Introduction to Time – Varying Electrical Networks
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Lecture 63
Applications of Inter-reciprocity: Analysis of Chopped Amplifiers (Continued)

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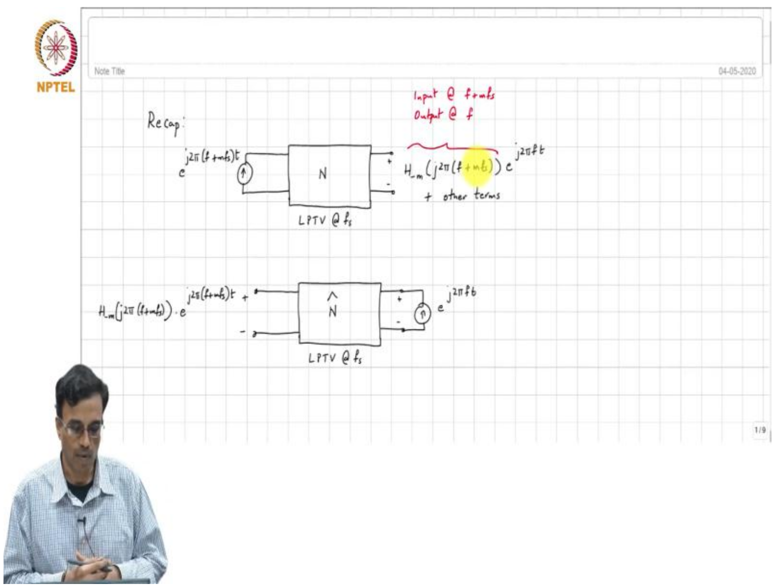

A quick recap of what we were doing in the last class. We saw the frequency reversal theorem and we were applying it to a practical case of chopped amplifiers. So, let us quickly recall what the frequency reversal theorem is and while, in this picture we show a current excitation and a voltage output, the same thing can be extended to other kinds of inputs and outputs. And one form of frequency reversal theorem is shown here.

So, this will, if you take a network N with is LPTV at f s, and you excite it with e to the j 2 pi f plus m f s times t, the output at frequency f will be H minus m of j 2 pi times f plus m f s times e to the j 2 pi f t, plus other terms. And as you know, this number quantifies the ratio of the input at f plus m f s and output at f.

It so turns out, as we have seen in great detail that if you excite the output port of the adjoint network, with a current e to j 2 pi f t, then the voltage developed here will be H minus m of j 2 pi f plus m f s times e to the j 2 pi f plus m f s times t. And this holds for all m and for therefore, if you want to find all H sub minus m of j 2 pi f plus m f s, rather than do this one at a time, where you keep changing m and measuring what the output is at f, it is much smarter to work with the adjoint network.

You excite the output port of the adjoint with your desired output frequency which is f and then automatically whatever develops across the input port, when you expand in a Fourier series, each of those coefficients will give you the H sub minus m of f plus m f s.

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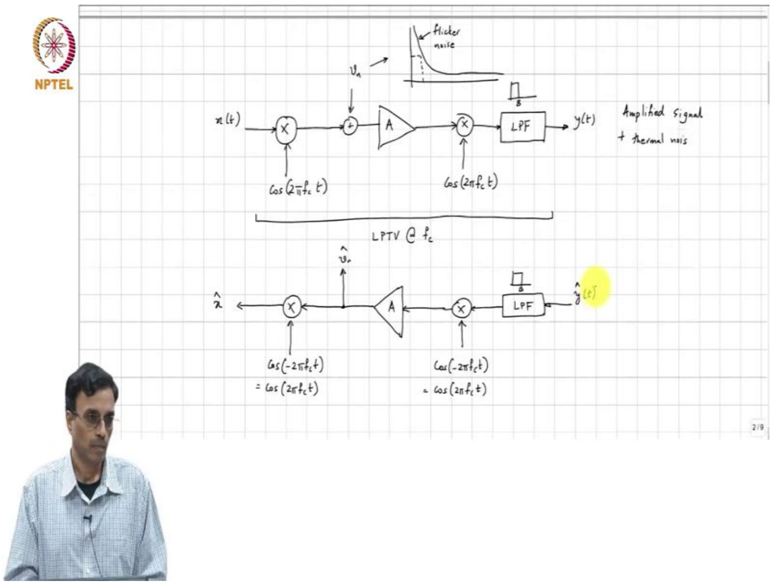

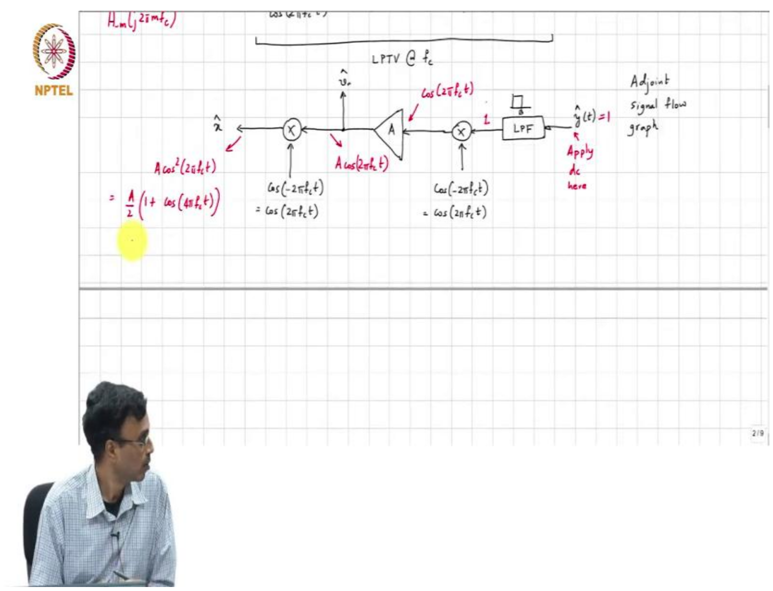

And to illustrate one of the applications of this theorem in a practical context, we were looking at chopping. And the idea behind chopping, at least the basic idea behind chopping is to, as you all know, it is an attempt to address the problem of flicker noise in amplifiers and this is the basic signal flow graph. Of course, in practice, there is going to many non-idealities but, the idea is that the spectrum of x of t will get translated to around f c.

And then gets processed by the amplifier where just remember that this v n which represents the input referred noise of the amplifier, basically has got a thermal noise flow, but at low frequency, it has got flicker noise. And unfortunately, the signal bandwidth is also at low frequency. So, this is a big problem as far as the signal to noise ratio is concerned because the noise spectral density at low frequency is not dominated by thermal effect but by flicker noise.

So, a way to address this is to modulate the input signal to a high frequency which we call f c, the chopping frequency, process or amplify the signal at that frequency and this frequency must be chosen clearly so that at f c and neighborhood of f c the amplifier's noise must be just thermal noise. And then, once you amplify it, the output is amplified but still around f c and then you again multiply it by cosine at f c so that it comes back to base band.

And as we saw the last time around, this is not enough, there is also unwanted artifacts, I mean basically the offset and low frequency noise will get shifted to around f c and there is also a 2 f c component. And all those can be easily eliminated by using a low pass filter. So this is the amplified signal plus thermal noise only. Now, because this is an LPTV system, so if you, for instance, if you put this whole thing inside black box, this is an LPTV system at f c and we would like to, and as with any LPTV system, there is always the danger of multiple input frequencies translating to the same output frequency.

And our output frequency of interest is simply the base band frequency which is close to DC. So, we would like to understand what all input frequencies can translate to base band. So, to do that, we recognize that there are two inputs and we do not know what frequencies can translate to base band. So, one way of doing this would be, for each input, you go on putting 0, f c, 2 f c, 3 f c, 4 f c and so on until you blew in f s and this will take a long time.

The smart way to do it is to use the principle of adjoint network and use frequency reversal. So, this looks like a signal flow graph. So, we can apply what we know about adjoint signal flow graphs and so we will draw the adjoint. And what will you do about the adjoint? We basically reverse the directions of all the arrows. So, this becomes cos 2 pi minus 2 pi f c times t which fortunately is the same as cos 2 pi f c into t because cos is an even function.

And what do we do here? Well, this is a summing node, so this becomes the pick-off point. So, this is v n hat, this is x hat and the amplifier is turned the other way around. And the multiplier is also turned the other way around and this again is cos minus 2 pi f c t which is cos, the same as cos 2 pi f c times t. And this is the low pass filter. It is turned the other way around. The bandwidth to the low pass filter is B, which is very low frequency. And so this filter also has got a bandwidth B which is very low frequency and this is y hat t.

So, since low frequency and DC are pretty much the same, we would like to figure out what all components can alias to DC at the output of this whole signal chain. So, what we do? What should we do? So, this is the adjoint signal flow graph. And what do you think we can do to, we will apply the signal at what frequency? What is f in our case? Base band, low frequency is the same as DC. So, we will basically apply DC at the, so you apply DC here. And what do we do? In other words, what we wanted to find are H minus m of j 2 pi 0 because we are interested in H minus m of j 2 pi m f s, for both f c, sorry f c, for both x and v n.

So, you apply DC here, what do you get here? Let us assume, so this y hat of t is 1. And the reason we are applying DC is that our output frequency of interest in the original signal flow graph is DC. So, if this is 1, what do you get here at the output of the low pass filter? You get 1. If you get 1 here, what are you going to get here?

You get cos 2 pi f c t, if you get cos 2 pi f c t there, what do you expect to see here? A cos 2 pi f c t and if that is A cos 2 pi f c t, what we expect to see at x hat? A cos square 2 pi f c t which is simply nothing but A by 2 times 1 plus cos 4 pi f c t. So, as far as the transfer function from x to y in the original signal flow graph is concerned, what conclusions can we draw from this analysis?

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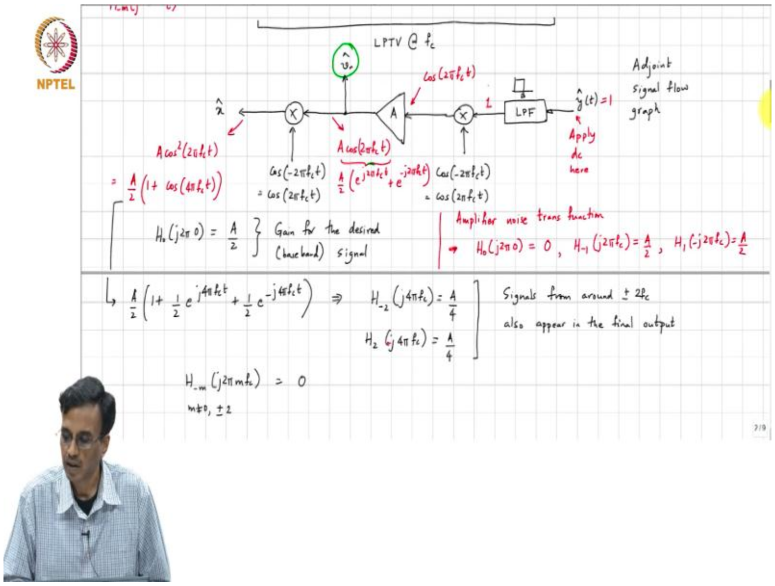

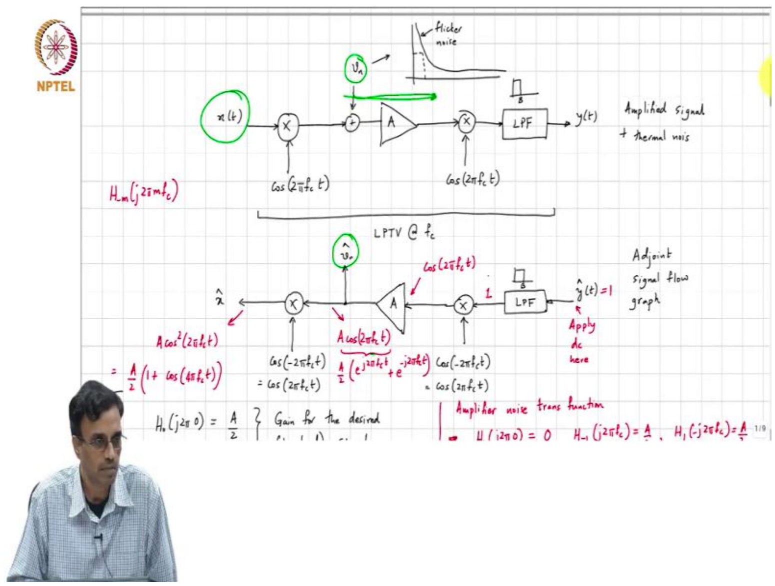

H 0 of j 2 pi 0 will be A by 2, we knew this already. This can also be written as A by 2 times 1 plus half e to the j 4 pi f c t plus half e to the minus 4 pi f c t, which means that H minus 2 of j 4 pi f c is, look carefully, is A by 4 and H plus 2 of j 4 pi f c is also equal to A by 4. And the conclusion therefore is that, while we knew already that our desired signal is going to get amplified with a gain which is A by 2, you also have signals from around plus minus 2 f c which can fold over and will eventually appear like in the same band as the desired signal at the output.

So, signals from around plus minus 2 f c also appear in the final output. What comment can we make about H sub minus m of j 2 pi m f c for m not equal to 0 and not equal to plus minus 2?

They would all be 0. Does it make sense? So, this basically, our adjoint analysis has taught us that we have to be careful about input signals which are around multiples, which are around 2 fc because that can also get translated to base band. And the mechanism actually quite, once you think about it, once you know the answer, it is easy to intuitively see why.

So, if you have an input here which is at 2 f c, when you multiply it by cos 2 pi f c, it will also translate to, there will be one component at 3 f c and there will be a component at f c, that f c component will get amplified through the amplifier and then, a part of that will get demodulated to, half of it will come to DC, another half of it will go to 2 f c. So, that is what you are seeing at the output.

Now, what comment can we make about the transfer functions from, what all components of the amplifier's noise will translate to DC at the output? Well, to see that, we just look the signal that is appearing at v n hat and that A cos 2 pi f c t which can be written, which can be expressed as A by 2 times e to j 2 pi f c t plus e to the minus j 2 pi f c. So, as far as the amplifier noise transfer function is concerned, what do you see? Well, H, what comment can you make about H 0 of j 2 pi 0? It is 0.

And does that make intuitive sense? Why does it make intuitive sense? So, basically it is saying that if you, what does this mean in English? It means that if v n is DC, the output is 0, and why does it make sense? If you put in DC, what is happening? Basically, the DC is getting amplified but then it is getting modulated to the chopping frequency and is subsequently getting eliminated by the low pass filter. So, that is why it makes sense that H 0 of j 2 pi 0 is 0.

Now, what comment can we make about H plus minus 1 of j 2 pi f c, I think sorry, this H 2 of must be, H minus 2 of and this must be minus, yeah. So, H 1 or H minus 1 of j 2 pi f c is A by 2 and similarly, H 1 of minus j 2 pi f c is also A by 2. Now, what is this telling us? That if the input has got some frequency components around f c, they will get translated to, will appear at DC in the final output. And that makes sense because if this is got, if v n has got a component at f c, f c is getting amplified through the amplifier and getting demodulated by the output multiplier and you just see, correct?

But we know that at, I mean by definition, we know that, there is that is only thermal noise there. So, thermal noise from around f c is going to appear in the final output at DC. So, this is an example of using, the advantage of using adjoint signal flow graphs. Now, let me do a little more complicated example. Now, except that, remember, this is okay for a block diagram but in practice there are many difficulties with trying to implement such a scheme. One is, how do you multiply x of t with cos 2 pi f c t. It is much easier to, so what is done in practice is the following.