Introduction to Time – Varying Electrical Networks
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Lecture No. 66
Time-domain implications of inter-reciprocity and the adjoint network

So, the next thing that I would like to, so that basically completes whatever I had to discuss, as far as reciprocity and inter-reciprocity in LPTV networks was concerned in the frequency domain, let us try and re interpret what happens in the in the time domain.

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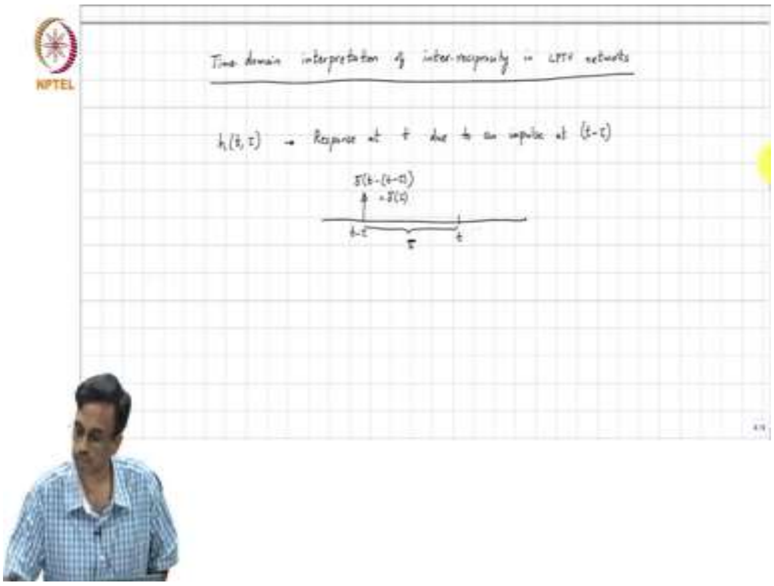

So, a quick review of notation. Remember, h of t comma tau was the response. Very good, response at t due to an impulse at t minus tau. So, in other words rather than pictures, this is the time of measurement. This is the time of application of the impulse, which is basically delta of tau, and this is t minus tau, and this is tau.

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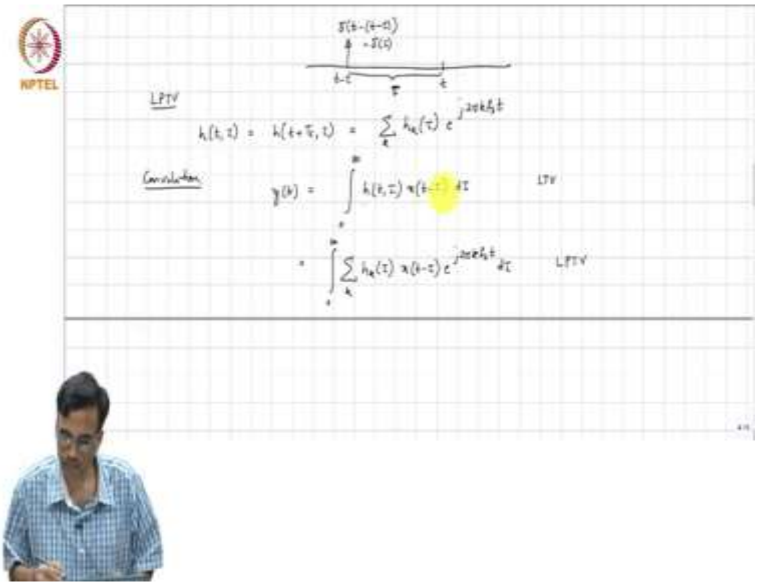

Al. Now, what comment can we make regard to an LPTV network? This is for a general linear time varying system. In an LPTV network h of t comma tau is nothing but h of t plus ts comma tau. Al. And then of course, we have the convolution integral y of t which is the output is nothing but integral 0 to infinity h of t comma tau, x of t minus tau dt. Is there a response at the output due to an arbitrary input? And this is again, for a general LTV system. Correct.

In the special case of an LPTV system, because h of t comma tau s is periodic with respect to ts if you change t by the say by that amount this can be written as a Fourier series, this can be expanded as a Fourier series and this is nothing but sigma over k. H sub k of tau. Because, how will you do, how will you find the Fourier series coefficients, you will integrate this over one period of t because t is the periodic variable. I mean this is the functions periodic in t. So, when you integrate with respect to t, t will go away. So, what will only remain is tau. So, h sub k of tau e to the j 2 pi times kfs times.

So, in an LPTV system therefore, this further reduces to integral zero to infinity sigma over k h sub k of tau x of t minus tau. I will x of t minus tau e to the j 2 pi, k fs times t d tau. Correct, because we are multiplying this with x of t minus tau. So, it does not make I mean, I can push the x of t minus tau before because it multiplies every term.

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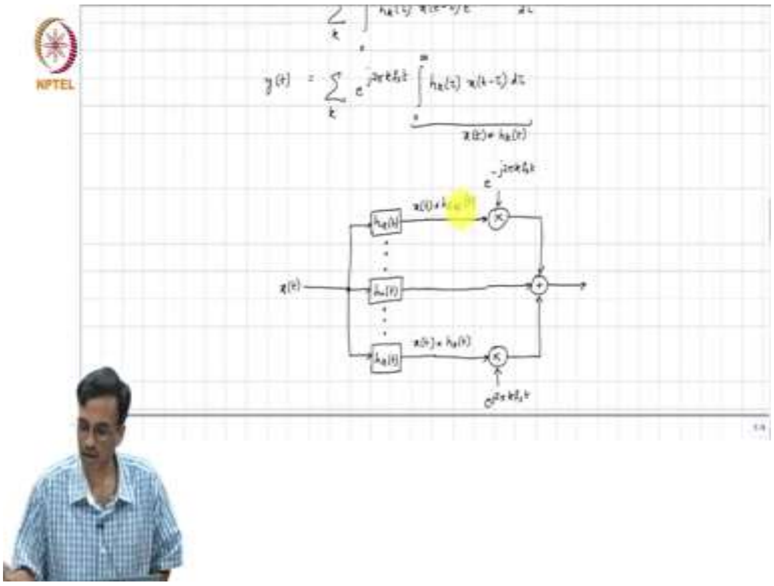

And I can I mean, by adding an integrating or integrating and adding is a same thing. So therefore, this is nothing but sigma over k integral 0 to infinity h sub k of taus x of t minus tau e to the j, 2 pi kfs times t d tau. Now, if you stare at this, what do you notice the integral is relative with respect to tau, whereas, this has got no tau in it, so this can be more outside the integral and therefore this is nothing but the sum over k e to the j, 2 pi kfs times t integral h sub k of tau, x of t minus tau, d tau. So, what does this remind you of?

Student: That is LTI system.

Professor: This is nothing but the convolution integral for an LTI system. So, what does this equation telling us? This equation is telling us that the time in the time domain you come in this is nothing but the time domain equal into the harmonic transfer function stuff that we saw earlier, you basically take x of t, pass them through linear time invariant filters. So, the impulse response of this filter is h 0 of t, this is h minus k of t, this is h sub k of t. And what are we doing to the kth branch?

Student: K might be I think better.

Professor: Very good. So, what we are doing the kth branch, we are going to multiply the, so this will give us x of t convolved. Let me write this spread it out a little bit. So, this is nothing but x of t convolved with h sub k of t al. So, this is x of t convolved with h sub k of t this is you multiply this by eighth j 2 pi kfs times t and you add them all up. Al? This is nothing, but e to the minus j 2 pi kfs times t. That make sense? So, this is x of t convolved with h sub minus k of t.

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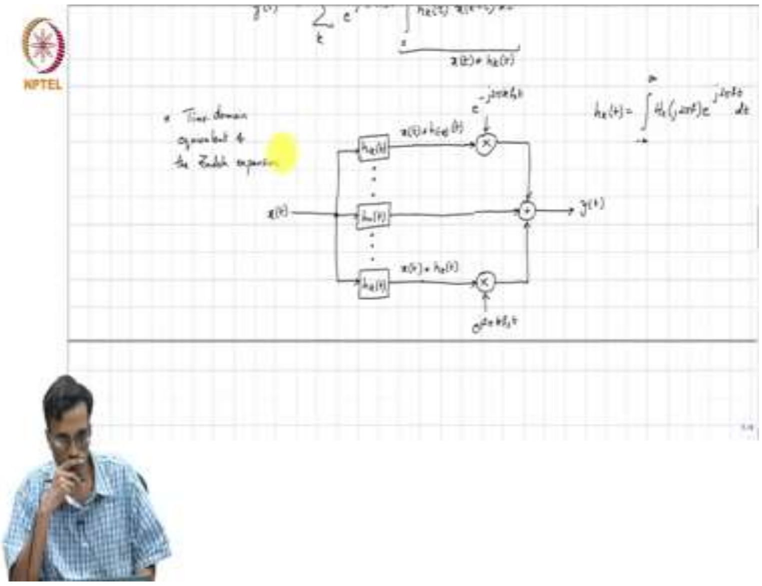

Well, this block diagram must be familiar. This h of t is nothing but h sub k of t is nothing but the Fourier transform, inverse Fourier transform of the kth harmonic transfer function. So, that is nothing but h sub k of j, 2 pi f, e to the j, 2 pi f t dt. This is nothing but the time domain equivalent of these are the expansion so I am going to, this is y of t is the time domain equivalent of the Zadeh expansion.

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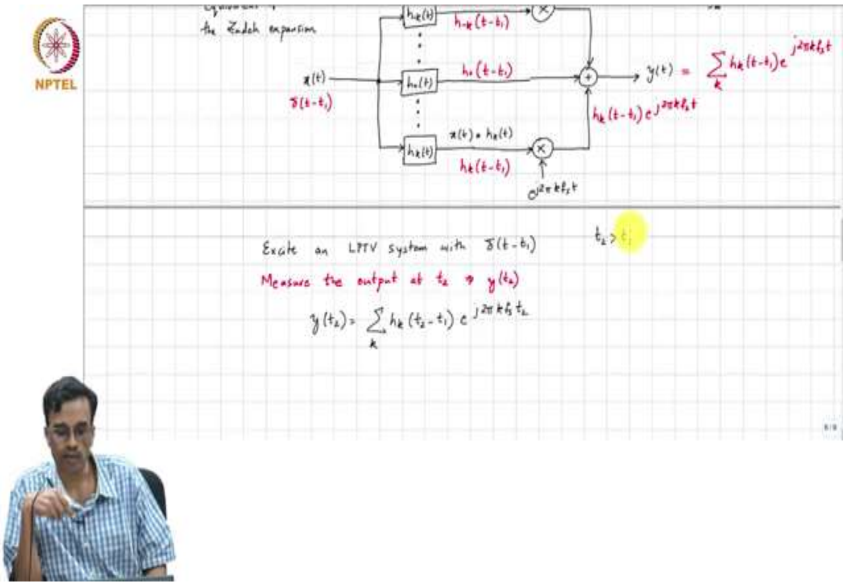

Al, now something that I would like to draw your attention to. Now, let us say we excite an LPTV system with an impulse at t1, correct. So, I am going to draw this so this is t delta of t minus t1. And let us say I am interested in measuring the output at t2. So, in other words, y of t2. And so, what if I excite the system with delta of t minus t1, what would be the output of the kth, excite the filter with an impulse a time invariant filter with an impulse at 0 it gives you hk of t, now you are exerting it with an impulse at t1 because the h sub k is time invariant, you basically will get h sub k of t minus t1.

Here you will get h 0 of t minus t1, here you will get h minus k of t minus t1. So, what will you get here?

Student: It is like a multiplication.

Professor: This is you will get h sub k of t minus t1, e to the j 2 pi.

Student: Kfs times.

Professor: Kfs times. So, what will y of t be?

Student: It is like sum of all those terms.

Professor: Is simply the sum of all these terms and sum over k, h sub k of t minus t1 e to the j 2 pi kfs times t.

Al. So, y of t2 therefore.

Student: Is summation of all.

Professor: A sum over all k.

Student: h sub k.

Professor: h sub k of t2 minus t1, times e to the j 2 pi…

Student: kfs times t2.

Professor: kfs times?

Student: t2.

Professor: t2. Al. And because the system has to be positive, which is larger t2 or t1?

Student: t2 is the answer.

Professor: t2 is greater than t1 because, you know a system is causal. So, therefore, you can only look at the response after?

Student: Applying a nip.

Professor: Applying a nip.

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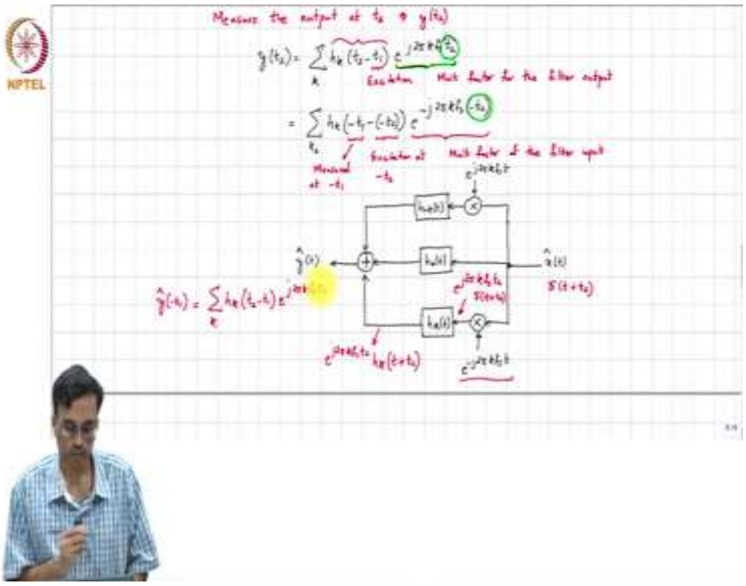

Now, let me show, we can interpret this in a different way. So, and again, this is very analogous to what we did with reciprocity. I can think of this same expression. As h sub k of minus t1 minus, minus t2 times e to the minus j 2 pi kfs times minus t2. Correct? I mean, numerically these two are…

Student: Equal.

Professor: Equal. Now, the question is how do you interpret this?

So, remember in this expression what was the t2, the t1 is the time at which.

Student: Larger.

Professor: In the first expression there.

Student: It is time at which it is updated.

Professor: So, this is the excitation, h sub k is the response of the filter of a time invariant filter excited at t1 observed at…

Stident: t2.

Professor: t2 and the output of that filter is multiplied by, the multiplication is that the…

Student: t2.

Professor: Is at. Is the input multiplied or the output multiplied?

Student: It is k the filter output is multiple.

Professor: Filter output. Multiplication factor for the filter output. Does it make sense? Now, if you want to interpret this equation, going, looking at the equation above what, at what time are we exciting the filter?

Student: minus t2.

Professor: So, this excitation at minus t2 at. And remember, and this is a multiplying factor. Now, do you think that multiplication must occur at the input or at the output? First equation here, you can see that we have multiply. I mean, we are observing the output at t2, and this factor is dependent only…

Student: t2.

Professor: t2. So, therefore, the multiplication is occurring…

Student: At to t2.

Professor: At the output of the filter. Here, we are exiting at minus t2 and this factor also contains minus t2. So, what does this mean?

Student: Multiplying by t2.

Professor: We are multiplying the. So, this is multiplication factor at the filter input. Do you follow? And what is this, therefore, how can you interpret this?

Student: Is the measured at the.

Professor: This is so, this is measurement at.

Student: Minus t.

Professor: Minus t. Al. So, how can you now you now draw the block diagram of this how will this look like? Just for argument's sake, I am going to put the input on the right side and draw the output on the left side. So, x of t, this goes into a bunch of arms, the kth arm whatever the filter is head sub k of t is the impulse response correct. And what is happening to the multiplication? At the output of the filter input or the input of the filter? At the input. And what are we multiplying the kth arm by?

Student: e minus j.

Professor: e to the?

Student: Minus j.

Professor: Minus j 2 pi kfs times t. Al. And so this is going to be h subzero of t al, and this is going to be h sub k of t, this e to the j 2 pi.

Student: Sir minus 2.

Professor: kh minus k, h minus k of t, e to the j 2 pi.

Student: kfs times.

Professor: Plus kfs times. Correct? And what is, what are we doing with all the outputs? You are adding them up. And this is let us call this by hat of t, let us called the x hat of t and then y hat of t. So, what is the saying? If I apply an input at t2, so an impulse at minus t2 so this is delta of t plus t2. Let us verify this again. So, what do you get here? This is going to be.

Student: e point minus j 2 pi kf times t2.

Professor: If you take an impulse and multiply it by w of t what do you get at the output?

Student: e power j 2 pi of k times fs into t2.

Professor: You get e to the plus j 2 pi kfs times t2, times.

Student: Delta.

Professor: Delta of t plus t2. Does make sense? So, what comment can you make about the output of the filter here?

Student: Again that is input.

Professor: It will be e to the j 2 pi k fs times t 2.

Student: 2 hk of.

Professor: hk h sub k of.

Student: t plus t2.

Professor: t plus t2. And so, therefore, if you evaluate this at minus t1 correct. So, y hat of minus t1 is going to be sum over k of h sub k of t2 minus t1 times e to the…

Student: j 2 by k times fs times.

Professor: j 2 by k times fs times.

Student: t2.

Professor: t2.

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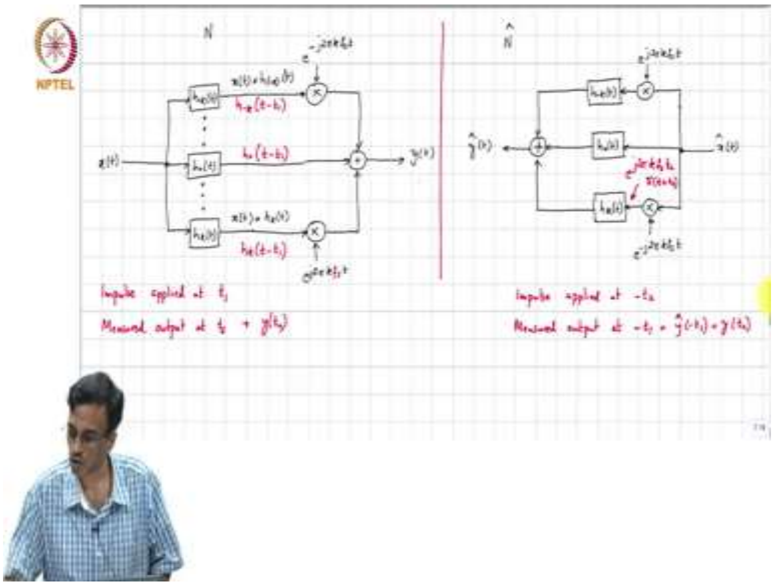

So, I may copy this and earlier. So, you apply an impulse at.

Student: e and after t.

Professor: So, impulse applied at t1, measure that measured output at t2. Measured output at t2. This will give you this y of t2 is exactly the same, as.

Student: Impulse applied at.

Professor: Impulse applied at.

Student: Minus t2.

Professor: Minus t2, and measured the output at.

Student: Minus t1.

Professor: Minus t1. And that was y hat of minus t1 and that is exactly equal to equal to.

Student: y by y.

Professor: Equal to?

Student: Both our input.

Professor: Y of t2. Does it make sense? And what do you see? What is the relationship between the diagram on the left and the diagram on the? What have we done? This is nothing but the adjoint or the inter reciprocal network of where the inter-reciprocal networks of.

Student: Each other.

Professor: Each other. It is clearer or it is not clear?

Student: Clear, sir.

Professor: So, this is the original network n and this is the inter-reciprocal network or the adjoint network n hat. Al.

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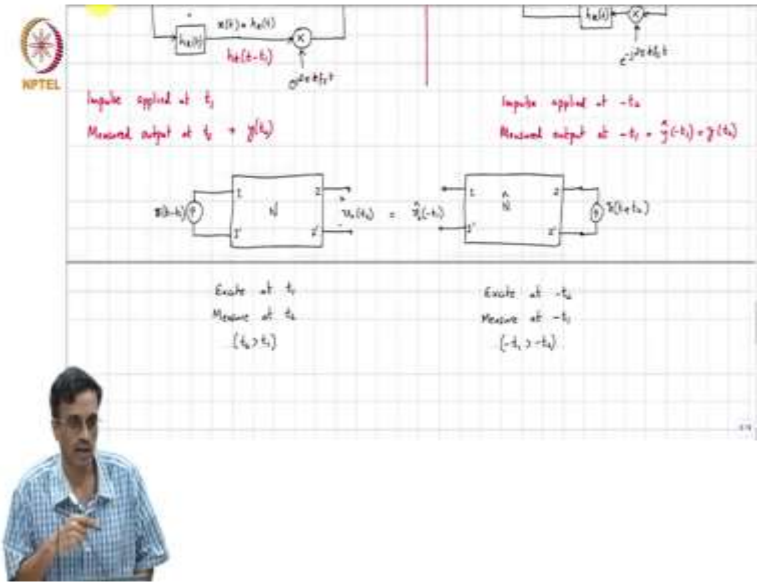

So, what is the moral of the story? So, if you take the network, original network n and you apply an impulse at t1 and measure. So, let us say delta of t minus t1 and vo of t2, and this is that adjoint network, you excite the…

Student: Output port.

Professor: Output port.

Student: Delta of t plus t 2.

Professor: Delta of t plus t 2. And you measure.

Student: Input at minus t.

Professor: vo hat of minus t1 you get the you get the.

Student: Some one.

Professor: Some one. So, vo of t2 turns out to vo hat of. So, you excite at t1 measure.

Student: t2.

Professor: At t2. And for this t2 obviously, must be greater than t1. Correct? You will get the same output if you exit the adjoint at the output port at minus t2, and measure at.

Student: Minus t1.

Professor: Minus t1. And so minus t1 is greater than minus t2. So, this is the time domain interpretation of inter-reciprocity in LPTV networks. Do you understand? Let us stop here. We will continue tomorrow.