Introduction to Time-Varying Electrical Networks

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Lecture 8

Inter- Reciprocity in linear Time Invariant Networks (contd)

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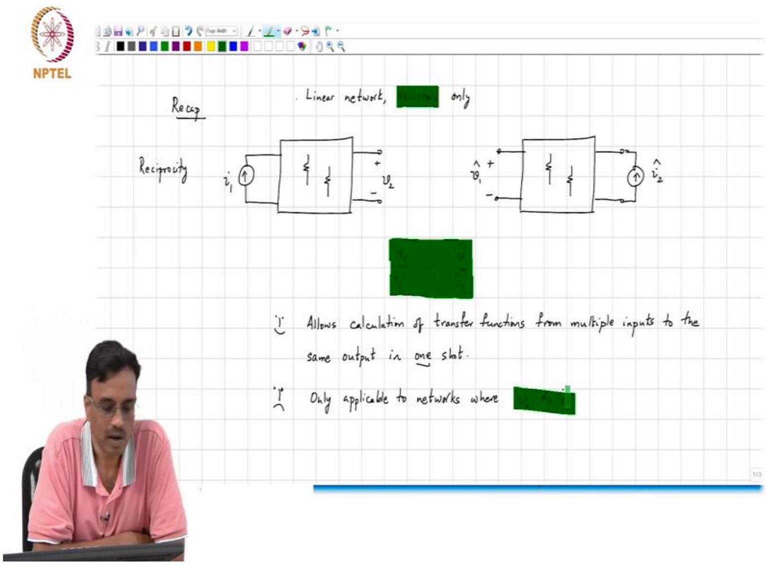

Good morning everybody. Welcome to this is advanced elliptical networks lecture four. A quick recap of what we were doing the last time around. We saw that if you had a network with only bilateral elements namely things like resistors, inductors and capacitors. Where you can relate either, the instantaneous voltage and the instantaneous current. By a resistance or the voltage phasor with the current phasor by an impedance. Then you can within codes interchange the location of the excitation and the response as illustrated in this picture.

And we saw last time that you can use Tellegen's theorem to show that v2 by i1 is the same as v1 hat by i2 hat in the network on the right. Now, so apart from you know the mathematical curiosity of this and its elegance. It is got a very practical application and that application is the following. Many times, particularly you will appreciate this when we start doing noise calculations. Many times, it turns out that you are interested in transfer functions from multiple inputs to the same output.

Now, that you know the simple-minded way of solving such a problem would be the first instinct as many of you pointed out in the last class. Would be to use network analysis to find the transfer functions from one source to the output, one at a time using superposition. Correct. It turns out as we again discussed in the last class that there is a cleverer way of doing this. If you exploit the fact that the network only consists of elements where you can relate the branch currents to the branch voltages.

You know in this fashion and that is the following. So, you excite the output port and then look at the voltages or currents you know through the independent sources inside the network and this way you solve the network only once, but in one shot we are able to get all the transfer functions that we want without having to evaluate the network multiple times. Now, that is you know, one of the advantages or one of the practical uses of reciprocity.

Well, there is a still an issue and that is that no, while this is great. You know most practical networks that we deal with as you know analogue circuit designers are networks that consist of controlled sources. Voltage controlled, voltage sources, voltage controlled current sources, CCVS and VCCS and clearly there every branch voltage cannot be expressed as a linear function of its branch current.

Because the branch voltage could be dependent on, the voltage across some other branch if it is a voltage-controlled voltage source and so on. So, this is a great seemingly useful idea. Unfortunately, its applicability seems to be limited in practice simply. Because you know we do not deal all the time with networks where every branch voltage is related to its branch current.

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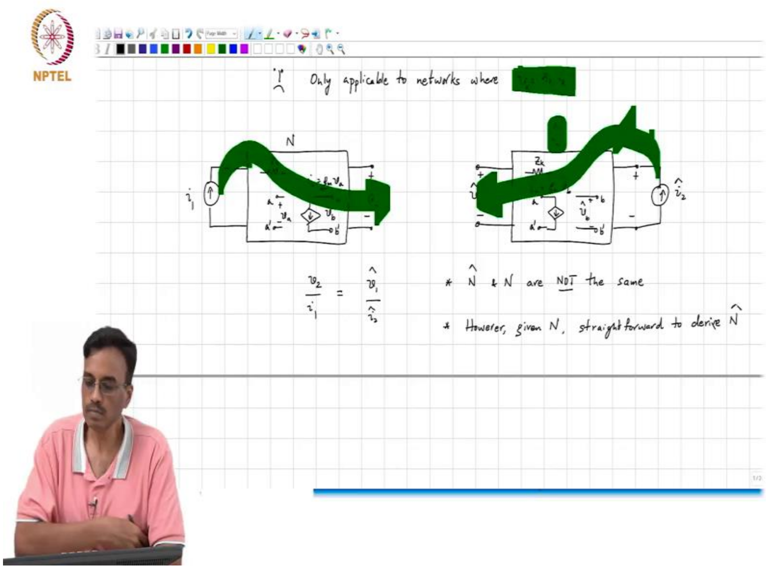

So, one way around it as we saw last time was to recognize that the problem with reciprocity as we saw it occurred only as far as the. So, this is i1, this is v2 and this is the network n and as far as the branches where you have, I believe we call these a and a prime and b and b prime this is v b prime and this is I b prime, I mean this is v a and v b, this is i b. Which happens to be g m times v a. This is r k or z k whatever.

So, when we did the, did try to relate the port currents of, when we wrote Tellegen's theorem we found that it all works out except for the contribution of this controlled source and we said well we found within ports a hack. We said well it seems pretty straight forward to fix that problem and that is well we just move the controlled source this way, so this is v b hat and this current i a hat is what we see last time gm times v b.

If we did this, then you know all the terms other than the port voltages and port currents when you write Tellegen's theorem just simply cancel out and we see that like with a network with bilateral elements only. We again have v 2 by i 1 is v 1 hat by i 2. So, this network as you can see, can you comment on whether this network and this network are the same.

Student: (())(7:33)

Professor: Are N and N hat the same? They are not the same because we have taken that controlled source and flipped the controlled and the controlling ports. That seems intuitively reasonable because in this in the network N signal flows from, signal flows like this and we have some transfer from v 2 to i 1 and if you want the same transfer from i 2 hat to v 1 hat it seems reasonable that the controlled source points the points the other way. So, I am going to just write down, so please note that N hat and N are not the same. However, given N what comment can you make about N hat.

I mean is it like rocket science to find N hat or what. It is just a matter of flipping the controlled and controlling ports in the N to arrive at. Now, if there were multiple voltage controlled current sources, what comment can you make about, what are you going to do to derive N hat?

Student: (())(09:33)

Professor: Very good. So, if you have multiple voltage controlled current sources what you want to do is simply flip the orientation of all the voltage controlled current sources to derive this N hat and another aspect that I would like to point out is that if gm is set to 0, whether you have N or whether you have N hat.

Please note, that the port impedances at between a, a prime and b, b prime remain the same. So, if you set gm to 0, what comment can we make about the impedance between a and a prime?

Student: (())(10:22)

Professor: It is an open circuit and likewise, between b and b prime, it is also an open circuit. What comment can we make about N hat if we set g m to 0? It is the same, very nice.

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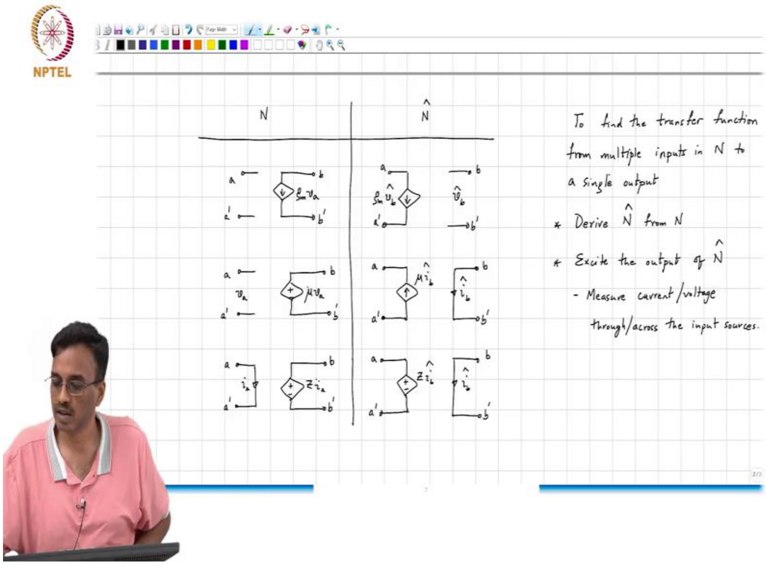

So therefore, in N if we had a voltage controlled current source. So, this is a, this is a prime this is b, this is b prime this is gm v a. Then in N hat we have a, a prime, b, b prime, v, v prime, this is g m. Now, this is a voltage controlled current source. So, I am not going to sit I mean here we did the derivation and found this you can do the same derivation for the other controlled sources. There I will give that as a part of the homework, so that you are convinced.

But I am going to only put down the final result here. Let say you know use our intuition to figure out what we are going to do. Let us if the original network had a voltage controlled voltage source. So, this is v a, this is mu times v a. What comment can we make about, what we need to do in N hat? This is a, this is a prime, this is b, this is b prime. I mean we can derive it but which I will leave for the homework. But, what should you intuitively expect based on our experience so far?

Student: (())(12:48)

Professor: Very good. So, remember the controlled and the controlling ports must be must be interchanged. So, b, b prime must be the controlling port and remember another aspect is that if mu is 0, you want the port impedances to all be the same. So, if mu is 0 between b and b prime in N, what comment can we make about the impedance we have? It is a short circuit.

So, this must be the controlling port and must be a short circuit and so this must be a current controlled source and what must be there between a and a prime in N hat?

Student: (())(13:39)

Professor: Well, its impedance must be infinite when mu 0. So, therefore this must be a current controlled current source. So, if this is i b hat, this must be mu times. A common cause for confusion is the direction of the current and what can we say about the direction of the current, how what is a good way of remembering what the direction is?

Remember, in the original network if you yank v a up, so if v a increases what happens to what tends to happen to v b, v b will increase. Assuming, mu is positive. Now, we turn the source the other way and so if v b increases, v a must tend to must also tend to increase. So, if v b increases i b that i b hat will tend to flow in the direction we have shown. For that direction we must expect the voltage between a and a prime to increase.

And how will voltage across a and a prime increase? When you push current into it. Correct. So, that is an easy way of remembering what direction you need to put the current source. Is that clear? So now, without further ado let us write up the other cases. So, one is a I mean in fact there is only one other case what is that other case? Current controlled voltage source. Because the current control current source is the same as the voltage controlled voltage source.

If you have a voltage control voltage source, we replace it with the current control current source in the opposite, with the controlled and controlling ports flipped. Likewise, if you had a current controlled current source you would replace it with a voltage controlled voltage source with the controlling and the control port flipped. If you have a current controlled current source, so this is i a, this is mu times i a, this is b and b prime. What should we do? This is a prime, this is a current controlled voltage source. So, this is say z times i a, b, b hat. Let us see what we should do. Any thoughts?

Student: (())(16:49)

Professor: So, which is the control port and which is the controlling port?

Student: b is the controlling.

Professor: b is the controlling port and what comment can we make about the, what kind of controlling port, is the voltage across b controlling.

Student: (())(17:12)

Professor: It is a current controlled port? So, this is i b hat and what comment can we make about the controlled port.

Student: (())(17:22)

Professor: It must be a voltage source. This is z times n. So, therefore in summary if you have an arbitrary linear network now. Because, the four I mean the three R, L, C passive elements plus all the four control sources. With these four you can form any linear network that you want. Now, if you are interested in finding the transfer functions from multiple inputs inside the linear network to a single output you do not need to despair. All that you need to do is what?

What is the, to find the transfer function from multiple inputs in N to a single output. What should you do? What is the first step? Derive N hat from N and how do you derive N hat from N? You simply look at this table, all the bilateral elements namely the R, L and C remain the same and all you need to do is flip the control sources as per this table which you know how to derive and therefore, you have N hat.

Then what do you do? So, excite the out excite the output of which port? of which network? Excite the output of N hat. With the appropriate current voltage excitation that the situation warrants and measure current or voltage through or across the input source. Does not make sense?

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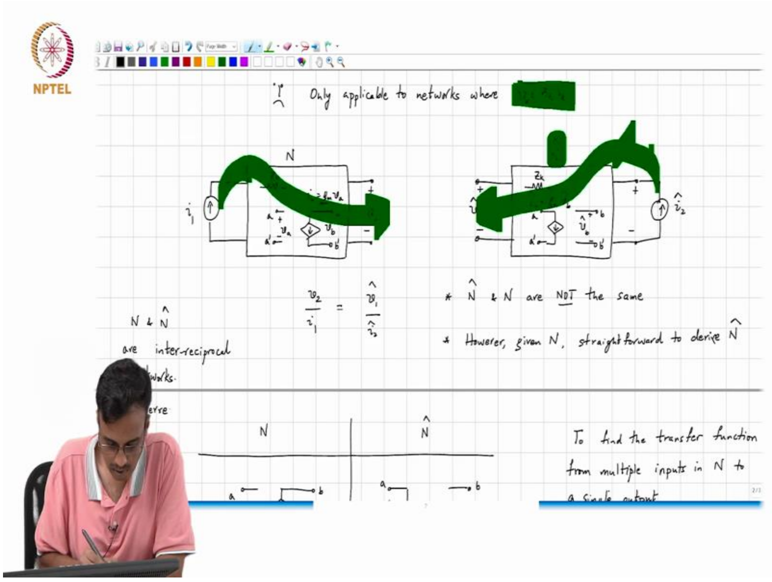

So, you know as he points out, it mean it does not make sense, to keep calling N hat, N hat, N hat. So, it sufficiently seems to be sufficiently nifty and important to have a name. So, I mean you can see that the ports in N and N hat are reciprocal. Except that N hat is not the same as N. But, can be derived from N. So, N and N hat are called inter reciprocal networks and this is called inter reciprocity.