Introduction to Time – Varying Electrical Networks
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Lecture 62
Applications of Inter-reciprocity: Analysis of Chopped Amplifiers

The next thing that, so we have seen, what you call, a fair amount of theory. Now, the question is why is all this stuff even useful. And to draw your attention to that, I am going to take a few examples.

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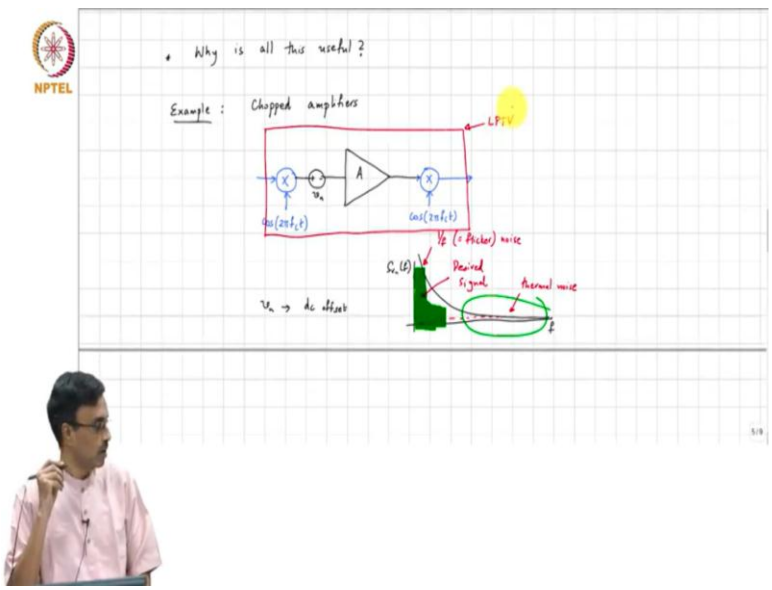

The first example I am going to talk about is, or rather, what we are going to talk about is why is all this useful? And so, there are, as I said there are many signal circuits which are time varying and in all these circuits, it is often a good trick to know, to able to analyze them with a lot of simplicity.

For instance, the first thing that I am going to talk about is chopped amplifiers. And before I get into the actual details, let me try and explain why chopping came about or is needed in the first place. So, it turns out that there are a lot of applications where the signals that are processed or amplified are very low frequency signals. Examples being signals that you acquire from the human body like ECG, KGE, whatever, there are all very slowly varying signals.

If you were trying to measure very small amount of weight, for instance, in a precision balance for instance, this is mostly done using some kind of Wheatstone bridge kind of sensors, you have strain gauges and then you are trying to amplify a very, very small difference in voltage. And of course, this voltage is a DC voltage and needs to be amplified. And the amplifier that you use, let us say, it is got a gain of A and it turns out that almost all amplifiers will have some, what is called DC offset.

So, this vn is an input referred voltage which has got DC offset. Plus, it also turns out that there is something called flicker noise. So, if you look at the spectral density of the equivalent input referred noise voltage, it turns out that it will have a shape like this. So, this is S v n of f and this is frequency and it turns out that this will keep going like this.

Remember, from out earlier studied, we kind of expect that, we are always expect to see thermal noise. So, this is thermal noise but in addition, there is this 1 over f or flicker noise. And it turns out that, I mean our understanding of the physics of flicker noise is, I think there is still a lot of work going on in that area. But as engineers, we have to understand that well this is something that one has to live with, it is not something that one can get away with. The only way to reduce this flicker noise apparently is to make the process cleaner and cleaner.

In MOSFETs, it turns out, that the flicker noise is because of trapping of carriers in the channel at the interface between the oxide and the silicon. And at any rate, the only way to fix this or reduce the spectral density of this flicker noise is to have much larger, is to have much cleaner manufacturing process which is very expensive. You can also reduce it by increasing the area of the MOSFET. And the problem with that is that well, speed becomes compromised. And to reduce flicker noise spectral density by a factor of 10, it turns out that you have to make the area of the device 100 times larger.

So, that is a losing battle. And so why is this flicker noise problematic? Well, if your signal, if this is your signal content, if this is the desired signal, then as you can see, what matters, the noise that matters is not the thermal noise but the flicker noise because in the signal band, it is dominant. And it can be many order of magnitude larger than the desired signal.

So, this is a serious problem and there seems to be no clean, within quotes, device level solution to this, other than saying that I need a much better device, which might be, A, maybe too expensive, B, it might be too, it simply might not be possible. So, you need, what you call, a more system level approach to be able to mitigate the effect of flicker noise.

Now, what do you do? The idea is the following. Well, you know that its only low frequency, its only the low frequency portion of the input referred noise of the amplifier is degraded by flicker noise. The higher frequency part, namely, this area here is basically only thermal noise. So, the idea behind one approach to avoid this flicker noise problem all together, is to say I will take the input signal, translate it to high frequency where we know that the input referred noise of the amplifier is only white, you go there and then you amplify the modulated signal at high frequency, you take the input signal, you modulate it to a high frequency, high enough that you know that the amplified noise is only thermal at that point. And you amplify it.

The amplified signal will still be at this high frequency and then you can demodulate the amplified signal back to base map. So, that is the basic idea and modulation is, as you all know, is simply multiplying with a sin wave. So, for example I am going to assume that this is cos 2 pi f naught t, I am going to call this f c t for a good reason. And I will multiply the output also by cos 2 pi f c t.

And so, this is, so let us see what happens in this case. One thing to observe is that, what kind of, if I now put all of this in a box, what kind of system is this? It is an LPTV system. So, even though the amplifier itself is time invariant, because of the multiplication, this becomes an LPTV system. So, let us see, first of all, how this stuff works. So, if this is x of t, let us assume, I am going to redraw this.

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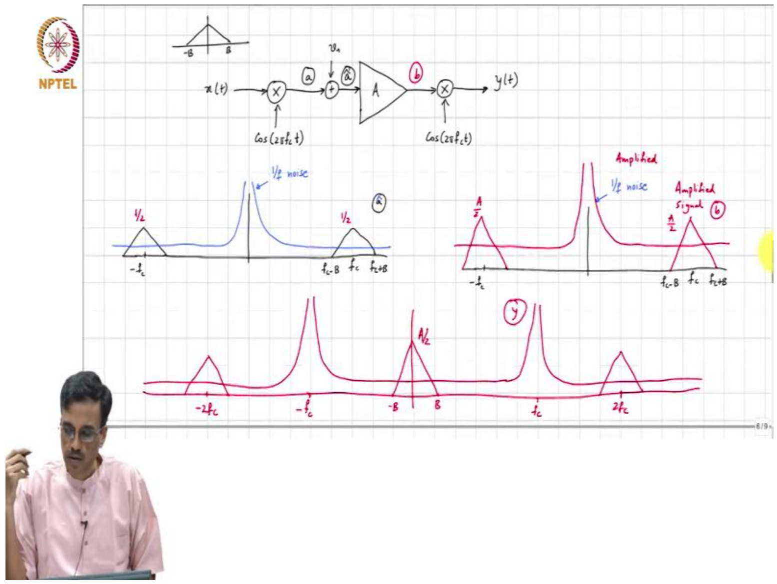

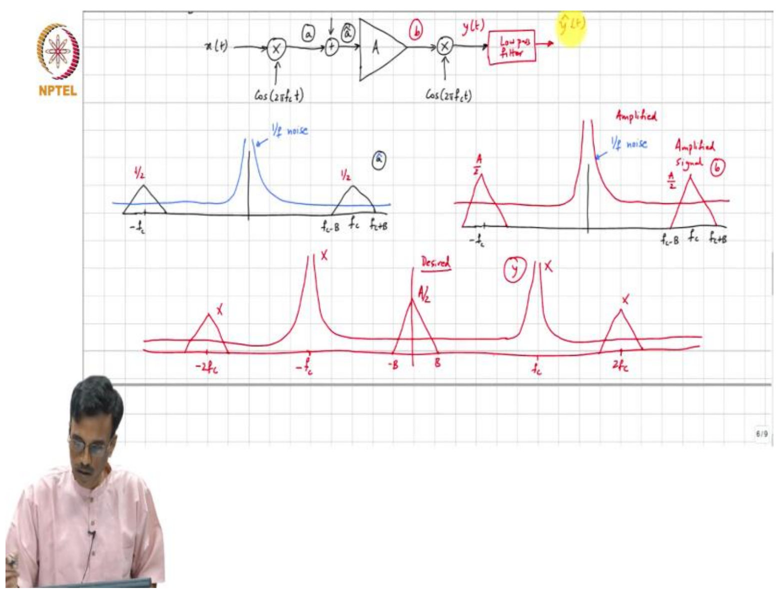

So, this is x of t, the input, f c is called the chopping frequency and this is the input referred noise of the amplifier or actually this will be the, if you are drawing a signal flow graph, this will be a summation, this is v n, this is going to be amplified by again A, this is cos 2 pi f c t and this is y of t. So, let us see what the spectra are at various points.

So, what comment can you make about the spectrum there? Spectra are at various points so what comment can you make about the spectrum there? Let us only draw the positive side or maybe we can draw the negative side also. So, let us say this is x of t, it is base band, with some bandwidth minus B, plus B. So, what happens when you multiply it by cos 2 pi f c t, it gets frequency translated to f plus f c and f minus f c. So, basically, so this is, I think we should draw this more to scale.

This is B and this is minus B. This is, let us assume that this is f c, this is f c plus B, f c minus B and similarly, this is f minus f c and we have the same thing, right? So, the next thing is what happens, well, we are adding, so this is the spectrum at A. What will be the spectrum at A hat? You have to add DC offset and flicker noise, so let us assume that we add flicker noise now.

So, this is 1 over f noise. So, this is the spectrum at A hat. Now, what does amplifier do? Well, the amplifier will amplify everything that it sees, so that goes there, this is amplified flicker noise and this is the amplified signal. This is b, this is what you see at b. Now, what happens, this is amplified signal. And now if you multiply this by cos 2 pi f c t, what do you think will happen?

This will get translated to the left and to the right. So, when it gets translated to the left, so let us say this is y of t, so we need to worry about and by the way, if this is x of f, the amplitude of this is half and this is also half and that is because cos is 1 half of e to the j 2 pi f c and yeah. So, this will be A by 2, A by 2 correct? And now when you translate this to the left, you basically will get A by 4. And you will get another base band tone when you translate it to the right which is also A by 4.

So, when you add the two together, you get A by 2. You will a tone, you get something at 2 f c, just one second, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, that is, you will get something at 2 f c, so this is 2 f c and you will get something else at, so this is the spectrum at y. So, this is minus 2 f c and this is B, this is minus B. But we have forgotten one thing. The flicker noise is also going to get translated. So, what is going to happen? You will see something at f c, and you will see something at minus f c.

So, this is minus f c and this is plus f c. So, we have a whole, this is out desired signal. But we have a whole bunch of undesired components. So, all this is something that you do not want. Fortunately, there are either at a frequency f c or much higher. So, what do you think you will do? We can filter them out, so what you want to do therefore, is to put a low pass. So, this is y of t and this is y hat of t.

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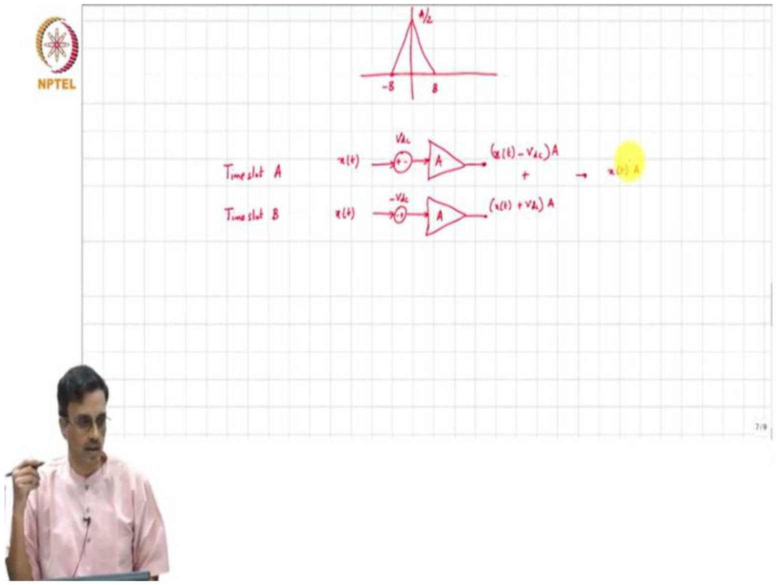

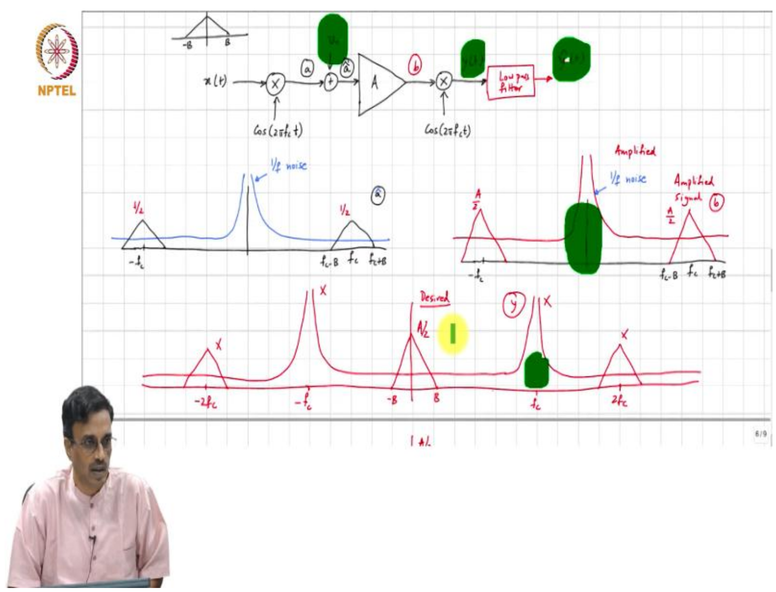

So, y hat of t is, take this and simply filter it with a low pass filter. So, this is minus B, this is B and the gain is A by 2. So, fortunately, as you can see, the even though the amplifier has got a lot of flicker noise, we have avoided the output, does not contain any flicker noise component. Well, we have seen everything in the frequency domain. In the time domain, it is easy to understand intuitively what is happening.

Well, flicker noise is concentrated at low frequency. So, you think of it as a slowly varying DC offset. So, if it is varying very, very slowly, and the slowly is with respect to what, with respect to the chopping frequency. So, if the amplifier is input referred, if the flicker noise is evidently a very low frequency phenomenon and you can think of it as an offset which is changing slowly with respect to frequency. The idea is that, well if you process the signal with positive offset, for a little bit of time and then, with a negative offset, for the next time slot, the offset has become positive and negative.

And whereas, and if you do the same thing at the output of the amplifier, so in other words, intuitively what you can do is, time slot A, you can process the, so if you put x of t, you get x of t minus v d c times A. In time slot B, you put x of t, you flip the sign of the amplifier, I mean you arrange things such that the effective offset has become minus DC. And that is easy to arrange. For instance, you could, you do not have to physically invert the amplifier, you could invert the input signal and invert the output of the amplifier also. So that effectively, you get a negative DC.

So, basically you get x of t plus v d c times A. And if you do this sufficiently fast and you average the two outputs, then you will be able to x of t into A. So, that is idea behind chopping. But because of the periodically time varying nature of the system, you can have a potential for, I mean remember that out output frequency of interest is low frequency. I mean the frequency of our base band signal, our base band signal resides here. That is the frequency range over which our base band signal resides.

But because this is an LPTV system, there is a chance that, other frequency, this LPTV at f c, so we should in principle, expect that all frequencies of the form output f plus L times f c will can potentially alias to out desired output. So, the obvious question is, first of all, what are all the frequencies that can alias, B, what is the strength with which they, what is the gain from those frequencies to this our desired output frequency. And we would also like to find what the actual transfer function is from, and what do we expect, what do we see before we do the math?

A low frequency input v of n, what is happening to it at y? It is getting translated to f c. So, we should expect that H 0 from v n to the output is actually 0, because if there is DC here, v n to y of t it is 0 and therefore v n to y hat of t also it is 0. We should the transfer function from v n to y hat to be 0, because y hat is basically removing everything at f c.

So, in other words, we have multiple inputs, we have two inputs here actually, x of t and v n and we would like to find the transfer functions from, we would like to find what all components of v n and x of t can potentially translate to low frequency or to DC at the output. Does it make sense? And therefore, this is going to be, I mean this is easily addressed by, when you have multiple inputs as well as multiple frequencies, it makes a lot more sense to use the principle of inter-reciprocity or the adjoint and we will do that in the next class.