-equilibrium situations, we know, we have seen this in last class so, basically now,  $n$  is going to be the total carrier density which will be  $n_0$  plus the excess, let me put small, excess electrons, the excess electron density.

$$n = n_0 + \delta n$$

And similarly,  $p$  is going to be.

$$p = p_0 + \delta p$$

This is what we have seen in the last class and we said that

$$np \neq n_i^2$$

So, these are excess carrier density that we are introducing like so, by shining light. We could get situations like this. So, we want to be able to represent this on a band diagram. And to do that we define a Quasi Fermi levels. So, define Quasi Fermi level, I simply call it as QFL, just for short notation. The definition we use is very similar to what we have seen in the normal equilibrium carrier density cases.

So, I simply replace equilibrium electron density by the total electron density that will be equal to  $n_i$  times exponential. In the equilibrium case, we had  $E_F - E_i$ . So, now, instead of  $E_F$ , I will put as  $E_{Fn}$ ,  $- E_i$  by  $KT$ .

$$n = n_i e^{\left(\frac{E_{Fn} - E_i}{KT}\right)}$$

Similarly, for holes, I will replace the equilibrium hole density by total hole density. And that will be equal to exponential. We might already be guessing here. So, instead of  $E_F$ , I will put it as  $E_{Fp}$  by  $KT$ .

$$p = n_i e^{\left(\frac{E_i - E_{Fp}}{KT}\right)}$$

So, what I am essentially doing here is; I am replacing  $n_0$  by  $n$ ,  $E_F$  by  $E_{Fn}$  and  $p_0$  by  $p$ ,  $E_F$  by  $E_{Fp}$ . So, these  $E_{Fn}$  and  $E_{Fp}$  are known as Quasi Fermi level. So, this is the definition. So,  $E_{Fn}$  and  $E_{Fp}$  are called Quasi Fermi levels. This is different from the regular Fermi levels. So, we saw that regular Fermi level is useful when you have equilibrium situations.

Whenever you have non-equilibrium situations, we use Quasi Fermi levels, which essentially capture the total carrier densities. So, this is how we define it. And why do we have two of this? In the regular situation, we had only one Fermi levels. We do not have one Fermi levels for electrons and Fermi levels for holes in equilibrium situation. We saw that you know, either of the expressions is giving the same answer. But if we go to non-equilibrium, we see that it does not work and we have to use what is known as Quasi Fermi level.

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**Quasi Fermi levels - example**

$n = n_0 \exp\left(\frac{(E_{Fn} - E_i)}{kT}\right)$      $p = p_0 \exp\left(\frac{(E_i - E_{Fp})}{kT}\right)$

$n_0 = 10^{15} \text{ cm}^{-3}$   
 $\delta n = \delta p = 10^{17} \text{ cm}^{-3}$

At equilibrium  
 $\delta n = \delta p = 0$   
 $E_{Fn} = E_{Fp} = E_F$

Non equilibrium is represented by two QFL's  $E_{Fn} \neq E_{Fp}$

$n = 10^{15} + 10^{17} \sim 10^{17} \text{ cm}^{-3}$   
 $p = 10^{15} + 10^{19} \sim 10^{19} \text{ cm}^{-3}$   
 $E_{Fn} - E_i = kT \ln\left(\frac{10^{17}}{10^{15}}\right)$   
 $\sim 60 \text{ mV} \times 7 = 0.42 \text{ eV}$   
 $E_i - E_{Fp} = kT \ln\left(\frac{10^{19}}{10^{15}}\right)$   
 $\sim 0.42 \text{ eV}$

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Let us take an example. The same situation, just note I have replaced here the expressions, instead of  $n_0$ , I am using  $n$  and then I am using  $E_{Fn}$  which is a Quasi Fermi levels for electrons. Similarly,  $p$  and then Quasi Fermi level for the holes. So, now consider the same situation, you have a doping density of  $10^{15}$ . But now, I have electron and hole excess carrier density as  $10^{17}$ . I will introduce by let us say, shining light.

So, we also saw that the Fermi level, the equilibrium Fermi level  $E_F$  was basically 0.3 eV above  $E_i$ . So, now where will the Quasi Fermi level for holes and electrons be? So, to do that let us estimate you know, in equilibrium case, of course, now, you can, if you substitute the equilibrium carrier densities in the expressions, at equilibrium  $\delta n = \delta p = 0$ . Therefore,

$E_{Fn} = E_{Fp} = E_F$ . Please you know, try to substitute these numbers, will check it out. It will turn out to be this that is in a equilibrium situation.

But now, when we want to talk of non-equilibrium situation, so we have; what is  $n$ ?

$$n = n_0 + \delta n = 10^{15} + 10^{17} \sim 10^{17} \text{ cm}^{-3}$$

Let us take it to be that. Okay, by the way, I am always ignoring the units, but please make sure that units are right.

$$p = p_0 + \delta p = 10^5 + 10^{17} \sim 10^{17} \text{ cm}^{-3}$$

This is carrier density.

Now, where will the Quasi Fermi level? We use the same approach as we taken for the regular Fermi level.

$$E_{Fn} - E_i = KT \times \ln\left(\frac{10^{17}}{10^{10}}\right) \sim 60 \text{ meV} \times 7 = 0.42 \text{ eV}$$

7 orders of magnitude changes in the carrier concentration, so basically 7. So, that will be how much. It is going to be exactly equal to 0.42 eV.

So, the Quasi Fermi level for electrons is at 0.42 eV from  $E_i$ . Similarly, you could do a calculation  $E_i - E_{Fp}$ .

$$E_i - E_{Fp} = KT \times \ln\left(\frac{10^{17}}{10^{10}}\right) \sim 0.42 \text{ eV}$$

So, what this is telling us is: whenever you have non-equilibrium situation, we can represent that by Quasi Fermi levels. Now, for this case, this Quasi Fermi level, I will represent somewhere here, the distance is going to be a 0.42 eV.

Similarly, there is a Quasi Fermi levels for holes. This distance is 0.42 eV,  $E_{Fp}$ , this is  $E_{Fn}$ . So, the conclusion what we are seeing is the  $E_{Fn}$  and  $E_{Fp}$  need not be the same when you have non-equilibrium. So, the conclusion would be; non-equilibrium is represented by two lets say Quasi Fermi levels  $E_{Fn}$  and  $E_{Fp}$ . So, I mean, people get confused. Students, sometimes, they are confused, it is simply a tool for us, even Fermi level as tool to represent how the carrier densities are in a semiconductor.

So, it might not so, you might not see it, you know, right now, when you look at this now, just 2 levels, you know, uniform semiconductor, does it matter. But once we have a device with p-n junctions and all that, these become very useful tool for us to visualise, just by drawing the diagram, you know, what is happening in the device, so, band diagrams. So, this is a definition. Let us take another example. This is the first case. Let me take another example.

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Quasi Fermi levels – example

$n = n_0 \exp\left(\frac{E_{Fn} - E_i}{kT}\right)$      $p = p_0 \exp\left(\frac{E_i - E_{Fp}}{kT}\right)$   
 $N_A = 10^{15} \text{ cm}^{-3}$   
 $\delta n = \delta p = 10^{10} \text{ cm}^{-3}$   
 $n = 10^{15} + 10^{10} \sim 10^{15}$   
 $p = 10^5 + 10^{10} \sim 10^{10}$   
 $E_{Fn} - E_i = 60 \text{ meV} \times 5 = 0.3 \text{ eV}$   
 $E_i - E_{Fp} = 60 \text{ meV} \times (-5) = -0.3 \text{ eV}$   
 $\delta n = \delta p = 10^{10} \text{ cm}^{-3}$   
 $E_{Fn} - E_i = 0.3 \text{ eV}$   
 $E_i - E_{Fp} = 60 \text{ meV} \times (-5) = -0.3 \text{ eV}$

Energy band diagram showing  $E_C$ ,  $E_i$ , and  $E_V$  levels. The Fermi level  $E_F$  is shown as a dashed line between  $E_{Fn}$  and  $E_{Fp}$ .

$E_{Fn} = E_F$  → Electron concentration is not significantly perturbed by the excitation.  
 As  $\delta n \uparrow$   $E_{Fn}$  moves closer to  $E_C$   
 $\delta p \uparrow$   $E_{Fp}$  moves closer to  $E_V$

At sample approaches equilibrium  $E_{Fn}$  &  $E_{Fp}$  approach  $E_F$

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Now, I will retain everything same in the problem except that now, I have changed the excess minority carrier densities to be  $10^{10}$ . So, now what happens? So, now you will see that.

$$n = n_0 + \delta n = 10^{15} + 10^{10} \sim 10^{15} \text{ cm}^{-3}$$

$$p = p_0 + \delta p = 10^5 + 10^{10} \sim 10^{10} \text{ cm}^{-3}$$

So, where will the Fermi level be?

We can easily compute that let us see,  $E_{Fn} - E_i$  is going to be now I am writing the answers; please if you are unsure, stop it, go back and verify and make sure that you get it right;

$$E_{Fn} - E_i = \sim 60 \text{ meV} \times 5 = 0.42 \text{ eV}$$

$$E_i - E_{Fp} = \sim 60 \text{ meV} \times \log \frac{10}{10} = 0$$

So, basically, what this is telling us is: the Fermi levels have become I will not draw  $E_F$ , the regular equilibrium I just draw the well let me actually put  $E_F$ .

Now, the Quasi Fermi level for electrons is also at the same location. Remember, see, you are getting the distance from the equilibrium sorry, the intrinsic level is 0.3 eV. So, basically, what we are seeing is  $E_F = E_{Fn}$ . And whatever the Quasi Fermi level for the holes, I should have written it as p here  $E_{Fp}$ . You see that  $E_{Fp}$  the Quasi Fermi level for holes is equal to the intrinsic carrier density. So, your  $E_{Fp}$ .

So, what does mean  $E_F = E_{Fn}$ ? What does it mean? It implies that electron concentration. It is not significantly perturbed by the excitation. You are essentially having a electron hole pairs

created by light and that excitation is not really changing the electron concentration that is why  $E_F = E_{Fn}$  and the  $E_{Fp}$  is changing.

In the previous example, when the concentrations for  $10^{17}$ ,  $E_{Fp}$  was much, much closer to the  $E_V$ . So, basically as the excess carrier density is increased,  $E_{Fn}$  goes towards, let us say we can even conclude this, as  $\delta n$  increases,  $E_{Fn}$  moves closer to  $E_C$ . Similarly, as  $\delta p$  increases,  $E_{Fp}$  moves closer to  $E_V$ , vice versa. Exactly what you would expect with a Fermi level, but you know in n type, the Fermi level will be above  $E_i$ , in p type will be below  $E_i$ .

Now, even in n type semiconductor when you have excess carrier density, you could have  $E_{Fn}$  and  $E_{Fp}$ , which are exactly the same expressions, just the definition slightly different. We are taking the total carrier density into account. One more example, let us say  $\delta n = \delta p = 10^7 \text{cm}^{-3}$ . So, I will cut the story short. I will say

$$E_{Fn} - E_i = .3eV$$

$$E_i - E_{Fp} = 60meV \times \log \frac{10^7}{10^{10}} = 60meV \times -3 = -0.18eV$$

So, what it is happening is: as you are reducing the excess minority carrier density, you are moving closer to  $E_F$ . So, now in this case, the third case I will put it in the red here. So, it is somewhere here this was  $E_{Fp}$  for  $\delta p = 10^7$ . So, you see, we have seen in the previous case like right, last lecture that as you as the excess minority carrier concentration reduces, sample approaches equilibrium.

So, we can conclude that as sample approaches equilibrium  $E_{Fn}$  and  $E_{Fp}$  approach  $E_F$ . So, we said that already. Because we said there is no physical reason I mean, if you see the way the Quasi Fermi levels are defined at equilibrium both are equal to  $E_F$ .