Introduction to Semiconductor Devices
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Lecture – 2.1
Effective Mass in Semiconductors

Hello, everyone, welcome back to Introduction to semiconductor devices. This is lecture 4. So, in the last lecture, we were talking about how we can visualise a 2D bonding model, which shows you how silicon atoms are bonded in a lattice. And then after that, we talked about how electron hole pairs are produced by various generation processes. And in the end, we covered the differences between direct and indirect bandgap semiconductors. So, today, we will try to understand more about carrier properties in semiconductors.

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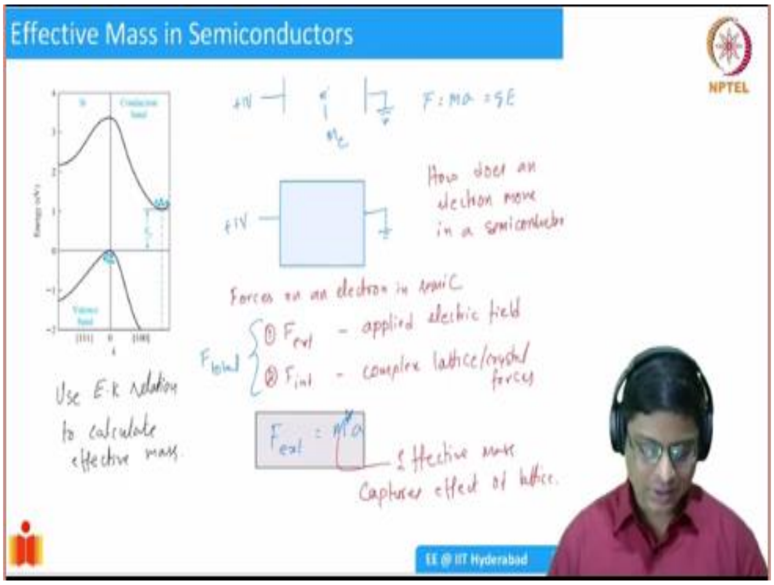

To do that, we will start with a concept known as effective mass in the semiconductors. So, we have already seen this E-k diagram. This is a indirect bandgap semiconductor, which is silicon. So, we ask ourselves the question, how does the carrier move in a semiconductor like this? For example, if I take a case of a electron moving between 2 metallic plates, let us just imagine a metallic plate with certain voltage.

And then I will apply another metallic plate. And then between them, let us say I put one of them at +1 volt, and the other one I ground it and now I place a electron in between that plates. So, of course, you know the electron moves in response to the field. So, how would you describe this? This we will describe using Newton's laws.

$$F = ma = qE$$

What is the mass here we use? We use the mass of the electron. This is mass of the electron. So, this is how electron behaves in free space. But now how does it behave in a semiconductor? So let us imagine a piece of semiconductor. So, I take this, I do the same thing. I ground one side. On the other side, I apply let us say positive 1 Volt. So, now the question is, how does the electron behave in this?

How does an electron move in a semiconductor? To answer this, we need to understand what are the various forces acting on the electron? So what are the types we can think of? Of course, you know, there are no let me say forces on an electron in semiconductor. Let us make it semiconductors semi C. So, we have first one, they have external force, which is essentially the applied electric field.

In addition, we have an internal force, forces due to the crystal, So let us say there, there are this f internal, which are essentially you know electron is going to feel you know, the crystal potential. The lattice potential it will feel, and it will also undergo some scattering events, you know. Because electron is, let us say, moving in one direction, it can undergo a scattering with a lattice site and then it can change directions.

So the, the forces are quite complex for us to visualise. But let us say this F internal essentially encapsulate all of them. And we say that this is basically a complex lattice, lattice forces or crystal, whatever you want to call it, forces. These are this F internal. So, now, how does electron behave in this environment? It is going to be quite challenging if you want to monitor every single electron which is moving in a crystal.

If you would like to monitor it is going to be quite challenging, it is impossible. So, what we will do is, we will introduce a concept known as effective mass. What we will say is just like the Newton's second law, F= ma. And we will write now, you know, basically this, both of them is F total. So, basically F total will be equal to mass times acceleration. It is the real mass.

But what we will say is, we will introduce a concept of effective mass and we say

$$F_{external} = m^{*}a$$

. So, basically, this particular m^{*} which you know we will call as effective mass. So, what does it do? With m star it is essentially capturing whatever is the effect of the crystal or lattice. All the forces and everything it will capture and then eventually it will simply give you one number which is going to be the proportionality between external force and of the electron, the acceleration of the electron.

So, m star is captures the effect of lattice. This is what it does. How can we do that? So, the way it is done is all this you know, lattice potential and how electron moves in that is all captured into the E-k relation. So, we will use this. So, we will use the basically the goal now is how to calculate the effective mass using, use E-k relation to calculate effective mass. This we can do because you know at least qualitatively we can think about it.

The E k relation is all result of what, how the energies are changing with momentum and all that in a crystal. So, E-k relation we will use. How do we do that?

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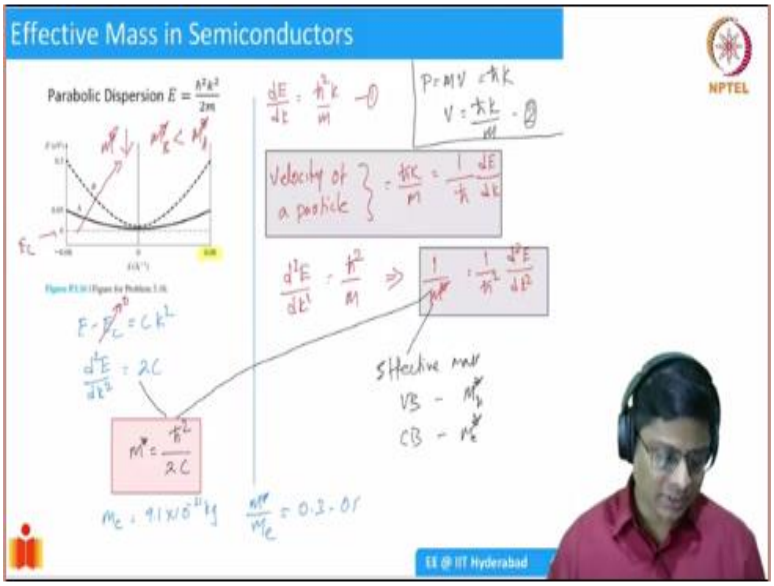

It is fairly simple actually. So, what you are seeing here is you know we already mentioned that electron has at least you know, electron in free space has a parabolic dispersion. And we also said that, ah when you look at the E-k relation for example, here, around the edges for example, here, the dispersion is still parabolic. So, we said these 2 in a positive and negative E k are in an essentially 2 different directions.

So, around the edges, band edges you know be it the top of the valence band or the bottom of the conduction band, it still exhibits, it can be approximated as a parabolic dispersion. So, we will use the ideas that we have learned in the past. So, this is just, let us say you have parabolic dispersion, how can we estimate the mass, effective mass from that. So, let us do one thing. Let us compute $\frac{dE}{dk}$.

If you do this, essentially it will be, I am taking the derivative of the dispersion. So, it will be

$$\frac{dE}{dk} = \frac{\hbar^2 k}{m}$$

So, now, you might recollect, you know that we define we know classically $p = mv = \hbar k$. And we said quantum mechanically that would be $\hbar k$. So, if you want to define a quantum mechanical analogue of velocity, what you need to do is simply say velocity should be quantum mechanically mass.

So, this is what you can do to define a quantum mechanical analogue of velocity. So, by combining, let us call this as 2. Let us call this as 1. By combining 1 and 2, we can write velocity of a particle travelling in a band, if you are talking about conduction band, you are talking about electrons, if you are talking about the valence band, then we will talk about the holes. So, velocity of a particle = $\frac{\hbar k}{m} = \frac{1}{\hbar}\frac{dE}{dk}$.

. So, this is fairly you know, simple. So, what we have to do is simply take the first derivative of the E-k diagram, E-k relation. If you do that you get the velocity of the particle. Once you know that, you know if you want to say at k = 0 you want to compute then you will substitute k = 0 and you get the velocity of the particle at the zero point or at the edge of the band you can get that

So, this is how we know we can actually calculate how carriers move in a semiconductor. We can do one more thing. We can take the second derivative. Let us try that. So

$$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m} \Longrightarrow \frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}$$

So what this is telling you is, so what this is telling you is that the effect, no, we call the second derivative inverse of the second derivative as the effective mass. That is all you have to do. So, this m^*, we will basically have, you know, this is the effective mass. So, if you are talking about valence band, relay, we will denote it as $m_h{}^*$. So, mass of the whole $m_h{}^*$, effective mass of whole.

You are talking about conduction band, we will say it is $m_e{}^*$, which is effective mass of the electron. So, this is how we can simply calculate effective masses. So, let me take an example. So, what I am doing here on the left is a problem from the textbook I just picked it up. And I will also give you a couple of more problems in the homework. You should try out.

So, what you have is parabolic relations, 2 parabolic relations two different semiconductors. Let us say A and B have the dispersion diagrams, which are given E k relations which are given here. So, now what is the effective mass or which one has a higher effective mass? So, we have seen that effective mass is inversely related to the curvature. So, higher the curvature of the band second derivative , you see the curvature right ?

So, if you have higher second derivative, higher curvature, you have lower effective mass. So, in that sense, if you look at this picture, you can immediately analyse that here from when you go from A to B, the curvature is increasing. So, effective mass m^* , should reduce So, basically, in other words, m^* of B should be less than m^*, of A, the reason we can say that is B has more curvature so, lesser effective mass.

We can also calculate exactly. This is qualitatively you can immediately see the curvature of the bands and decide this, we can also calculate it. So, here, let us say there is a dispersion relation given and then the case given the number k at the edge of the band is.08 per Angstrom. So, we can calculate it, how do we do that? Again, simple mathematics. So, basically, this is a parabolic dispersion.

You could write it as simply e is equal to I can write it as

$$E - E_c = ck^2$$

equal to, I do not know what the coefficient is, but I know it is parabolic, so, what I am doing,. E_c basically edge of the conduction band. Sometimes these bands can be shifted to account for that we are doing.

In this case so, in this particular case, E_c is going to be $E_c = 0$. Because conduction band minimum is at 0, this is basically E_c here. So, we do not need to worry about that. So,

$$E = ck^2$$

So, what is the effective mass? We have to take the second derivative. So, let us take the second derivative of this guy.

$$\frac{d^2E}{dk^2} = 2C$$

So, what I can do is, I can combine my relations you know I have this relation from here and then I have this relation. I can combine them and I can define basically

$$m^* = \frac{\hbar^2}{2c}$$

I think this should be fairly correct. So, h cross square by d by dK square so that h cross square by d. This is the relation. So, given any dispersion we can calculate the effective mass.

And one more small point I want to know before I move to the next topic. That is when we say effective mass so, we know m_e is basically mass of electron is 9.1×10^{-31} kgs , this is massive electron. So, whenever we say effective mass many times you see that in the textbooks they refer to the ratio of $\frac{m^*}{m_e}$ So, $\frac{m^*}{m_e}$ is what a lot of times you see in the textbooks.

So, it will be some number like maybe you know 0.3, 2.5 something. It can depend. These are very much dependent on the orientation in the crystal line you know, a lot of factors actually. So, basically we just refer to the ratio and that means, we are just you know, to get the actual mass, effective mass you have to multiply by m_e. Please make sure you remember that is all. So, this is about effective mass.