Introduction to Time - Varying Electrical Networks
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Lecture 52
Input impedance of the 4-path filter at f_s

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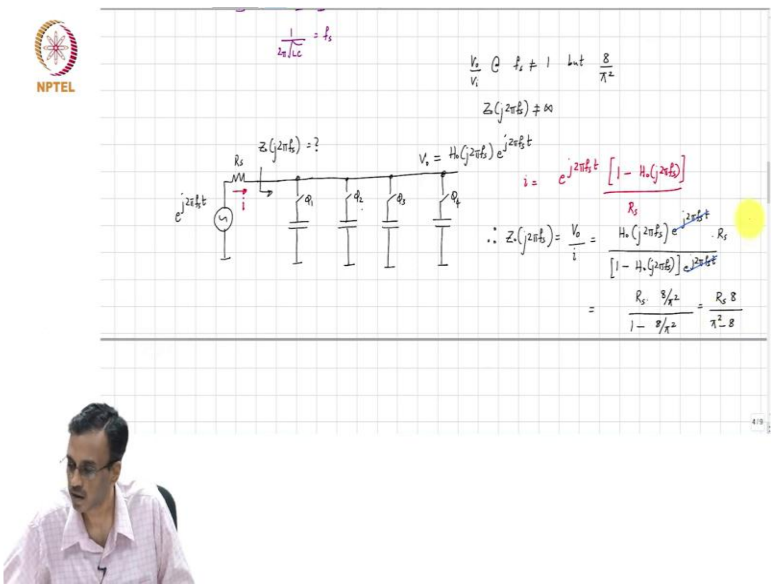

Now, another thing that I would like to draw your attention to is the following. So, this is vi, this is vo. So, when we look at an LC resonant network we always say that at resonance or at the center frequency the impedance of the offered by the LC parallel network is infinite because the inductive impedance and the capacitive impedance cancel each other. What should we expect now intuitively?

Student: Here it should be large.

Professor: It should be large but it will not be infinite. Why?

Student: Because gain is not there.

Professor: Very good. So, basically we should expect that the we know that we vo by vi at fs is not 1 but 8 over pi square. So, therefore we should expect that z naught of j 2 pi fs, yeah, is not infinity, but presumably some large value. Let us calculate what that large value is. So, if this is vi, this vo is nothing but at the frequency fs is nothing but H0 of j 2 pi fs times.

So, if vi is e to the j 2 pi fs times t then this is H0 of j 2 pi fs times e to the j 2 pi fs times t. And therefore, what comment can you make about the current flowing through the resistor? That is all. So, i is nothing but e to the j 2 pi fs times t times 1 minus H0 of j 2 pi fs that is the difference in voltage divided by RS. And therefore what is z naught j 2 pi fs?

Is nothing but the voltage v0 by, we are trying to find looking an impedance of the set of 4 capacitors which are periodically switching. The voltage we naught is known and the current i is also known. So, v naught by i and therefore v naught is nothing but H0 of j 2 pi fs times e to the j 2 pi fs times t, this divided by e 1 minus H0 of j 2 pi fs times e to the j 2 pi fs times t times RS. Which therefore it is easy to see that this goes away and this is going to be RS times H0 is 8 over pi square divided by 1 minus 8 over pi square which is RS times 8 divided pi square minus 8.

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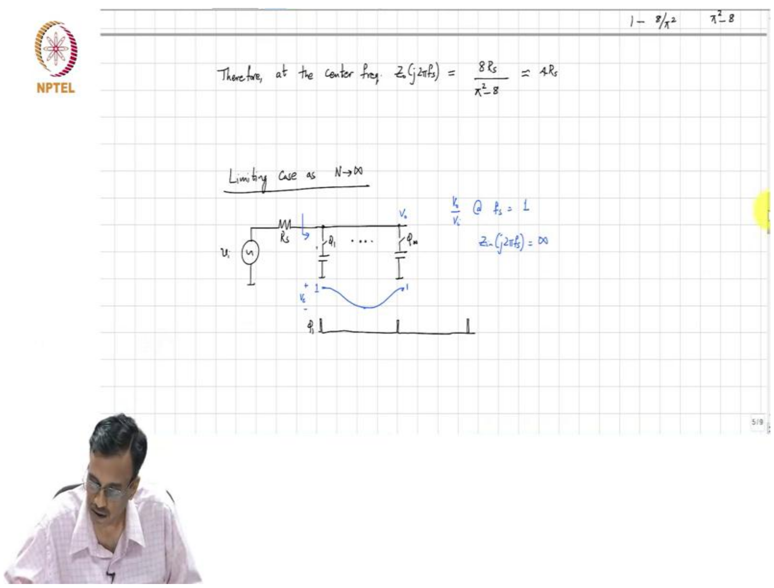

So, therefore, at the center frequency the looking in impedance of this so called switch capacitor resonator is 8 RS by pi square minus 8 which is approximately pi square is about 10, 10 minus 8 is 2 is roughly about 4 RS. If the gain was infinite, then it should be infinite. So, if you go on increasing the number of phases. So, for example, if you made an 8 path filter where each phase is now on for a Ts by 8 what comment can we make about that all these quantities and what comment can you make about the gain for instance?

In the limit as the number of phases become infinite what comment can you make with respect to the gain at fs? In the limiting case let us say you have an infinite bank of capacitors, each with an infinitesimally small width clockwork.

Student: Average will be 2 by pi.

Professor: No, average will not be 2 by pi, it will be exactly what that input voltage is at that point. So, the voltage across the different capacitors will be the same as the input sine wave. So, if you look at the output voltage, it will be a sine wave which whose amplitude is the same as that of the input. So, the gain will be 1. If the gain is 1, the, yeah. So, the limiting case, so this is vi is RS and then you have phi 1 let us say blah, blah, blah, blah.

So, and then each one of these is infinitesimally, so this is an example phi 1. And if you put a cosine here what will happen, this will sample the cosine at one point. So, the voltages across these capacitors will actually do, will do this. So, they will go from, so this is these are the VCs. So, this will go all the way from 1 to I mean back here. So, the voltage here is therefore going to be exactly, yeah, so Vo. So, as N tends to infinity, Vo by Vi at fs is going to be 1 and Zin of j 2 pi fs is going to be infinite.

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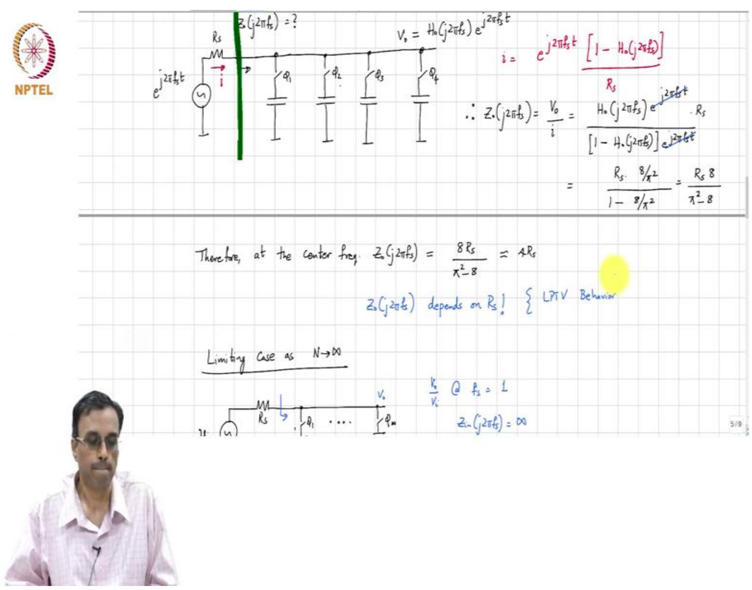

Another thing that I would like to draw your attention to with this case is the following. See one strange thing that we are seeing here or seemingly strange thing is that the looking in impedance of this circuit, yeah, the looking at impedance towards the right, it depends on RS which is something that we will never see in a time invariant circuit.

So, z naught of j 2 pi fs depends on RS which is something which is this is due to LPTV behavior. The only thing we can say is that the z naught has contributions from frequency, frequencies other than that at fs because its time varying behavior and basically those components are all responsible for the, they depend evidently on RS and therefore it is not surprising that it depends on RS.

The only valid treatment of impedance I mean you cannot, I mean this z naught which relates the fundamental voltage to the fundamental current is only one part of that Z that matrix which relates the harmonic components of the voltage developed across the network to the harmonic components of the currents being injected. So, and therefore, you can have a component at fs due to down conversion of currents at frequencies of the form fs plus multiples of fs.