Introduction to Time – Varying Electrical Networks
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Lecture No. 69
LPTV networks with sampled outputs: A continuous-time delta-sigma data converter

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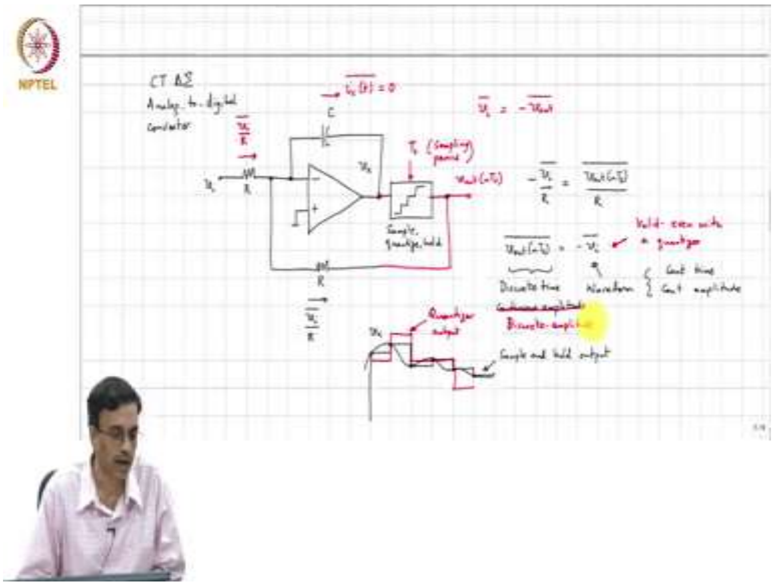

Again, very industrially relevant system that I am going to talk about is basically what I like to call what's called continuous time delta sigma data converter. I will give you a quick introduction for those of you who have not seen this before. So, let us say I mean, here is something that you have seen before, and this is not a delta sigma converter.

This is c, this is vi, and this is v out and this is R. So, the op amp is ideal and there is negative feedback around the op amp. Now, let us assume that the circuit is working, meaning that none of the voltages or currents inside this network go to infinity. So, that, if that is the case what comment can we make about the average current through the capacitor?

Student: It is vi by R.

Professor: If you have.

Student: Average current is 0, sir.

Professor: The average current through the capacitor is 0. If the average current through the capacitors is 0, what comment can we make about the average current here and the average current here?

Student: Both has to be same.

Professor: Both had to be exactly the same. And what is the average current flowing through the input resistor?

Student: Vi by R.

Professor: It is vi by R on average. So, vi averaged by R. What is the current flowing through the feedback resistor?

Student: Vi by R.

Professor: Yeah, so v out minus v out by R. And because the average current to the capacitor is 0, it must follow that vi average must be equal to minus the average value of vi. However, the saying is that the dc gain is minus 1. So there is nothing new here.

Now, what I am going to do. The next thing I am going to do is break the loop like this, and put a sample and hold here. And so, this is sampled at the sampling period is ts. And let us say this therefore, I would now called it v out of nts. So, in other words, what this is doing is this is sampling the output of the op amp at every ts and holding that for the rest of the.

Student: Cycle.

Professor: Cycle. So, this is what we do next. So, what does this mean? What comment can we make now about the average current through the capacitor?

Student: Has to be 0, sir.

Professor: Has to still be 0, assuming the system is stable? That basically means that the current through the input resistor, is must be on average, must be exactly the same as the current through the feedback resistor on average. So, this is going to be still vi average by R flowing that way. But what are they accomplished by putting the sample and hold in the feedback path? The output.

Student: Only vi samples are attached.

Professor: So, vi, on average by R is vout of nts on average, divided by R. Correct, so that must be a minus actually. In other words, so, we out of nts on average is nothing but minus vi on average. So, what have we accomplished by doing this? We are now able to relate the property of a waveform, which is continuous. And of course, continuous amplitude to discrete time and continuous amplitude.

And remember, so, as I said this is called a continuous time delta sigma, analog to digital converter. And analog to digital conversion is convert something which is continuous in time and amplitude to something which is discrete in time and amplitude, and with this we have come halfway there. We have discretized time, but amplitude is still continuous.

So, the next thing logical progression is to basically put a quantizer here. So, you have so, sample quantize and hold. In other words, if vx here was doing something like that the output of the sampler just the sampler would have sample and hold would have done this. So, this is the output of the.

Student: Sample hold, sir.

Professor: Hold output. And this is the quantized output. Now, what comment Can we make about the average current to capacitor the moment to introduce a quantizer?

Student: On average it will be 0.

Professor: The current through the capacitor must still be 0 on average, and therefore, vi on average by r must still be the same current I mean the same current must still flow through the feedback resistor except that now, and therefore, vout this must still be valid even with a quantizer. So, valid even with a quantizer. And therefore, but now, vout of t is now no longer continuous amplitude, it is.

Student: Discrete.

Professor: Discrete amplitude. So, we now have a system which is discrete, both in.

Student: Amplitude and time.

Professor: Amplitude and time, and therefore, it is a digital quantity. And it is related to the average property of the analog waveform vi of t and therefore, this is an analog to digital converter. So, if the input waveform vi of t, vi of t is slow enough then on average vi remains the same as vi of t and therefore, the output sequence remains. I mean so basically you have converted from.

Student: Low frequency compared to the output.

Professor: The average if you take the output sequence which is quantized, both in amplitude and time and you average the output sequence you will get a good approximation to the average value of the input waveform, and this is the principle behind, this is what is called a continuous time.

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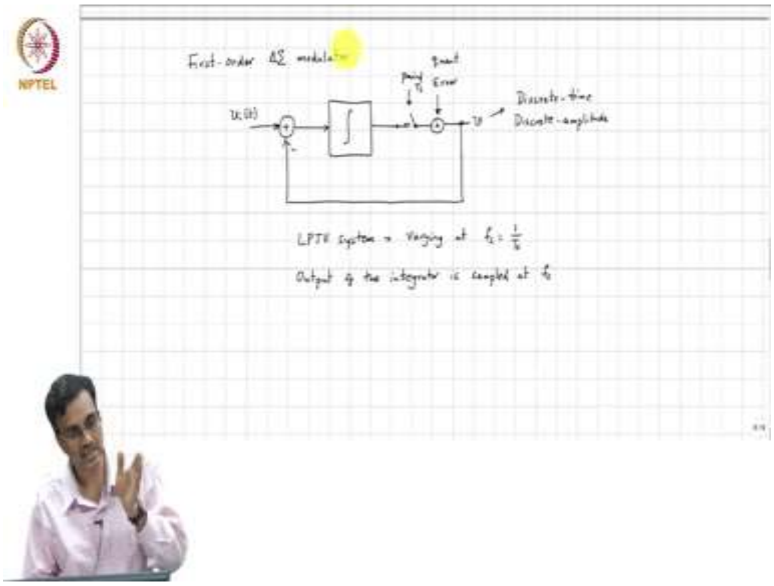

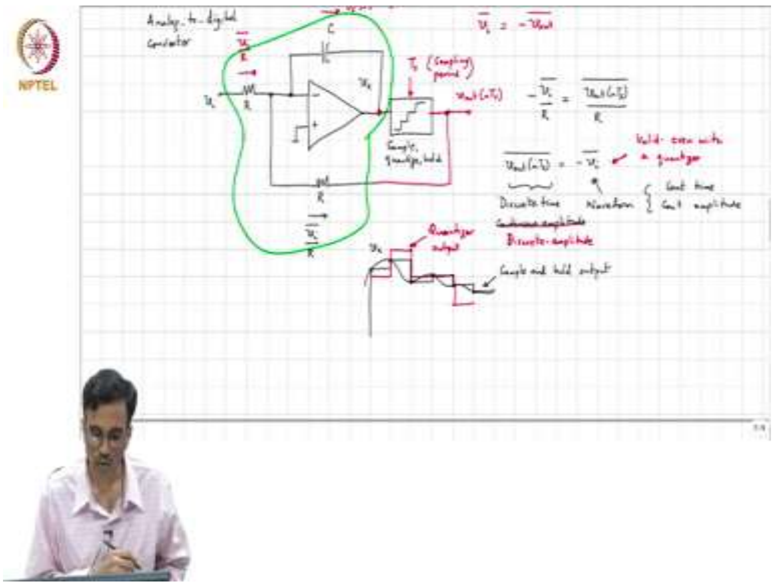

So, there is a sigma converter. So, if you think about it in a block diagram form, we have an integrator. The output is sampled and quantized and quantization is basically you can think of it as some error and you feed it back. So, remember this is nothing but integrator with two inputs. So, this is nothing but a continuous time integrator with two inputs whose one input is the input to be digitized the other input is the output of the quantizer. And so, the quantizer is represented by a quantization error. And if I call this v and so, this is vi of t, and therefore, v is discrete time, discrete amplitude.

And so, as if you look at it as now, if you look at it as a system you can see that this is a periodically. This is a sampler so, this is a linear periodically time varying system, and it is varying at. So, let us say this is period is ts varying at fs actually, which happens to be 1 over ts. And when is the output relevant? What are we doing? We are taking this continuous time waveform.

Student: We are sampling it.

Professor: and we are sampling it at. So, the output of the integrator is sampled at fs, which is the same rate.

Student: System is varying.

Professor: At which, which is the rate which is the same as that at which this system is varying. And so, this is what is called a first order and it turns out that delta sigma modulators or delta sigma data converters are extremely important to practice and you are probably carrying maybe 20 of them in your phone right now. Use all the way from sensor interfaces to wireless transceivers.

So, to cut a long story short therefore, in many, many practical useful applications, we have LPTV systems of course, but what is relevant is the sampled output of an LPTV system. And what is more, the system is sampled at the same rate at which.

Student: The system is varying.

Professor: The system is varying.

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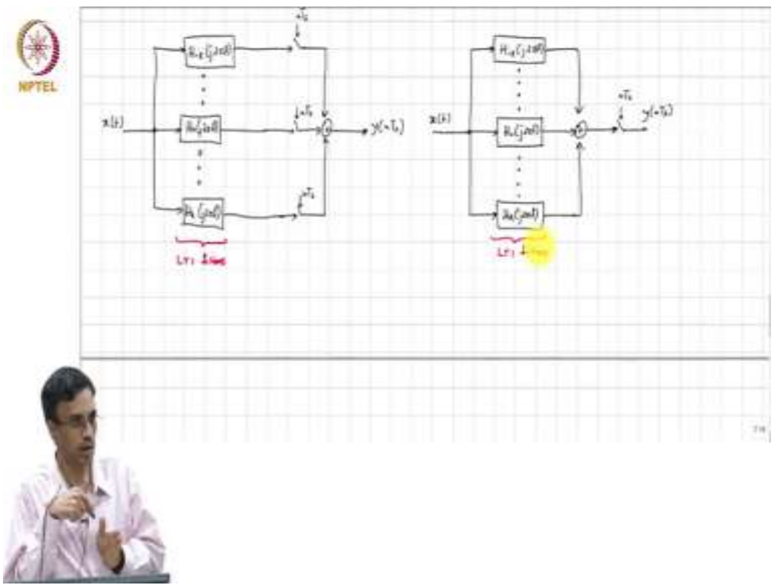

So, in other words, so, let us start off with see what is so special about this, such systems. So, we have the LPTV system where we can basically represent it by its Zadeh expansion. So, we have h 0 of j 2 pi f then we have h minus k of j 2 pi f this is e to the minus j 2 pi k k fs times t h sub k of j 2 pi f. So, this is vi of t. This is, I forgot about e to the j 2 pi k fs times t and this is we are going to call this x of t and y is t. This is the LPTV system. Now, what are we interested? We are interested in.

Student: Sampled value of output.

Professor: In principle we are interested in sampled value of the output. So, this is being sampled at. So, we are interested in y of nts. And just to refresh your memory, these are all linear time invariant filters. And well we are adding multiple, the outputs of multiple branches, and then sampling. You might as well, I mean, adding and sampling is the same as sampling and adding. So, what we are going to do is this is this. Then what is happening here, we are multiplying two quantities right and then sampling the output.

Student: So, it is equivalent.

Professor: It is equivalent to sampling the individual inputs to the multiplier, and then multiplying the two samples. So, if you sample this therefore, you can just basically say, you can move. If we sample this into t, now if I sample this and n times ts what do I get? e to the minus j 2 pi k fs times nts. So, what do you get? fs times ts is 1. So, what do you get?

Student: 1.

Professor: 1. So, this is simply equal to 1. So, likewise in all these branches you will basically get all of this will become 1. So now, what do we, we are sampling the outputs of many filters and then adding the samples, this is equal into moving the addition first and then sampling later, so this is equivalent to h sub 0 or j 2 pi f minus k sub minus k of j 2 pi f, h sub k of j 2 pi f, correct. This is x of t, this is y of nt. So, what is this now? And remember what are all these?

Student: LTI system.

Professor: They are all LTI systems.

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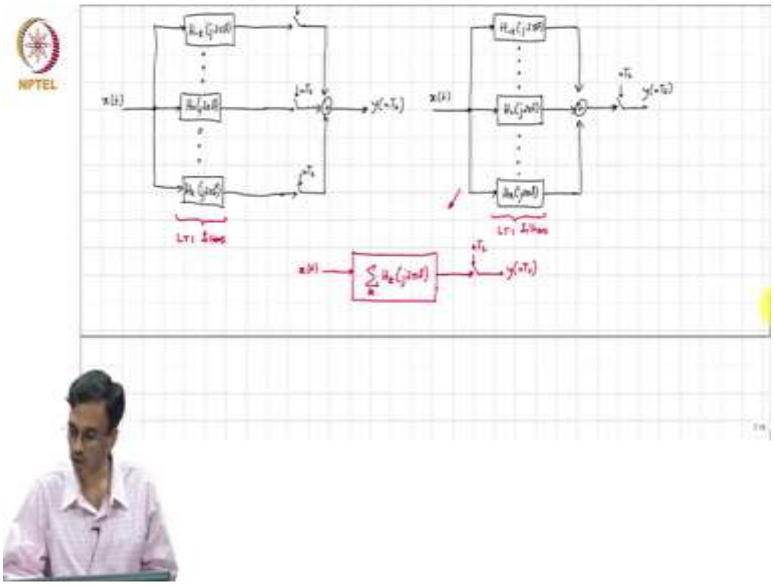

So, if you are only interested in the sampled output, you can think of the LPTV system as. So, this is nothing but x of t going through sigma over k of h sub k of j 2 pi f and the output is sample.

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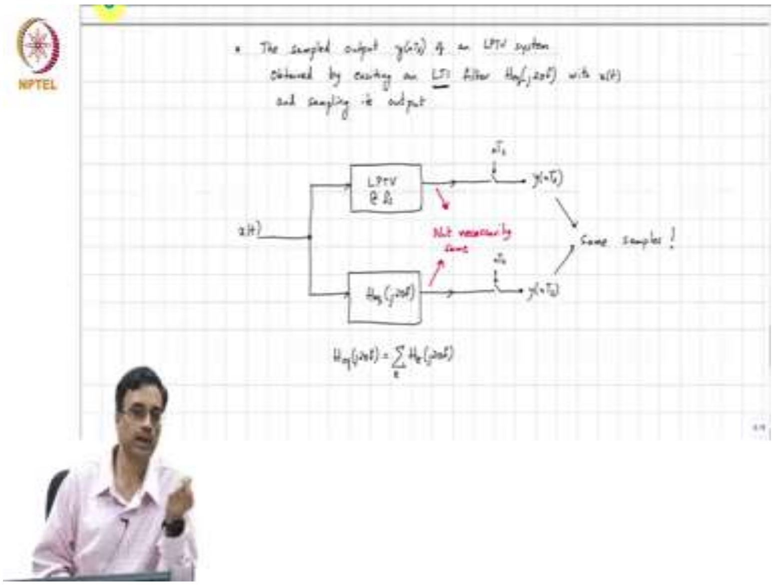

So, in other words, the key conclusion is that the sampled output y of nts of an LPTV system, which is also varying with the same frequency, can be thought of or can be obtained by exciting an LTI filter. Filter, let us call this h equal and have j 2 pi f with x of t, and sampling its. In other words, if I had the same input x of t, so this is the LPTV at fs, and this is a equivalent linear time invariant filter, where h equivalent. I mean, obviously, has equivalent depends on the LPTV system has chosen to be h sub k of j 2 pi f.

And if I sample the outputs in the same nts, I will get the same samples. And this is, this makes life a lot easier. Because if you are only interested in the samples, there is no need for you now to worry about all these multiple harmonic transfer functions and all that stuff. It is simply straightforward. I mean, it is, if you find h equivalent of j 2 pi f, which is simply the sum of the harmonic transfer functions of the LPTV system, then, you can determine the output samples without any problem.

Remember one thing though, that only the samples are the same, these are not necessarily the same. You can have different waveforms there, but when you sample it at n times ts you get the same samples. Does it make sense? And so, this makes life easy in many situations, particularly noise analysis. If you are interested only in samples and so on so we will discuss this in the next class.

The key takeaway today is that if you sample the output of an LPTV system at the same rate, then those samples are indistinguishable from what you would get. If you sample the output of an appropriately chosen linear time invariant filter and what is that appropriately chosen linear time invariant filter, it is that filter whose frequency response or whose transfer function is simply the same as the sum of the harmonic transfer functions of the LPTV system.