Introduction to Time – Varying Electrical Networks
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Lecture No. 70
LPTV networks with sampled outputs: The equivalent LTI filter

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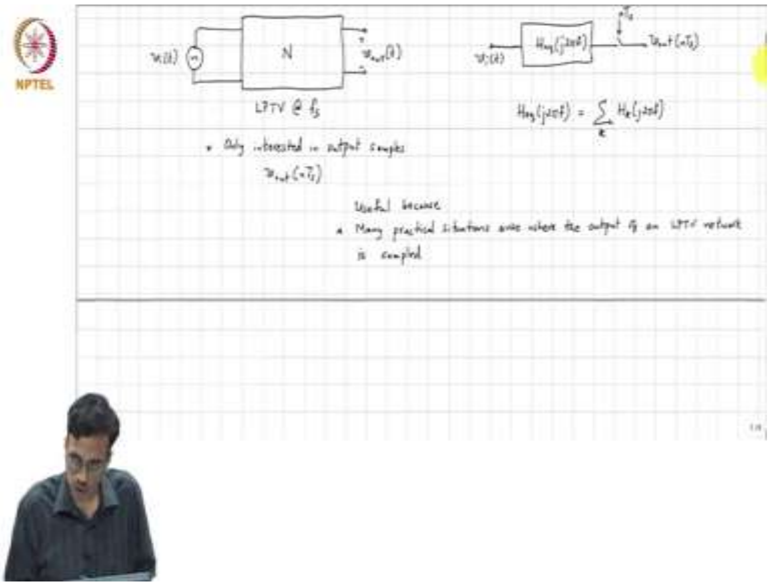

A quick recap of what we were doing in the last class. We saw that if you take an LPTV network and which is LPTV at fs and you excited say with voltage vi of t and look at the output vout of t. And let us say we are only interested in samples in output samples, vout of n comma ts. So, the key points to note are the sampling rate is exactly the same as the rate at which the system is varying.

Then we saw that you can think of v out of ts as taking vi of t processing it through a linear time invariant filter to the transfer function h equivalent of j 2 pi f and sampling its output at nts and h equivalent of j 2 pi f is nothing but.

Student: Summation.

Professor: The sum over all k of.

Student: H sub k.

Professor: H sub k of j 2 pi f. That is all. It is simply the sum of the.

Student: Harmonic transfer function.

Professor: Harmonic transfer functions. So, why is this, a useful result? Well useful because many practical situations arise where the output of an LPTV network is sampled. And we saw several cases yesterday, for example, we saw the sample and hold, we saw switch capacitor circuit, we saw continuous time delta sigma converter. And so, it is only the sampled output of the system which is of interest.

And in such situations, it does not make sense to kind of work with the continuous time output because the continuous time output has got, a lot more information than the samples. If you are only interested in the samples, it makes sense to work with a time invariant system where all calculations, convolution, everything, there is much more easy to do. So, the summary is that this is a very useful result in practice.

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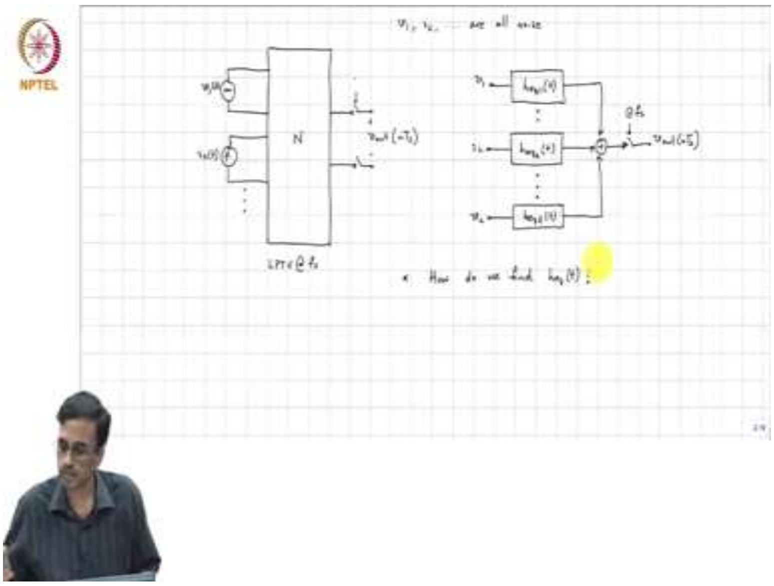

And many times, and because you have so many practical systems, many times you also have the situation where you have multiple inputs. So, you have v1 of t i2 of t blah, blah, blah. And this is a network n and it is LPTV at fs, and the output is sampled at.

Student: Fs.

Professor: At fs. So, this is vout of nts. So, in many cases and a very practical situation that occurs is that when v1,i2, etcetera are all noise. Because in a practical LPTV circuit, it is very likely that you have multiple noise sources. And therefore, you would like and the output is a sampled output. So, you would like to find the transfer function from each of those continuous time noise sources to a sampled output.

And it mean, now that we know this principle of the equivalent transfer function, LTI transfer function, you basically for instance, have say h equivalent 1 of t which is the impulse response corresponding to h equivalent of j 1 of j 2 pi f. So, and this is v1. You h equivalent 2 of t, i2, blah, blah, blah, h equivalent l of t and we have v slash if. So, with our multiple input sources you have multiple outputs. So, you sample this at fs and so, v out of nts is simply the sampled output of the sum of multiple linear filters being driven by different sources. So, the question now is, what are the question that we need to ask now?

Student: How we can target.

Professor: Well, for the obvious question now is how do we. Well, this is fine to say, from every input to the sampled output, you can think of it as the input driving an equal and linear time invariant filter. The question is, how do we find h equivalent of t? We will assume, we will do this for one input and then the same procedure we will follow for all the inputs.

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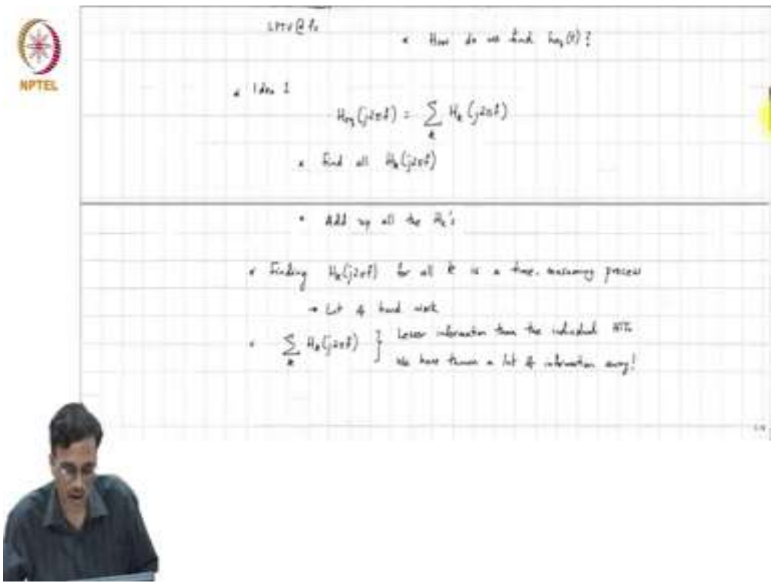

So, remember, I mean, we can think of many ways of doing this. The first so, idea 1, let us say is to say, well we know that h equivalent and j 2 pi f is sum over k, h sub k of j 2 pi f. So, this is basically suggesting. So, find all h sub k of j 2 pi f, and then find their sum. So, then add up all the h sub case. So, what comment can we make about what is your what do we think? I mean, this is a smart way of doing this or a dumb way of doing this?

Student: Finding all h fk by given we have to relate everything.

Professor: Right.

Student: Then use their simplicity.

Professor: So, the remember that to first find all the h sub k of j 2 pi f. So, finding for all k is a time consuming process. So, in other words, you need to do a lot of hard work. And then, of course, once you find them adding them all up is, that is easy. But what comment can we make about the amount of information in the sum of h sub k, versus the information in the individual h sub k is all put together, which has got more information.

Student: Individual harmonic.

Professor: So, basically, there is finding the individual harmonic transfer functions is a lot of work. And you do get a lot of information because you are getting each one of those functions individually then what are you doing? You are adding all of them and the sum obviously, has got lesser information, then the individual parts. So, you are working so hard to get all this information, and then throwing most of it away because you are only interested in the sum. So, this has got lesser information than the individual harmonic transfer functions.

So, we have thrown a lot of information of it. So clearly, if we are only interested in the some of the harmonic transfer functions, it is evidently not a very useful thing to do. It is not a smart thing to do. Well, the next thing is to use something that we saw earlier with respect to reciprocity.

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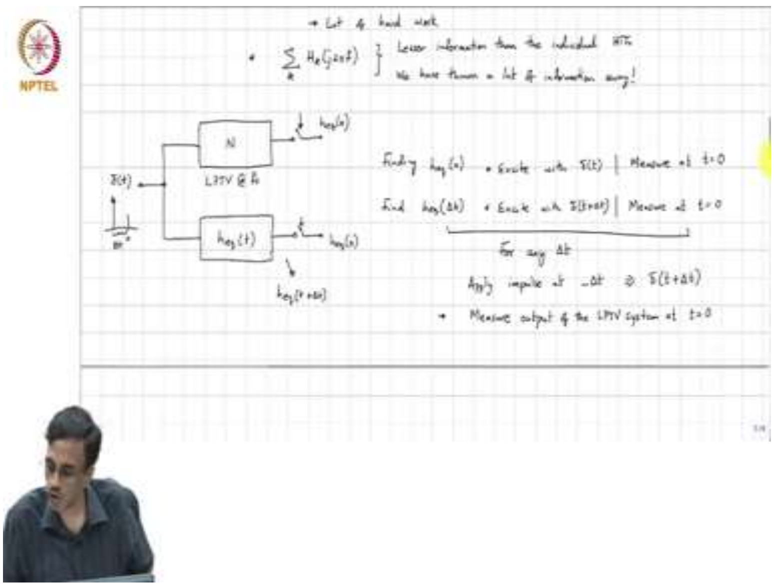

Remember and before that, I would like to draw your attention to another way of doing this. So remember, that if we had the same input exciting both an LPTV system, and an equivalent LTI system. So, rather I would like to call this, talk about this in a time domain h equivalent of t. And if we sample both these outputs, the idea is that I mean, the fact that the lower arm is equivalent to the upper arm means that the two samples will be the same.

Now, the question is what do we do about? How do we get this h equivalent of t. So, let us say, we put in, let us say we excite the input with, let us say, we are only interested in finding h equal to 0, just for argument sake. So, if I am only interested in finding h equivalent of 0, what will I do? What do you, I mean, what can we do?

Student: Time t.

Professor: Time?

Student: T equal to 0.

Professor: 0 what?

Student: We will sample.

Professor: We will sample. But what are you going to do the input?

Student: Input will be ts up to 0.

Professor: Do you understand the question? The question is if I am only interested in finding h equivalent of 0 that is the value of the impulse response h equivalent of t.

Student: T compared to g.

Professor: Very good. So basically, if you are only interested in finding h equivalent of 0, you excite with delta of t. And what do you get here?

Student: H equivalent of 0.

Professor: You will get h equivalent of t. And if you sample at 0 you get h equivalent of 0 here is what you would have gotten if you knew the, h equivalent of t. But the only access you have is I mean, you only have the LPTV network, and you are trying to find h equivalent of t. So, if you sample the output of the LPTV, network at t equal to 0, so what will you get. This must also be equal to h equivalent of 0, measure at t equal to 0.

Now, if you want to find, so let me write this down properly. So, if you want to find he h equivalent of 0, you excite the LPTV network with an impulse at t equal to 0, and you measure at t equal to 0. Now, if you want to find let us say, h equivalent of delta t, where delta t is some small positive time. What do you think you will do?

Remember, this is only, I mean, we are sampling only at multiples of ts, but we want h equivalent of delta t, where delta t is smaller than ts. What do you think we can do? We cannot change the sampling instants, because you are only sampling at.

Student: Of ts, multiples of ts.

Professor: Multiple of ts. So, you cannot say. I mean, you cannot say you put delta of t and you measure at delta t. What else can you do?

Student: You can shift the impulse of delta t.

Professor: To the left or right?

Student: To the like positive time we can say. Like.

Professor: You can only sample at 0 or ts or to 2 ts or and so on. If you shift the impulse to the right.

Student: T plus delta ts we have sample.

Professor: Not even t plus delta ts. So, now think and tell me clearly what should you do to the input impulse so that when you sample a t equal to 0, you will get h equivalent of delta t?

Student: T plus delta t we have to apply.

Professor: So, you have to apply an impulse at.

Student: T plus delta.

Professor: So, you excite, with delta of t plus delta t. In other words, you are applying an impulse at.

Student: Delta t before.

Professor: Delta t before. So, if you apply, this is 0 and you apply an impulse at minus delta t, then what do you get.

Student: H equivalent of t plus delta t.

Professor: You will get h equivalent of t plus delta t. When you sample it at 0, what will you get?

Student: H equivalent to 0.

Professor: So, you will get you will get your measure at t equal to 0. Does it make sense? Now, if you want so, delta d can be.

Student: Less than.

Professor: If you want the impulse response at any amount of time, for any time delta t this is valid for any delta t, correct. So, if you want for instance h equivalent of ts, what will you do you will apply the input impulse at.

Student: Ts before 0.

Professor: Yeah, ts before 0 is what time?

Student: H of minus ts, delta of t plus ts.

Professor: Which is that for you apply the impulse at.

Student: Minus ts.

Professor: Minus ts. If you want the if you want to find the impulse response h equivalent of t at a particular time say ts, then you will apply the input impulse at minus ts and sample the output at?

Student: 0.

Professor: At 0. Does it make sense? So, therefore, for any delta t apply impulse at minus delta t so that, that is that which basically means that you have delta t the Dirac impulses is apply is appeared 00 is applied at delta of t plus delta t, and sample or measure the output of the LPTV system at t equal to 0. So, if you want to find the h equivalent of t for all time, what will you do?

Student: You apply impulse at 0.

Professor: You apply an impulse at 0, measure the output at 0. Apply an impulse run a second experiment where you apply an impulse at minus delta t where delta t is a very small increment in time you measure the output at 0, then you do 2 delta t and so.

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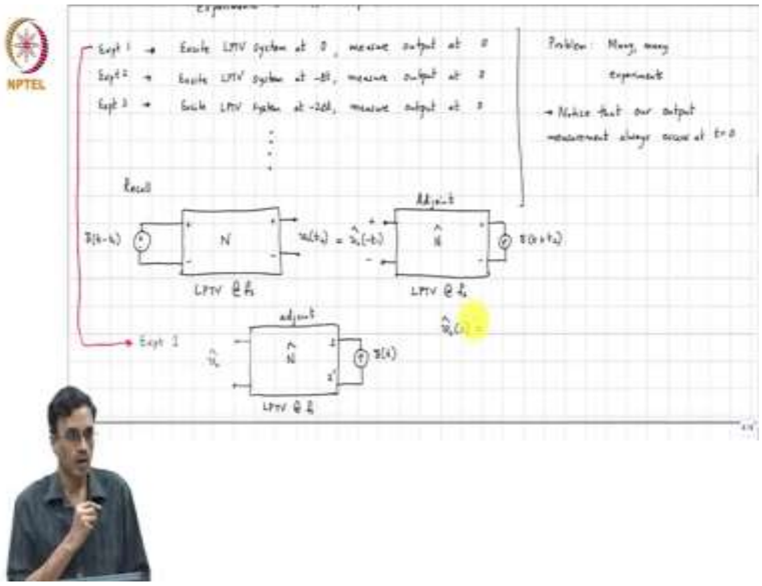

So, experiment therefore, to find h equivalent of t excite, the LPTV system at 0, measure output at 0, excite experiment 1 then experiment 2 is excite.

Student: LPTV system.

Professor: The LPTV system at. When I say excite, it means that all applied Dirac impulses at minus delta t measure output at 0 at experiment three excite the LPTV system at minus 2 delta t measure output at 0, and so on and so on and so on until, do infinite number of experiments, so that you will be able to get the response for all t. As delta, I mean, if you I mean, as delta t tends to 0, you will basically get.

Student: Like all the.

Professor: You will be able to get all the value. Now, what is the problem with this? The principle is okay. What is the problem? I mean, well, you have to run many, many experiments. And, but what do you notice? In all the cases the exciting input is being changed, but the output is always being measured at the.

Student: 0.

Professor: At the same time. So, this basically, now should ring a bell. And now, we should be able to explore it. Remember, whenever the you have multiple do an experiment with multiple inputs, and you have the same output it always makes sense to exploit reciprocity, and that is now what we are going to do next. And the key point is to notice that our output measurement always occurs at t equal to 0.

So, we now exploit or recall that if you have an LPTV network at fs, which is LPTV at fs you apply say this is a voltage at a current source it does not matter. If you apply an impulse at t1 and measure at vo of t2. Another if you take the adjoint network and excited delta of excite the output port at.

Student: T plus t2. T plus t minus t2.

Professor: The excited t plus t2 and measure vo hat of minus t1 and it turns out these to be the same. So this is the adjoint. Now, what do we, how do we apply this principle? Well experiment 1, we excite the LPTV system at 0, you measure the output at 0, so you will get the same output if you take. So, in our experiment, so, experiment 1 which is this fellow here, we will take the adjoint network, this is the adjoint.

We excite the output port with at 0, because we are measuring. We are exciting at 0, we are measuring at 0. So, if you take the adjoint, you inject an output impulse at 0 and vo hat. So, vo hat of 0 must be equal to?

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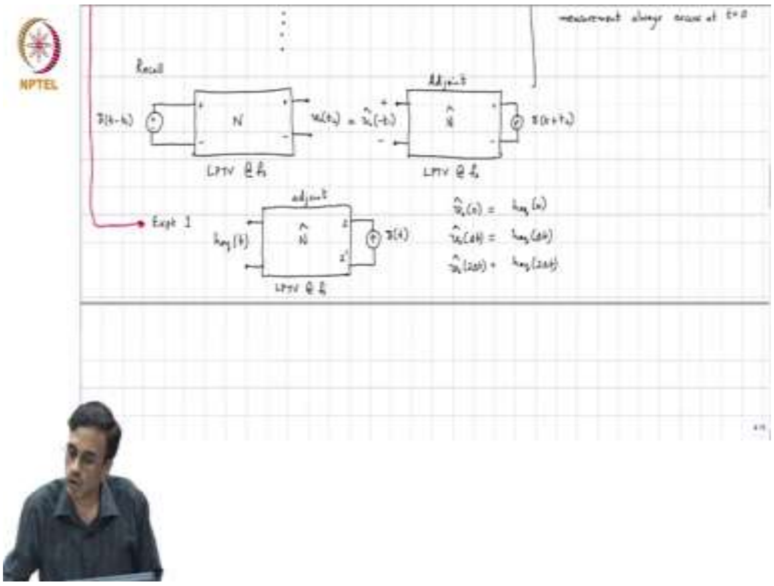

So, vout of 0 that you measure is basically h equivalent of 0. Does this make sense? Now, whatever experiment 2?

Student: You apply the concept of t plus delta.

Professor: You must apply the in the original. In the original experiment you apply, if you had to do it with n, you would apply an impulse at minus delta t and measure the output at 0. So, in the ad join network, what should you do?

Student: We have to apply the impulse at 0 at output port and visit the output at the input port at delta t.

Professor: Exactly. So, but applying the impulse at 0 at the output port, we have already done. So, if you simply measure vo hat at delta t what do you get therefore?

Student: H equivalent of delta t.

Professor: You will get h equivalent of delta t. Does make sense? And similarly, vo hat of 2 delta t is basically h equivalent of 2 delta t and so. So, what is the moral of the story? What do you, what is the waveform? So, if you excite the adjoint network with delta of t, what is the waveform that you will see here?

Student: H equivalent of t.

Professor: You will see he h equivalent of t. Does it make sense?

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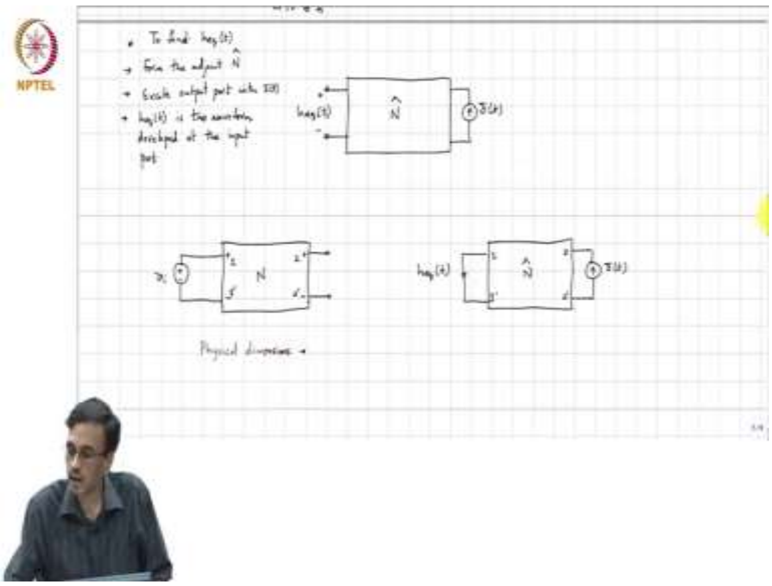

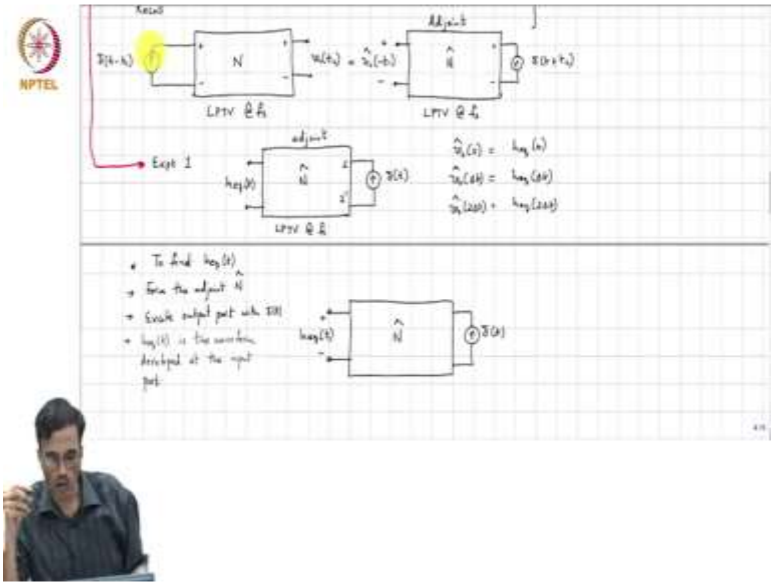

So, as we have seen with in earlier cases, the adjoint is a great way of simplifying the analysis or reducing many experiments into one. So, this is the bottom line therefore, is that you form to find h equivalent of t, you form the adjoint n hat. You excite the output port with delta t and port with delta t. And just simply so h equivalent of t is the waveform developed at the input. So, h equivalent of t is what developed here. And this was this was when, this situation corresponds to.

Actually, in our original network, we had a voltage input. Let us, I think I think I should make it I will just make it current input that is easier. So, this is actually a current source and the output is a, the output of the transfer function of interest is you are excited with the current you measure the voltage. So, in the adjoint, you excite the output port with a current and measure the voltage developed at the input port.

Now, of course, the other corollaries are pretty straightforward. This is, if the network is being driven by a voltage source and the output is a voltage source, then to find h equivalent of t or what should you do. Well, you find that joint so this is 1 and this is 2, 2 prime. And you excite the output port with a current impulse at t equal to 0 and measure the current in the input. So, this current here will be proportional to h equivalent of t. Remember, the h equivalent of t in this case where you have voltage input, voltage output physical dimensions are.

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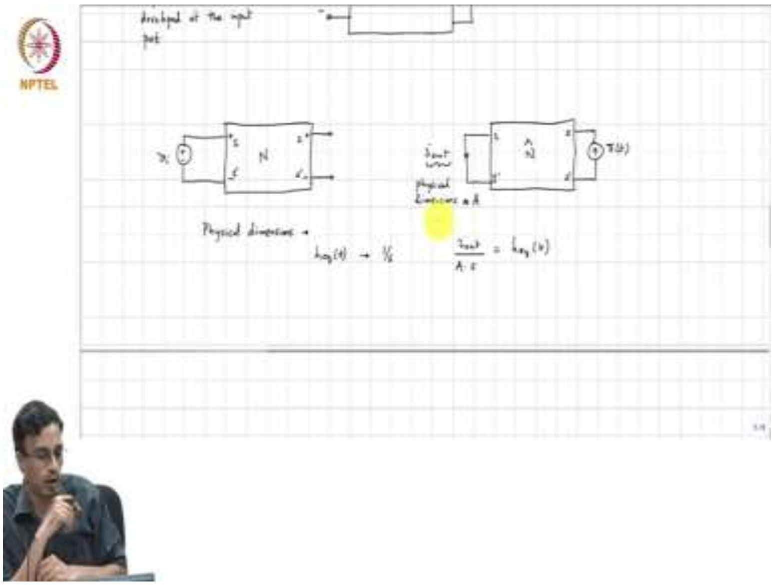

What are physical dimension. We have discussed this already in connection with impulse response long ago. Physical dimensions of h equivalent of t is 1 by second. But this is actually a current which is being developed. So, you have to divide this by ampere second. So, this is if delta of t this is an input current so this, the dimensions of this are basically the physical dimensions of this are amperes.

So, if you have wire and you are measuring current, you will measure amperes, but impulse response has got dimensions of 1 by second. So, to get the so let me kind of be a little more careful here. So, this is iout whose has physical dimensions or amperes so to get from iout to h equivalent of t what should you do, iout divided by amperes time second is h equivalent of t. Does it make sense?

So, the output current waveform has the same shape as the impulse response, but only the y axis has got one is hertz I mean, per second is hertz and iout is amperes. So, you just have to divide by ampere second which is basically cool. So, this is the same thing that we saw in connection with the impulse response when we hit the noise in linear time invariant networks.

And likewise, if we have a current input and a voltage output there, I mean we have already seen current input voltage output, voltage input, voltage output, current input, current output then the adjoint will be a voltage input and voltage output. And similarly, if you have a voltage input current output then you have a current input voltage. I mean I think is the, it is very simple.