Introduction to Time - Varying Electrical Networks
Professor Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology Madras
Lecture 51
Computing H_0(j2pi f_s) for a 4-path filter

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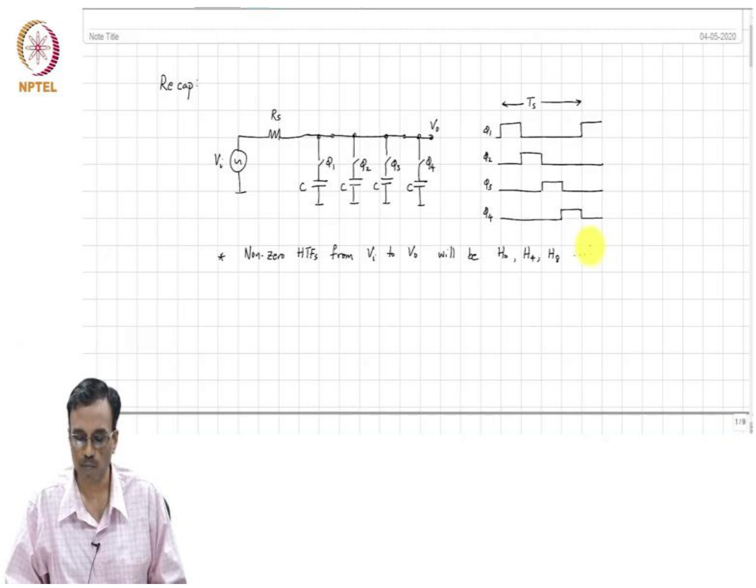

A quick summary of what we were looking at the last time around. We were looking at applications of the N-path principle and one of the applications that we were considering was what is called the N-path filter and an example of an N-path filter that has basic principle is the following.

So, you have an input source vi, you have a capacitor bank with 4 identical capacitors and this is the output view. As we were talking about this the last time around, if the input, the intuition is behind the circuit is the following, at the time, at this present time let us not worry about how this circuit came about in the first place.

So, the 4 clocks are basically look like this, so this is Ts and this is phi 1, phi 2, phi 3, and phi 4. So, clearly this is a periodically time varying circuit and if you stare at it carefully because the 4 resistors; capacitors are equal, if you look at the transfer function from vi to vo, it is apparent that, it is not only time varying at Ts but there is the network looks exactly the same every Ts by 4.

So, the only nonzero harmonic transfer functions from vi to vo will be H0, H4, H8, etcetera. And it turns out as we saw the last time around again just intuitively that this behaves like a bandpass filter and the response of the bandpass filtered has a peak at multiples of fs.

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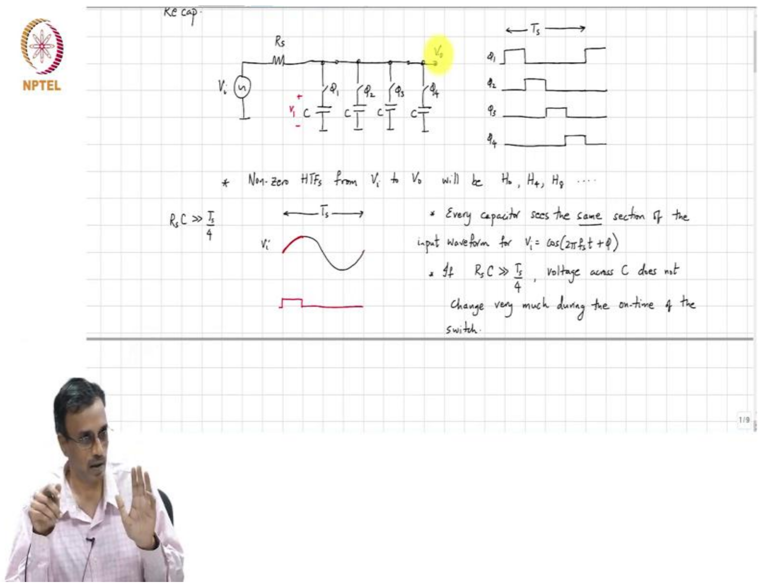

And to see that, well, if you look at a sinusoid let us say vi looks like this, so we have been assuming that one time period is 4 units. So, let us say vi is this and obviously this is also happens to be the time period at which the switches are opened and closed. So, if you look at this voltage across this capacitor let us call this v1 you can see that as far as that first capacitor is concerned, during every clock period it only sees this portion of the waveform.

And after this, so phi 1 is recall that phi 1 is high only during this period. So, it does not see the rest of the waveform and every cycle it sees the same window of that input sine wave, so basically every, there are two mechanisms that are happening to establish the voltage across the capacitor.

One, the input voltage is trying to charge the capacitor through this resistor Rs, and two, the whatever voltage is there on the capacitor is going to get discharged through Rs, so when the two become equal the capacitor voltage will remain constant in steady state. And it turns out that what is you choose this Rs times C which is the RC time constant corresponding to the charging of the capacitor to be much, much, much larger than Ts by 4.

And what does this mean? So, here I have chosen a sinusoid but for any frequency, I mean any phase shift with respect as long as the frequency is fs you will see that the same thing will happen, the, this capacitor, each capacitor will see the same section of the input waveform in all clock cycles. And if Rs times C is much, much greater than Ts by 4 then by definition the voltage across C does not change very much during the on time of the switch.

On the other hand if, so therefore you can see that the voltage across the capacitor will build up to a large value as long as the input is at fs or close to fs, if the input deviates significantly from fs that capacitor, the current going into the capacitor for in one clock cycle is say corresponds to one section of this input sinusoid, during the second clock cycle it corresponds to some other section of this input sinusoid, so on average therefore the voltage across the capacitor developed this is very small.

And this voltage vo is simply the voltage across that a particular capacitor when it is actually being charged because when the switches are off the capacitor is disconnected from vo and therefore the voltage stored on the capacitor has no bearing on the output waveform when the switch is off.

So, that is the intuition for bandpass behavior and we have also seen that the obviously the bandpass filter center frequency depends on Ts or the clock frequency fs and therefore changing the center frequency is simply changing the clock frequency. So, now you have a bandpass filter that whose center frequency is very well defined by the clock frequency that is the intuition.

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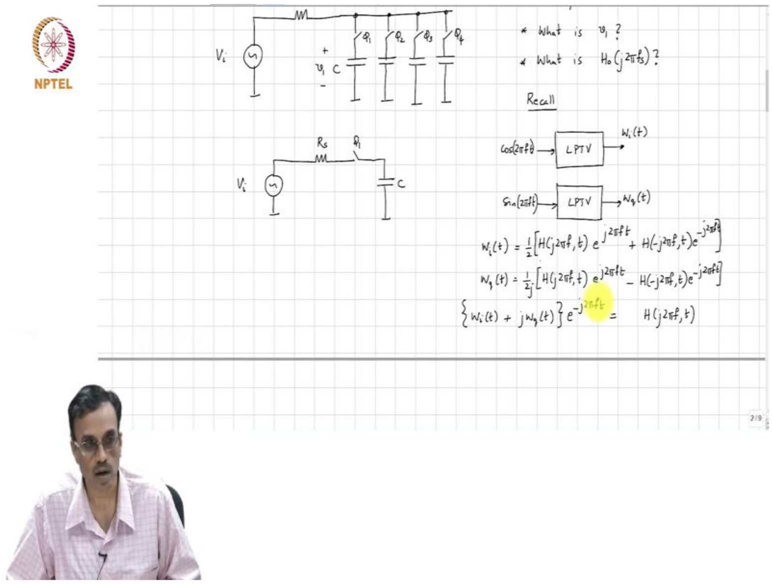

Now, let us get in this class let us get a little more quantitative than that. So, what we will do today is. And so let us take, let us first figure out what the voltage v1 across remember that Rs times C is much, much greater than Ts by 4. So, the question we are trying to answer is what is v1 that is the voltage across the capacitor C.

To do that, well, when v1, when the capacitor, the first capacitor is connected to the source vi which the equivalent circuit basically looks like this and we are interested in finding what is H0 of j 2 pi fs and this is the harmonic, zeroth order harmonic transfer function from vi to vo when the input frequency is fs.

So, vi, and remember that if you have an LPTV system and you excite it by cos 2 pi ft let us say you get Wi of t the same system if you excite it with sin 2 pi f t you get Wq of t and Wi is nothing but, of t is nothing but H of j 2 pi f comma t which is the time varying gain of the LPTV system, in this case this H of j 2 pi f comma t will be periodic with frequency fs.

So, this times e to the j 2 pi ft plus H of minus j 2 pi f comma t e to the minus j 2 pi f t and why is that? Well, cos 2 pi ft is nothing but e to the j 2 pi f t plus e to the minus j 2 pi f t by 2, this is the response to e to the j 2 pi f t, this is the response to e to the minus j 2 pi f t. So, in a similar fashion Wq of t is nothing but half H of j 2 pi f 1 by 2 j e to the j 2 pi f t minus H of minus j 2 pi f comma t e to the minus j 2 pi f t.

And how did we, if we add these two you will get Wi of t plus j Wq of t is nothing but H of j 2 pi f comma t times e to the j 2 pi f t, I am going to bring, we did this already, we are going to bring that this way and so what do you see.

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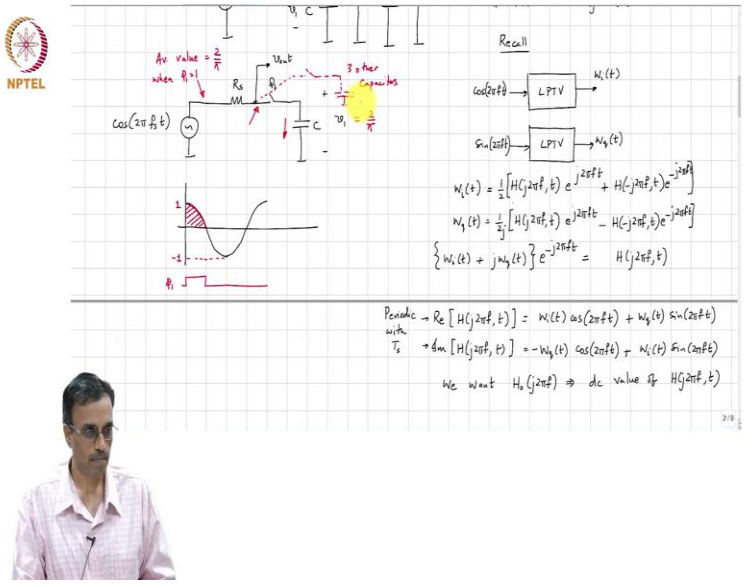

The real part of H of j 2 pi f comma t is nothing but Wi of t cos 2 pi f t plus Wq of t sin 2 pi f t and similarly the imaginary part of j 2 pi f comma t is nothing but minus Wq of t cos 2 pi f t minus or plus Wi of t sin 2 pi f t. Now, so if we want H0 of j 2 pi f which is nothing but the dc value of H of j 2 pi f comma t.

Remember because this LPTV both will be periodic with period Ts. So, we want the dc values of the real part of H of j 2 pi f comma t and the imaginary part of H of j 2 pi f comma t. And to do that what do we do? We should excite the LPTV system with a cosine and with the sin and therefore look at, and look at the two outputs Wi and Wq. And from there we can go and calculate these things.

Now, so therefore, the first job therefore is to excite this with cosine 2 pi fs times t because we are interested in finding the harmonic transfer function at an input frequency which is fs. And so this will give us, sorry this will give us that v1 and this voltage should basically give us I mean that is the eventual voltage we are interested in. Remembering that there are 4 other capacitors, 3 other capacitors connected to that node.

Let me just draw that, so first job is to find v1. So, what is the average current flowing through this capacitor? If you have any capacitor in steady state in any circuit what is the average current flowing in the capacitor is 0. So, cos 2 pi fs looks like this and this is phi 1, this is 1 and this is minus 1.

So, when the switch is closed it is only this part of the waveform that is relevant. And if the average current flowing through the capacitor is 0 that basically means that the average current flowing through this resistor during phase 1 must be 0. If the dc current flowing across the resistor is 0 then what comment can you make about the average voltage here versus the average voltage there? Both should be equal, so during, when phi 1 is high what is the average voltage at the left end of the resistor? It is simply, yeah very good, that is the average value of that quarter cycle.

That is basically, so, the average value is 2 by pi when phi 1 is 1. And therefore, what comment can we make about the average value on the right side of the resistor? It must be 2 by pi so basically v1 therefore must be equal to 2 by pi. So, the voltage across the capacitor is roughly constant. Why do we say it is roughly constant?

Because the we have assumed that the Rs times C is much, much larger than the on period of the switch, so the voltage across the capacitor in steady state does not change very much during that small duration. And therefore, it must follow that since the average current through the capacitor must be 0, it must follow that v1 on average must be 2 by pi.

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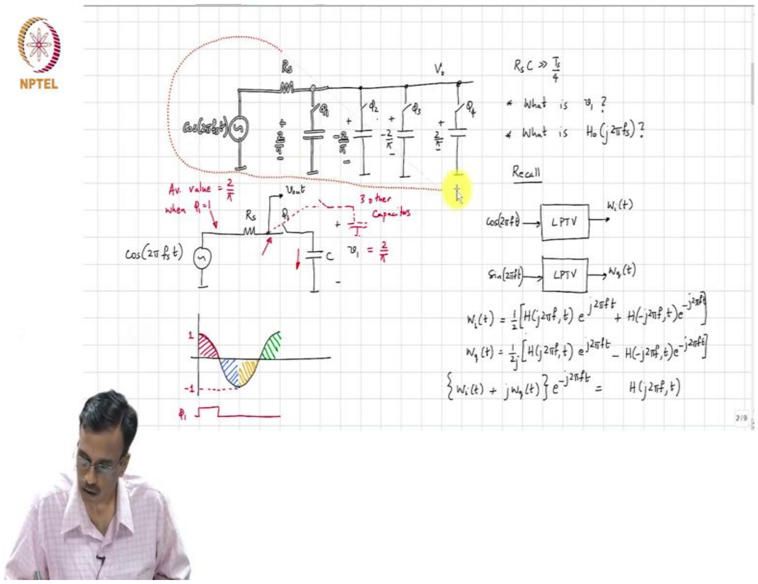

Now, by the same token what comment can we make about, what comment can we make about phi, the voltage v2? It will be as you can see this voltage so when this, when you excite this by cos omega t cos 2 pi fs times t the voltage here is going to be 2 by pi, the voltage here is going to be I am going to remove the reference to, this voltage is also, this voltage is going to remain, is going to be minus 2 by pi because you are going to be averaging that quarter cycle.

The third capacitor is going to be have an average value which is also minus 2 by pi and that is because we are going to be averaging that quarter cycle. And the 4th capacitor is going to have an average voltage which is 2 by pi because that is going to be averaging that quarter cycle. So, this, so I am going to copy and paste this.

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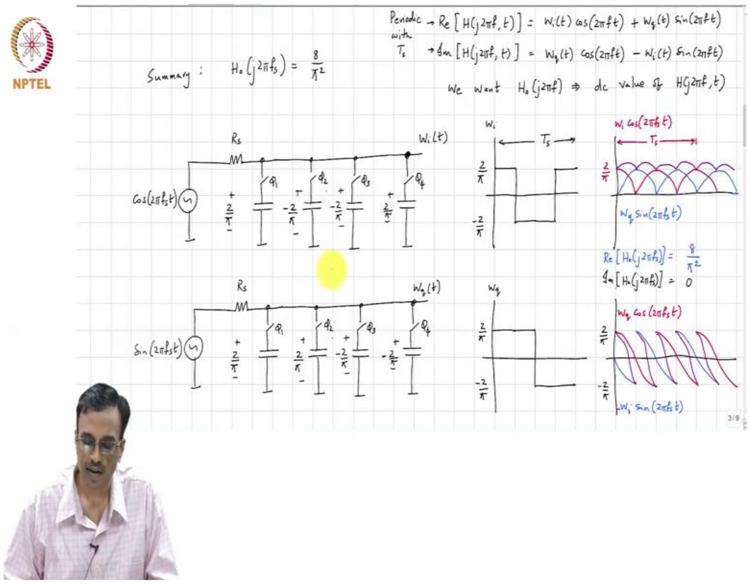

So, this is Wi of t and what do we have for W? When we, when the input is a sine wave as is needed to get Wq of t. Well, during phi 1 it is 2 by pi during phi 2 we are going to be averaging that quarter cycle, so that is going to be 2 by pi and then we are going to have minus 2 by pi and again minus 2 by pi.

So, what is Wi of t? So, what does this waveform look like? When phi 1 is on, so Wi of t basically looks like this, when phi 1 is on the voltage is going to be 2 by pi, for the next quarter cycle it will be minus 2 by pi, for the third quarter cycle it is going to remain minus 2 by pi and for the 4th quarter cycle it is going to be 2 by pi. And this is Ts that make sense.

So, this is Wi, what comment can we make about Wq? During the first quarter cycle it is going to be 2 by pi then second quarter cycle also is going to be 2 by pi, third quarter cycle is going to be minus 2 by pi and so this again, this is Ts and this is Wq. What we want however is not simply Wi or Wq but Wi times cos 2 pi fs times t.

So, let me, so if I plot Wi cos I am going to do that in red Wi cos 2 pi fs times t, what do you get? Well, during this period, you are going to have something like that, it looks like that. During the second and the third quarter periods what will it be, the cos would have gone negative but then this has become negative, so this becomes something like this, so this is 2 by pi and then again the cos would become positive and this keeps continuing. And remember that 4 units on this scale is Ts.

Now, what about Wq sin 2 pi f t? Wq times sin 2 pi f t will basically do this, so will be the sine wave in the first half cycle, in the second half cycle it will be the inverted version of the sine wave and this will keep continuing. So, this is Wq sin 2 pi fs times. So, these two, I mean they are interested in the average value of Wi cos plus Wq sin, rather than add the waveforms and average them you might as well average the individual waveforms and add them.

So, if you add so the average, so the, so H real part of H0 of j 2 pi fs is nothing but, what comment can we make? If the amplitude of the sine wave was 1, its average value over a quarter cycle will be 2 by pi, but now the average is, the amplitude is 2 by pi, so what will be the average? If the amplitude was 1 the average over quarter cycle is 2 by pi but if the amplitude itself is 2 by pi what will be the average or quarter cycle?

Student: (()) (30:25)

Professor: It is 2 by pi the whole square. So, basically as far as the red waveform is concerned its average value is 4 by pi square as far as the blue wave form is concerned its average is also the same thing, so the real part of H0 of j 2 pi fs is nothing but 8 over pi square. Now, let us go and find out what the imaginary part of, imaginary part of looks like this and we need Wi sin 2 pi fst, how does that look like? Wi looks like that so in the first half cycle we need yeah it does this and then gets multiplied and so on.

So, what comment can we make about the average value of Wq cos minus Wi sin? It will be 0, so the imaginary part of H0 of j 2 pi f s therefore must be 0. So, therefore, the summary is that H0 of j 2 pi fs is 8 by pi square, couple of things that I would like to draw your attention to. To find the real part of H of j 2 pi f s we have to actually add the red waveform and the blue waveform. And if you add the red in the blue waveform how will that resultant waveform look like?

Student: (()) (33:36)

Professor: No, if those two are straight lines, they will be a constant, they are not straight lines, so they will basically, if you add the two you will get something like, yeah, you will not get twice because they will, you will get the maximum around that point, so you will have get some wave form. And similarly, if you add Wq cos and minus Wi sign what will you get? Yeah, at some point they will cancel, you will get a waveform like that. The reason I am drawing those waveforms is that what comment can you make about the periodicity of the real part of H0?

Student: Ts by 4.

Professor: Ts by 4, so it is periodic with Ts by 4 and that was something that we expected, because this is a 4 path filter, so even though the switches are opening and closing periodically with period Ts, the gain as far as vo is concerned is it behaves like as if the system is varying at 4 times the sampling rate. So, this is, so that is one thing.

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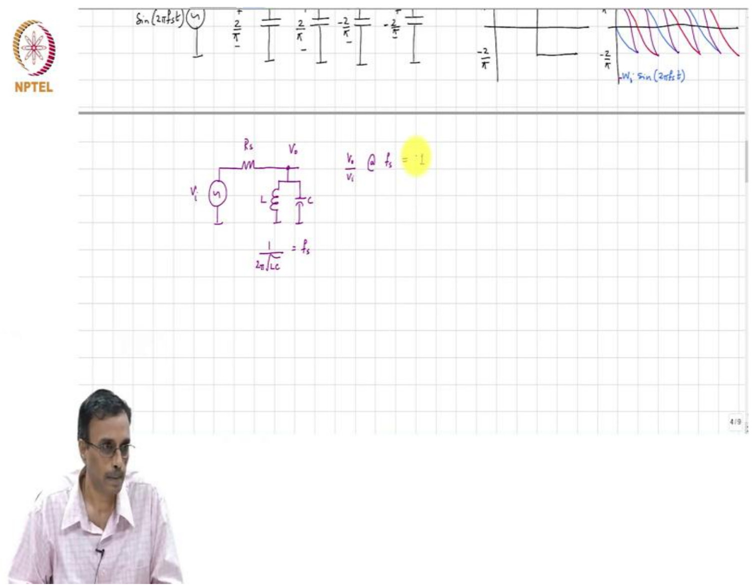

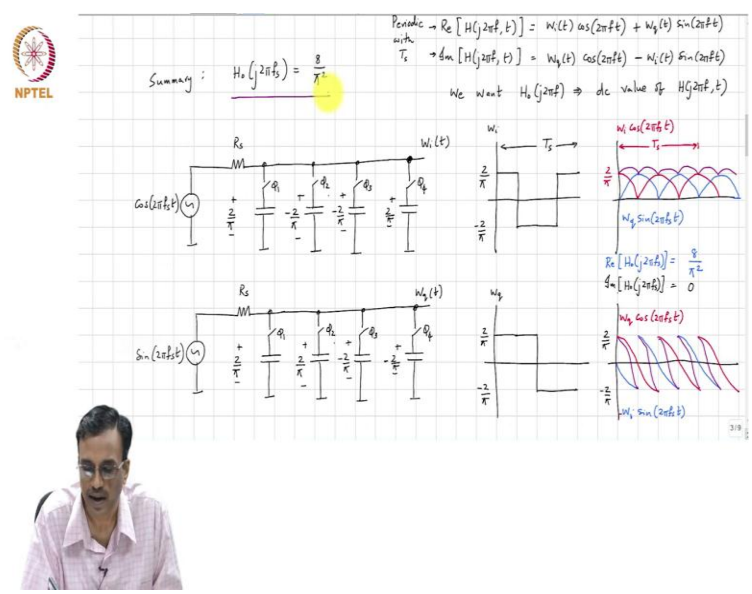

Another thing that I like you to draw your attention to is the following. Remember that if you had an LC network, what comment can you make, I mean remember that a second order bandpass filter, it will look like this and what comment can you make about vo by vi at, so let us say 1 by 2 pi square root LC is chosen to be fs. So, what is vo by vi at fs? At the resonant frequency of the LC network, the LC parallel network will be an open circuit and therefore what will be the transfer function from vo to vi? It is simply v1, I mean the important point is that it is real.

Similarly, we see that here also it is it is real, it is not quite 1, pi square is about, I would say about close to 10, so you can see that there is some loss involved and that basically is a deviation from a passive LC network, one of the many deviations from a passive LC network. The interesting thing is that the at 2 pi fs, I mean when the input frequency is fs, the harmonic transfer function is basically 8 over pi square.