Introduction to Time - Varying Electrical Networks
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Lecture 59
The frequency-reversal theorem for inter-reciprocal (adjoint) LPTV Networks: derivation

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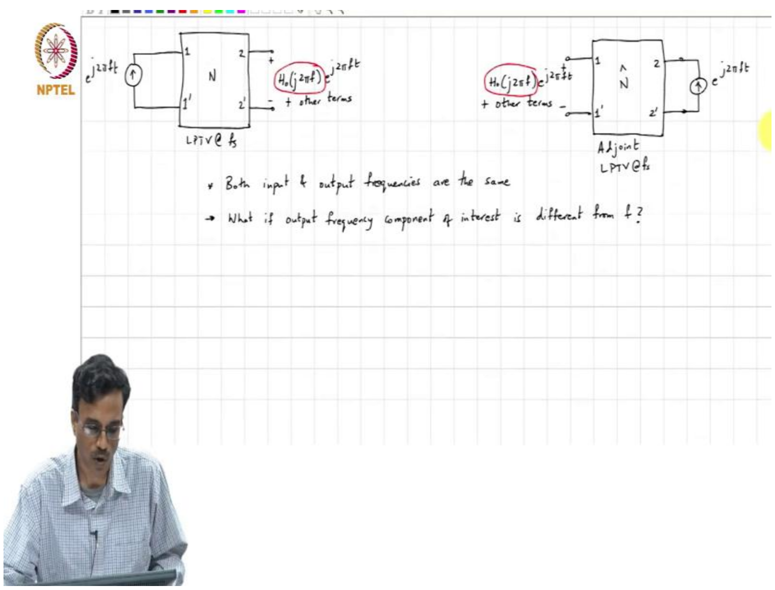

A quick recap of what we did in the last class, we derived the transfer function theorem with regard to inter reciprocal LPTV networks and the basic idea was the following or rather I should say the result is the following.

Let us say you have a network n, this is LPTV at fs and you excite port 1 of LPTV network say with the current e to the j 2 pi ft and the voltage that you obtain at the output port for instance apart from having a whole bunch of other components, it has also a component at f and if we denote the gain by H sub 0 which is the zeroth order harmonic transfer function of the network from input current to output voltage this is H naught of j 2 pi f e to the j 2 pi ft plus other terms.

And what we found yesterday was that if you created the adjoint network which is also LPTV at fs and you excited the output port with e to the j 2 pi ft the voltage developed at the frequency f across the input port will be H 0 j 2 pi f e to the j 2 pi ft plus other terms.

The key point is to observe that these two transfer functions are the same and there are well known rules for how one can obtain n hat from n and we saw how we do that, we basically if you have an element in n what you need to do is find an element which whose MNA stamp is the transpose of that element and it is just simply once that it is just simply a matter of element to element replacement in the network. Now, so the key points were that both input and output frequencies are the same so the obvious question is what if the output frequency we are interested in is different from f.

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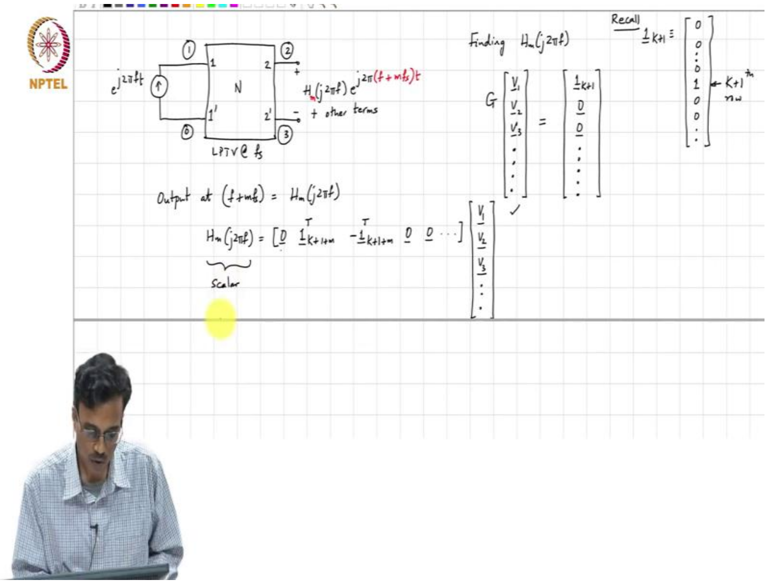

So, in other words what we are interested in therefore is this is the network n, so if we are interested not in H 0 but say H let us use another, let us use m and in which case this will become the output frequency of interest will be f plus mfs times t and the question is how can we exploit or how can we understand reciprocity in this case.

And again to find to find H sub m, how do we do this? Well, it is pretty much same old, what do we do we write the g matrix so again we use we assume that this node is 0, this is 1, this is 2 and this is 3, g times the voltage vector, so that is basically v1, v2, v3 and a whole bunch of other terms equals the current source vector that is going to be exciting the input port and that must be 1, very good, so this must be 1 capital K plus 1.

And remember that 1 capital K plus 1 is denotes is defined as a vector with all zeros except the K plus 1. And what will all the other terms, column vectors in this the side be, they will all be 0, where each of them is each 0 is basically a 2K cross 2K plus 1 cross 1 matrix. So, as usual therefore, the once you get, so if you solve the set of equations you will get, you will be able to get the voltage vector but that is not the vector that you actually interested in, you are only interested in v2 minus v3 and even in that you are not interested in all the terms, you are only interested in the what do you call the strength of the nth tone at f plus mfs.

So, the output, so at, what do you call f plus mfs which basically is equal to H sub m of j 2 pi f is simply given by what we need to do, we need to multiply this by, we are not interested in v1, we are interested in v2 and which term of v2 are we interested in? So, very good, so you will be interested in the K plus 1 plus mth term, so this is K plus 1 plus m transpose.

And remember what is this 0 here? What kind of vector is 0? The 0 there that is a, it is a row vector of size 1 row and 2K plus 1 columns and likewise this 1 sub K plus 1 plus m transpose is also a row vector. The next one will be minus 1 capital K plus 1 plus m transpose 0 0 0 etcetera. And this multiplied by v1 v2 v3 will give us our desired transfer function which is H sub m of j 2 pi f.

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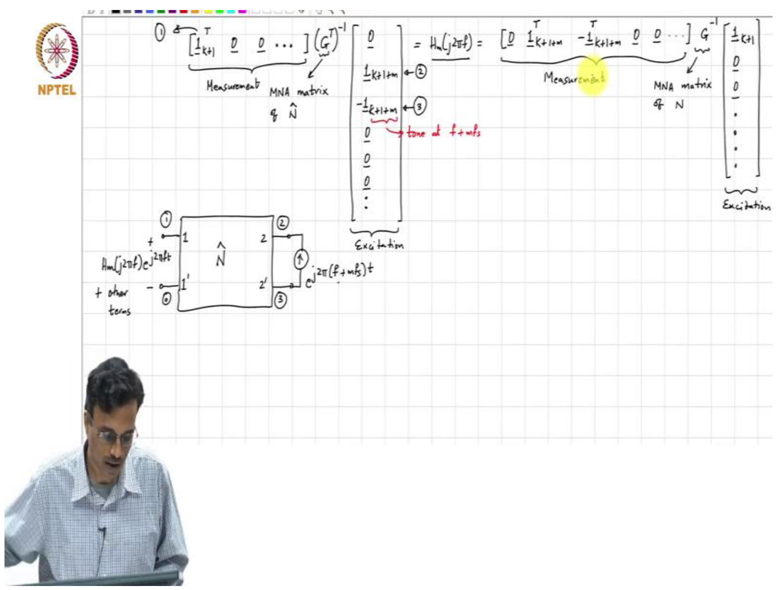

And as usual this is a scalar and therefore does not change if transposed, so I can also write this as H of j 2 pi fm so if I transpose both sides of this equation what I will get is by the way what is this guy here now, this chap is nothing but let me this is nothing but g inverse times, yeah, let me copy and paste this.

So, the voltage vector can be found by just simply computing the solution of that matrix so g inverse times the excitation vector which happens to be 1 capital K plus 1 and so on. So, if you transpose it I can reinterpret this so if I transpose both sides of the equation I can, we can see that H sub m of j 2 pi f can also be written as 1 sub K plus 1 transpose 0 0 blah, blah, blah times g transpose inverse times 0, I am not writing the transpose on the zeros because it is understood.

So, this is going to be 1 K plus 1 plus m, this is minus 1 K plus 1 plus m and 0 0 0 and so on. Does make sense? Now, how do you interpret this you can interpret this the equation on the left or the terms on the left in the following way you can think of this as the in the original network this was the excitation, this was the MNA stamp of MNA matrix rather corresponding to the network n and this is the measurement.

You can now interpret, if you interpret the order of the terms the same way this is the excitation and g transpose is the MNA matrix of a new network which from our past experience is simply the adjoint of the original network and this is the measurement matrix.

Now, how do we interpret in network form what would this mean. So, basically we have our network, the network what we call it is n hat and where do we add the excitation? So, this is node 1, this is node 0, this is node 2 and this is node 3, the excitation is between port 2 and 3, I mean between nodes 2 and 3 and what is the frequency of excitation?

So, basically the excitation is now a current source between nodes 2 and 3 and more importantly the excitation is at a frequency this is e to the j 2 pi f plus mfs times t, so this corresponds to node 2 and this corresponds to node 3 and this corresponds to tone at f plus mfs and what comment can you make about the measurement? The measurement is now at port 1 and what component are we interested in measuring?

No, no, what is this saying? This is you are measuring the voltage at node 1 and we are interested in measuring I mean and the quantity of measurement the frequency of interest is f1. So, this is basically saying that this is f, so what you will get what the equation is telling us is that if you excite port 2 of the adjoint network with an input at f plus mfs and observe the output at port 1 at the frequency, at the frequency f what is the ratio of the, what will you get, what is the magnitude, of what is the strength of the tone you will get at f?

What is this equation telling us? The input tone is at f plus mfs if you apply an input tone between nodes 2 and 3 at the frequency f plus mfs and you look at the voltage at port 1 with whose component is at frequency f that transfer function is basically H sub m of j 2 pi f which is plus other terms. And what is H m of j 2 pi f? It is also the transfer function from an input frequency at f to an output frequency at f plus mfs in the original network.

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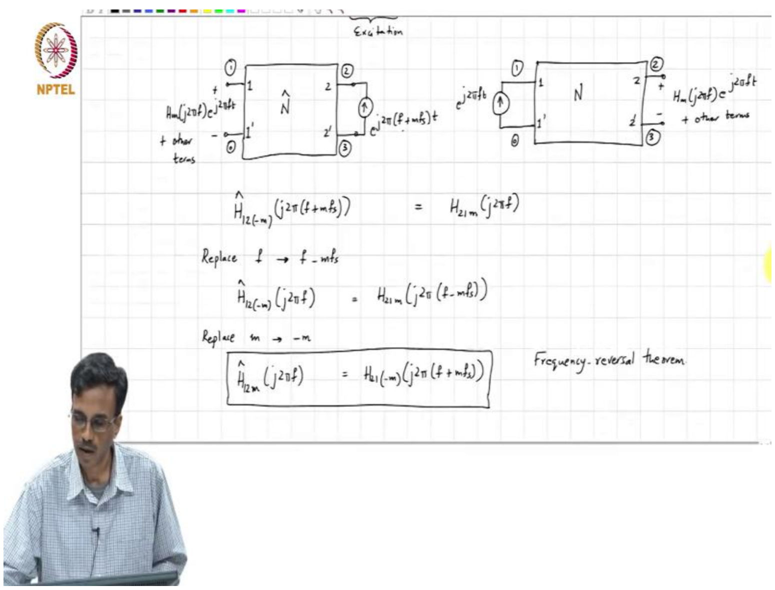

So, this is the, let me draw that, so this is the original network, this is H sub m of j 2 pi f times e to the j 2 pi ft plus other terms. So, what this is telling us is that, I mean and by the way what comment can you make about the if you apply a tone at f plus mfs and look at the output at f, what, I mean how will you, that is nothing but, it is if you look at the network n hat that is the harmonic transfer function from the output port, sorry H hat of the output port is 1, the input port is 2.

And what is the index of the harmonic transfer function? Minus m of what frequency? j 2 pi f plus mfs. So, that is because the input frequency is f plus mfs, the output frequency is f plus mfs minus mfs which is f and so this is basically H12 hat minus m of j 2 pi f plus mfs and this must be by inter reciprocity, this must be the same as H21 of m of j 2 pi f.

So, this is telling us that not only are the ports of excitation and where you measure the response they are interchanged, but in the input frequency and the output frequency are also are interchanged in the adjoint network. So, this is this is one form of the theorem. Now, if you replace f with, so now with f minus mfs in the above equation, I mean f is an arbitrary frequency, so I can replace it with anything.

So, then that this will become H hat 12 minus m of j 2 pi f must become H21 m of j 2 pi f minus mfs and then m is also arbitrary, so I can replace m with minus m. And therefore, you will get another form, I mean is basically the same it is basically interchanging n and n hat, H hat 12 m of j 2 pi f equals H21 minus m of j 2 pi f plus mfs. So, this is we will discuss why this is important in a little while, but so I will box the theorem, this is called the frequency reversal theorem, and that is what we are going to see next. So, what I am going to.