Introduction to Time - Varying Electrical Networks
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Lecture - 7
Inter-Reciprocity in Linear Time-Invariant Networks

So obviously, it would be great if we could find a fix to, find some way of being able to use this concept of reciprocity in networks that contain controlled sources.

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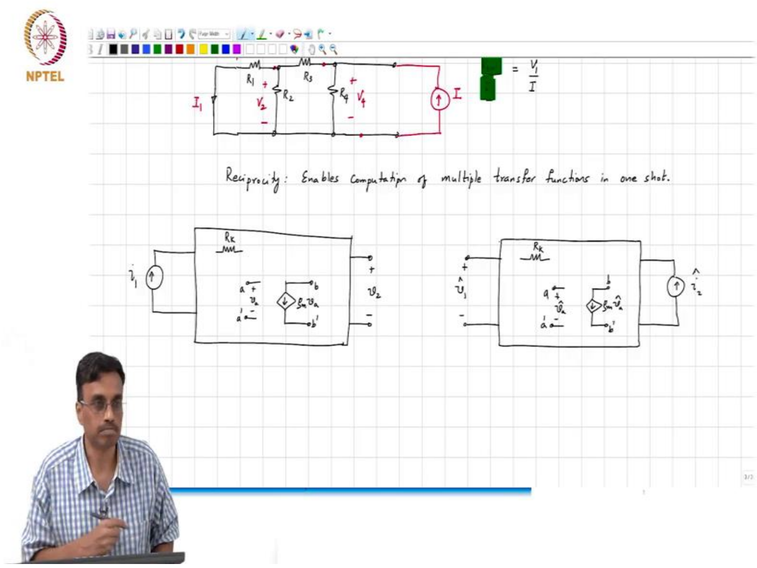

Now, let us see what we can do. And again, let me start with an example where we have, as usual, we have our front with network N. And as usual, let us try and excite the network with a current i1 and measures voltage v2.

And now, we say, apart from resistors, there is also a controlled source. I mean it makes sense to start with simple things one at a time. So let us say there is one controlled source. Once you figure out what you can do with one controlled source, you can figure out what you can do with multiple control sources.

So let us say you have a voltage-controlled current source. So let us call this a, a prime, b, b prime. And this is gm times va. So this is our network N, and now, we say, okay, let us try our luck as you know, fortune favors the brave.

So we just basically say it is a free country, let me try and apply Tellegen's Theorem to what we did with the passive network, where we interchange the location of the excitation. So we will call this, say, i2 hat, this is v1 hat.

There are resistors, which I call R sub k. And let us see what happens here; a prime, this is va. This is va hat actually and this is gm times va. It is b and this is b prime.

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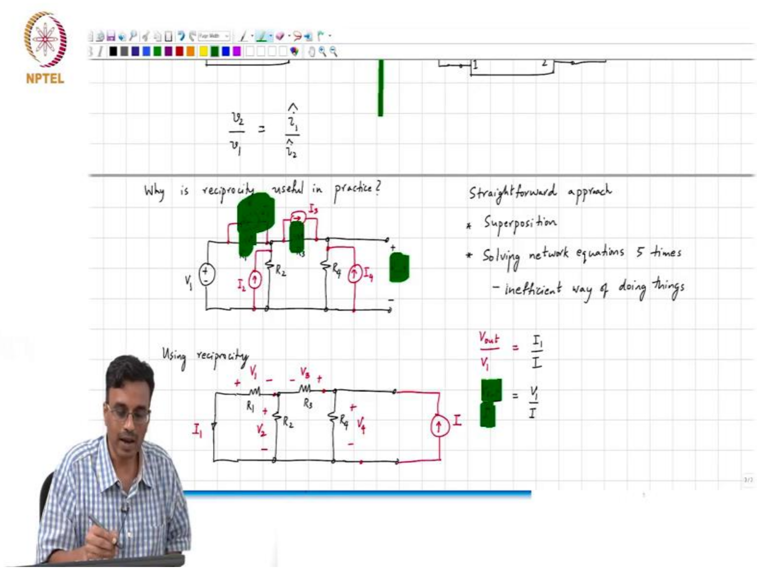

I would like to find this, now, since we brought it up, let me you know kind of digress a little bit. It is always confusing to figure out whether I flows this way or that which way. So what I usually like to do is the following. So if you apply V1 here, it is easier with this example.

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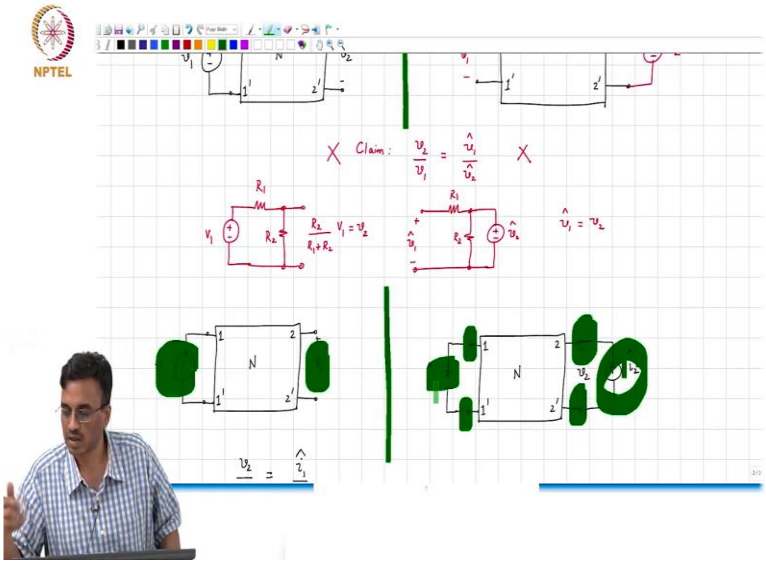

If I apply V1 here, let us assume, I mean and V2 develops here. So if on the other hand, if I apply i2 hat here, there is some voltage developing with this terminal greater than this terminal. So you would, in the absence of a short, you would expect that this voltage will be larger than this voltage.

So the moment you make the short circuit, current flows which way? Current flows downwards. I mean this is not rigorous by any means but this is a if you like to think of it, this is some kind of "mnemonic" which will prevent you from making errors because I mean you do not want to sit and write Kirchhoff's and Tellegen's Theorem every time you want to find out whether the sign of this is positive or negative.

So what do you call, what do you, I want to think of doing is when you replace the excitation, I mean the measurement port with the excitation or there is a voltage developed in that direction, which would cause, normally cause this voltage to be higher than that, the lower potential, and therefore, when you short it the current would flow from the upper terminal to the lower terminal. So that is just the way I like to remember it, you can come up with your own

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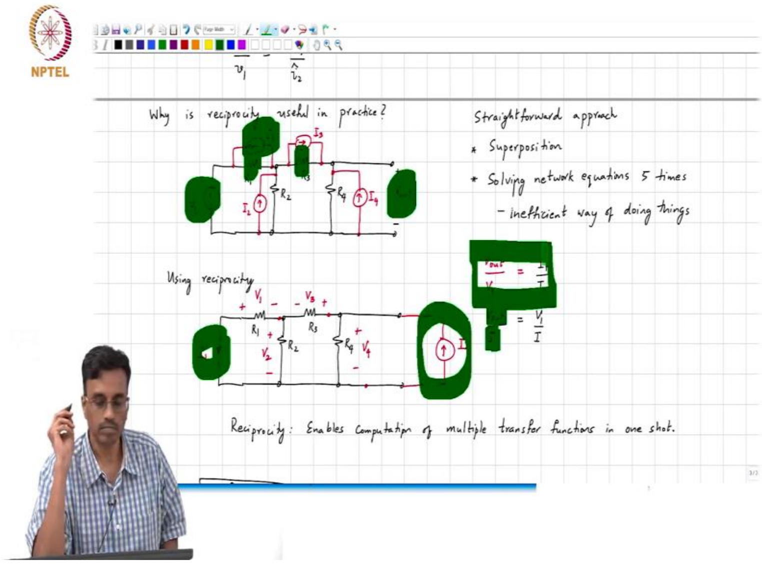

So now, given that, if I apply V1 here and if I get a positive voltage V out, it turns out that if I apply a current like this, the current must flow in this direction for this theorem to be valid.

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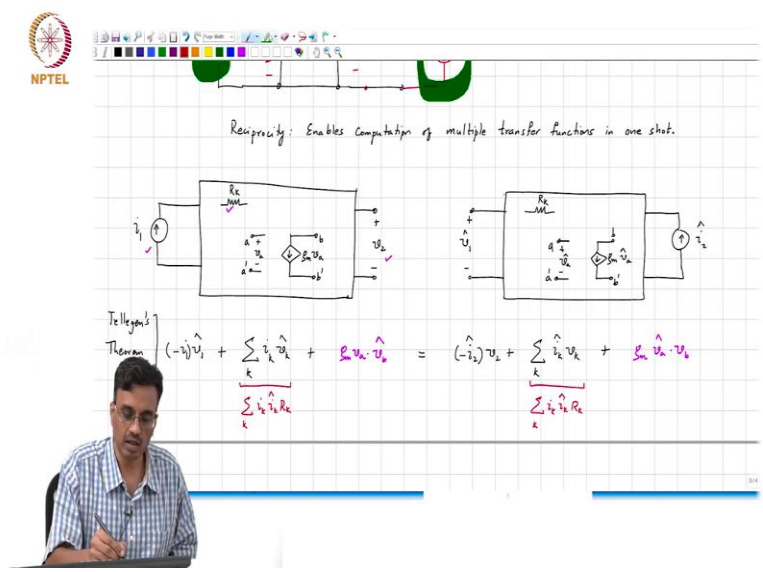

So coming back to, the limitations of reciprocity as we have seen so far stemmed from the fact that we only had elements which typically R, L, and C elements inside the network. And while we like R, L, and C, in electronics, we are dealing with controlled sources all the time. And we would like to see if we can make any headway with with controlled sources.

And that is what we are going to see see next. And as I said, fortune favors the brave, so be just be brave and write the same equations. The same approach that we took for the passive case and see where that leads us. And if all things work out, we are a happy family and go home. If they do not work out, we figure out why they do not work out and fix it, see if we can fix it.

So now, help me with this now. So i1 times v1 hat plus i2 which is, what is i2? I mean, we were just writing Tellegen's Theorem now. i1 times v1 hat plus i2, what is i2? 0. So v2 hat is irrelevant, plus sum over all internal branches, ik vk hat, that would be all that would, we would have to do on the left-hand side if there was no controlled source, now we have a controlled source.

So what do you think we should have here? Plus. So this will, so this will cover, we have covered this, we have covered this port, we have covered all the resistive branches; what should we cover now? The only thing left is the controlled source.

So ik, that is the current in the, in the port here, on the port A is what? It is voltage-controlled current source, so the current in the, in this port is 0. So that is of no concern, plus, gm va times vb hat. So that would be vb hat. And this must be equal to?

Student: (())(08:59)

Professor: Oh, sorry. Sorry, yeah. Thank you. Thank you, right. So what is the right-hand side of the equation? i2, so i1 hat. What is i1 hat? i1 hat is 0, so we do not have to worry about that. So what we need to do is we need to worry about i2 hat. It is actually minus i1 times v1 hat, so this minus i2 hat times, times v2 plus some over all resistors inside, which is ik hat times vk plus gm.

Well, we need to do ik hat. Here ik 0. As far as the first port is concerned, the A port is concerned, the current is 0. So we only have to worry about the, what, the about the B port, and that is what now? gm times va hat times vb.

And these, what comment can we make about this character and this character? This is simply nothing but sum over k of ik ik hat times Rk, and this is sum over ik ik hat times Rk.

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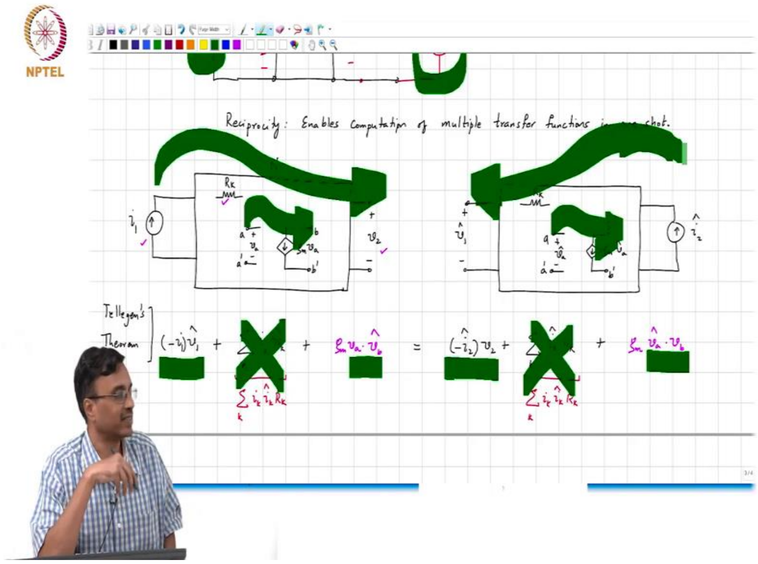

So what content we make about these two quantities? They cancel. Like before, no surprise. And we will be able to get our desired relationship between the port voltages and currents except that, there is this little irritant, which is va times vb hat on this side, and this is va hat times vb.

Are these two the same? I mean, can we move the hat from a to b and then you know be happy? I mean, are these, are those two equivalent; I mean, the quantities in magenta, are they equal? There is no reason why they should be.

So and now the question is what do we do, we scratch our heads and say oh, well, it almost works except for this, this extra term. Now you say, okay, well, I mean, if you think about it a little bit and then say oh, well, it was naive for us to expect that it would work in the first place.

Remember what is reciprocity saying; in English, what does it mean? You, you have a network, you apply an excitation here, you get some response; you apply the same excitation here, you get back the same response here.

But if you have controlled source, the job of the controlled source is to make sure that the output is controlled by the input, but it does not mean that you go and apply the same input the output port and you get a response at the input port.

I mean the job of the controlled source is to do that, so it would be terrible if actually reciprocity worked with the controlled source because the controlled source’s job is to allow signal to flow only in one direction. And reciprocity is saying that you know, it flows in this direction and this direction the same way.

So if you have to have any hope of making this work, I mean, what reciprocity you were hoping to get is to find that the transfer from here to here is the same as the transfer from here to hear.

If that were to happen, you can see that from A to B, signal flows like this. B is influenced by A. If you want the same transfer function when you apply the excitation on this side, here the signal still flow, can only flow in that direction. So that clearly, it is apparent why that does not work.

So what is your hunch, the first thing that you would like to do to try and see, I mean, if you wanted to have the same transfer function from the right or the left, what would you try? Well, I mean, the hunch is to say, okay, well here, the signal flows from left to right, whereas the excitation flows from, I mean, we excited on the right and we expect it to flow to the left, the same amount as it does on the left hand side. But the, clearly, there is a difference in the orientation of the controlled source.

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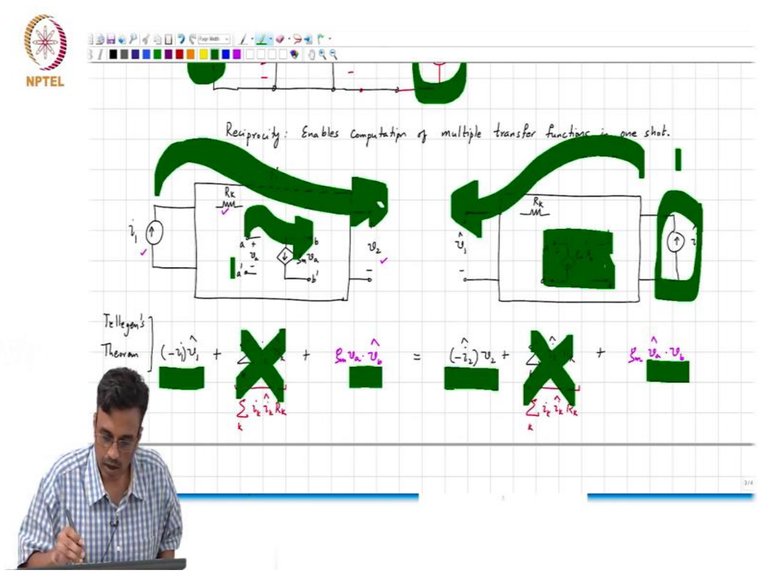

So let me see if I, if I flip the orientation of the controlled source in this direction. This is gm times vb hat, and see what happens. At least it seems to make intuitive sense because here you apply the, the excitation on the left and the controlled source has a controlling port on the left and a controlled port on the right.

It seems reasonable that given that the controlled, the controlled source is the source of all our problems, if we put an excitation on the right and measuring the response on the left, then you know the, what do you call, it seems reasonable to flip the orientation of the controlled sources. And we just try. At this point is just a hunch.

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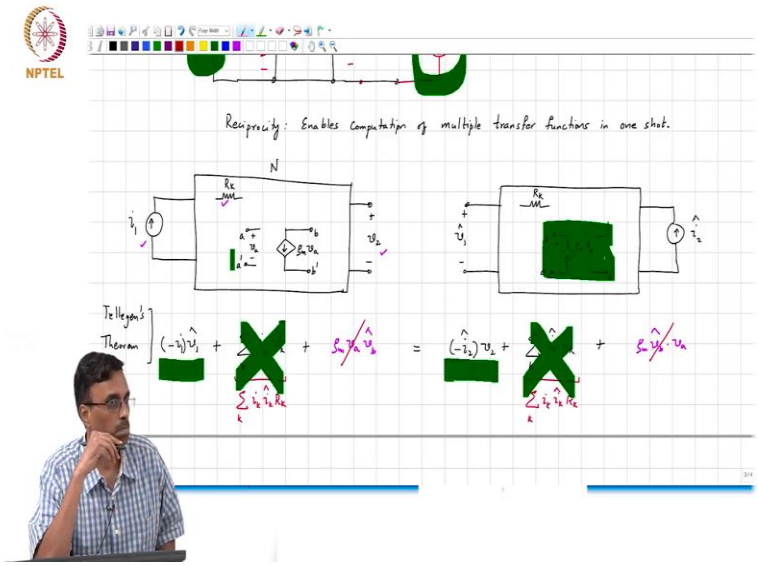

Now, so what comment can we now? So nowthe Tellegen’s Theorem expressions, which expressions will change? Which are the only terms that will change? The ones in, sitting in magenta. So help me now figure that out.

So i1, I mean ia is still, ia is still 0. So and so therefore, ia times va hat is 0. ib is gm times va times vb hat, very good. And on the right hand side, what do we see? ia is gm vb hat times va plus ib hat is 0, so that goes away. So what do we see now? Well, we are now able to, thank god we are now able to cancel of these two terms. Does it make sense? Okay.

So basically, this means that, well, we have an original network with some controlled source inside. However, if you interchange the location of the excitation and the response, not for the same network, but another network where you simply flip the orientation of the voltage-controlled current source, then you will find that you know v2 by i1is the same as v1 hat by i2 hat. Does it make sense?

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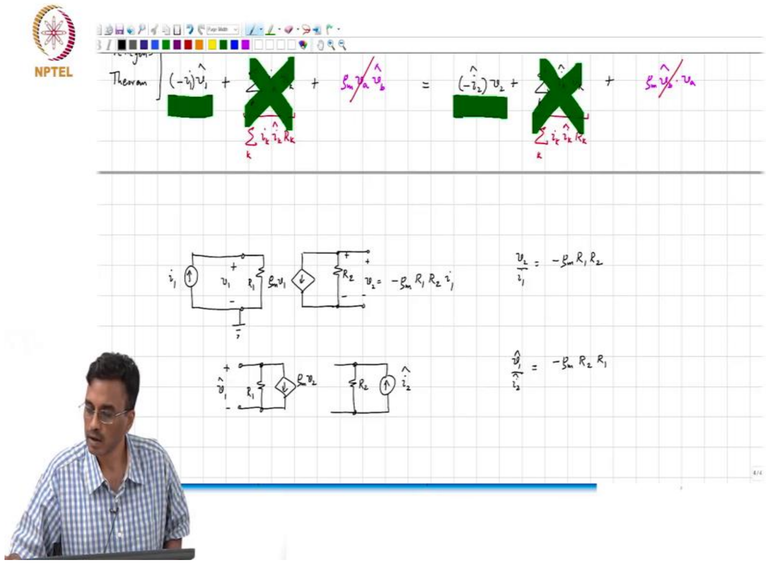

So here, let me take a quick example. The most trivial example is of course just one controlled source. Let us say this is our, sorry, so this is gm b1 and this is R. So what is v2? Sorry, we put in the current, so let us call this R1, i1, let us call it R2. So what is v2? Minus gm R1 R2 into i1.

Now, what am I going to do? I mean, basically, all the resistors must remain the same. Remember, what did we do for the controlled source? We just flipped its orientation. So what should I put here? The controlled port becomes controlling port and vice versa.

So this is R1, this is gm v2, this is R2, and this is v1 hat, and this is i2 hat. So v2 by i1 in the first case happens to be minus gm R1 R2. v1 hat by i2 hat turns out to be what? Minus gm times R2 times R1, and as you can see the transfer functions are the same.

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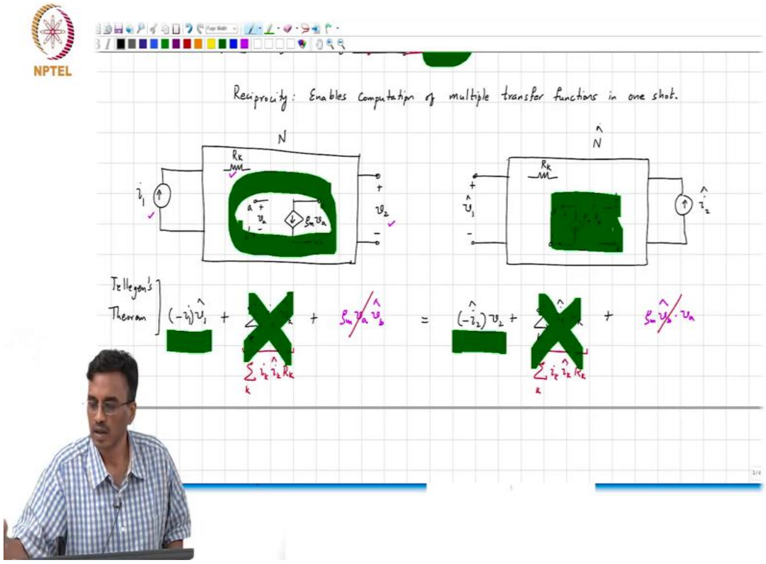

So I mean, so if you had one voltage-controlled current source, we just flipped that one voltage controlled source the other way around. And we have another network N hat where reciprocity like relationship is well.

Now, if you have 100 voltage-controlled current sources, what would you do? You flip the orientation of all the 100.

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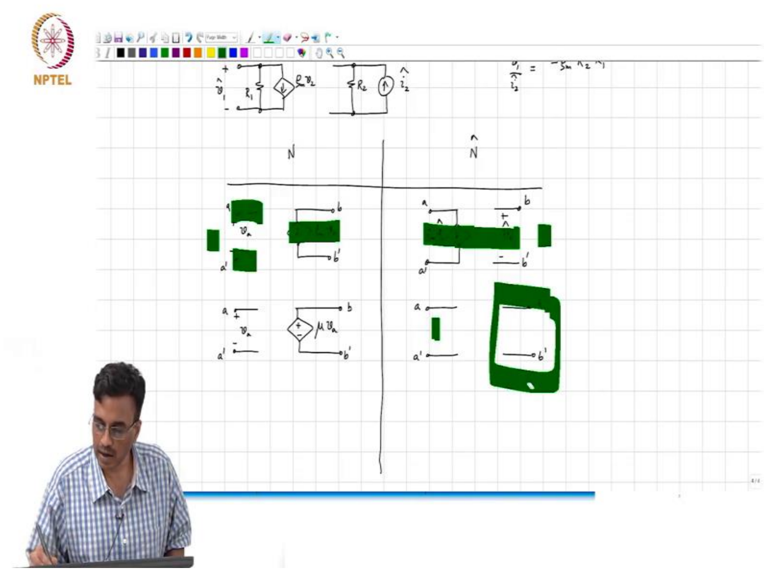

Now, if you had, so in other words, the rules are the following. If you have a voltage controlled, so this is N. You had a voltage-controlled current source in N. To get a reciprocity-like relationship, we must flip the orientation of the current sources, controlled sources as shown.

Now, the next obvious question is what? What is the next obvious question? I mean, this is not the only controlled source we know. We know, we know more controlled sources. The next thing, for instance, what happens when you have a voltage controlled voltage source?

So a, a prime; b, b prime. So this is va, and let us call this gain mu times va. What do you think you will do here? I mean, use the same intuition as before. The, what do you call the controlling and the controlled ports must be reversed in direction. So what comment can you, so which must be the controlling port?

Student: (())(25:48)

Professor: Pardon.

Student: (())(25:50)

Professor: It must be, must be port B. Another aspect that I would like to draw your attention to is the following. What comment can you make about port impedances here and here? So in other words, if you, if you set gm to 0, what comment can you make out the impedance here, across a and a prime. Infinite, it is an open circuit and likewise with b and b prime.

And what about here? It is the same thing. Now, so we know two things. One is we know that this is now the controlling port and this is the controlled port.

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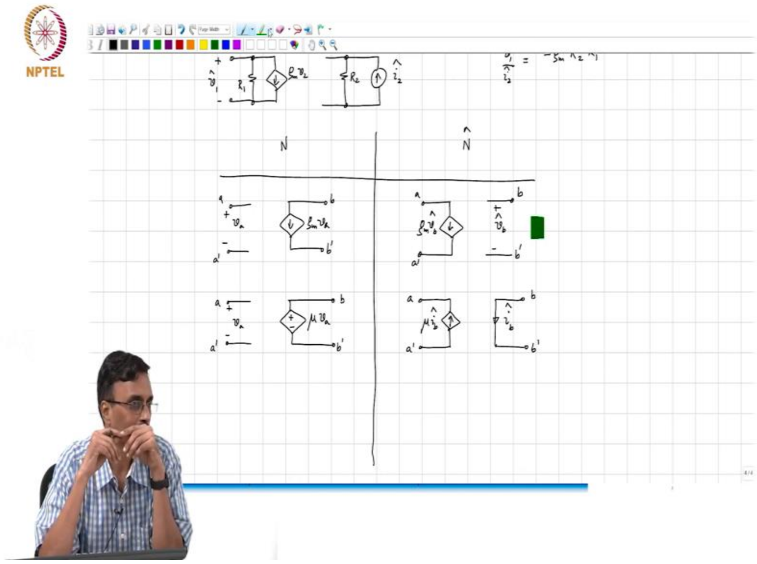

We also like to make sure that the, when mu is set to 0, when mu is set to zero, the port impedances are the same in both directions, in both, in both cases. So when mu is set to 0, what happens to va, is it an open circuit or a short circuit?

On the left hand side, when mu is set to 0, I mean, is the port impedance between a and a prime, is it, it is in finite, what about between b and b prime? It is 0. So b and b prime is controlling port, whose impedance is 0.

So what must, what kind of controlling port must it be? It must be sensing a current. So this is ib hat. And what the impedance between a and a prime on the right-hand side must be in finite when mu is 0. So what, what kind of controlled port must that be? Current source, very good. So this is mu times ib hat. I will stop.

Student: (())(28:33)

Professor: Pardon.

Student: (())(28:37)

Professor: Yeah. Yeah, they work for every single value of mu. All I am saying is that again, to figure out, especially when you have voltage-controlled voltage sources and when you have current controlled current sources, it is easy to forget which side is whether it, you need to put a voltage controlled voltage source on the other side or I mean, a tempting thing to do would be to put a voltage-controlled voltage source on the other side where the, where the controlling and control ports are flow flipped.

The way to remember that is that when you set the controlling parameter mu to 0, the impedances on both sides must be the, must be the same on the left side as well as on the right side.

Student: (())(29:30)

Professor: Pardon.

Student: (())(29:28)

Professor: Okay. So I mean, I think, that comes back to his question, which is, so what. I mean we have done all this, so what, why is this useful? And again it comes back to the fact that in many practical situations, one is interested in finding the transfer function from multiple sources to a single output.

In the passive case, one use the reciprocity to advantage. So you took the same network, you just simply interchange the location of the excitation. And in one shot you are able to calculate all these transfer functions that you are looking for. Here now what would you do?

All that we are going to do as we will show in the next class is that we have this original network with controlled sources. We want to find the transfer functions from multiple sources to one output.

What we are going to do is to form this new network. That is still in our heads. It need not be done in practice. This is just a technique for computation and determination. We are going to create something with, where all the controlled sources are flipped as per our recipe we have shown here.

We need to still consider two more controlled sources, and in that network, we are going to interchange the excitation port with the response port. And as we will see going forward, it basically boils down to, I mean rather than use original network, we new, use this new network, where the reciprocity kind of relations hold, and therefore, again, with one shot, you can get all the transfer functions. Does it make sense people? You get it? Good.